OPTGEN – METHODOLOGY MANUAL

Version 6.0

May 2010

Table of Contents

1 OBJECTIVE 4

2 PROBLEM FORMULATION 5

2.1 Objective 5
2.2 Modeling overview and variable description 5
2.3 Dictionary of variables 6
2.3.1 Sets 6
2.3.2 Indexes 7
2.3.3 Constants 7
2.3.4 Variables 8
2.3.5 Notation 8
2.4 Formulation 8

3 DECOMPOSITION METHODOLOGY 12

3.1 Deterministic problem formulation 12
3.2 Characteristics of function $w(x)$ 13
3.3 The approximate investment problem 14
3.4 Decomposition algorithm 14
3.5 Geometric interpretation of the algorithm 15
3.6 Alternative expression for the approximate investment problem 16

4 APPLICATION TO THE EXPANSION PLANNING PROBLEM 18

4.1 Application of the decomposition methodology 18
4.1.1 Operation problem 18
4.1.2 Evaluation of Benders Cut 19
4.1.3 Approximate expansion problem 21

5 EXPECTED VALUE CRITERION 23

6 ADDITIONAL MODELING ASPECTS 25

6.1 Discount rate and calculation of investment decision costs 25
6.1.1 Evaluation of investment decision costs 25
6.1.2 Understanding the file outdisbu.csv 28
6.2 Associated project constraints 29
6.3 Precedence constraints 30
6.4 Mutually exclusive projects 30
6.5 Project reinforcements 31
6.5.1 Evaluation of the Benders cut for reinforcement projects 31
6.6 Plants with multiple generators 32
6.7 Evaluation of reference costs 33

6.8 Evaluation of the unit operating cost of thermal plants 34
6.9 Emission constraints 34
6.10 Fuel availability constraints...35
7 BIBLIOGRAPHY 36

1 OBJECTIVE

OPTGEN© is a computational tool for planning the expansion of generation and interconnections of energy systems.

Chapter 2 formulates in detail the expansion planning problem. Chapter 3 presents the solution methodology, based on decomposition techniques. For simplicity of presentation, and without loss of generality, we use a deterministic formulation. In chapter 4, we describe the application of the methodology to a deterministic expansion problem. Finally, chapter 5 extends the methodology to the stochastic case.

2 PROBLEM FORMULATION

2.1 Objective

The objective of Optgen is to determine a least-cost investment schedule − sum of investment plus expected value of operation costs − for the construction of new generation capacity (hydroelectric and/or thermoelectric) and of regional interconnections.

The expected value of operation costs is calculated for a set of inflow scenarios. An optimal operation policy is calculated for each scenario (deterministic optimization) and the expected value is calculated as the weighted average of the optimal operating costs (the weights are the probabilities of each scenario).

The optimal operating policy calculation for each inflow scenario takes into account operating constraints such as load supply; water balance in the reservoirs; maximum and minimum generation; reservoir storage limits; and limits on power exchange among regions.

The optimal investment schedule calculation represents investment constraints such as minimum and maximum dates for the investment decision; sets of associated projects; and sets of mutually exclusive projects.

2.2 Modeling overview and variable description

The system is composed of a set H of hydro plants and a set I of thermal plants located in N regions, or subsystems. Each subsystem has an energy load; there are interconnections among subsystems which allow energy exchanges. A subset of those hydro and thermal plants plus interconnections form the existing system; the complement is the set of candidate projects J. For each stage t and for each project j we have an associated investment decision $x(t,j)$. This decision $x(t,j)$ is a binary variable, equal to 1 if the decision is to build project j in stage t; otherwise, it is equal to zero.

The moof represents a set of constraints associated to those investment decisions:

- Minimum and maximum decision dates;
- Mandatory project constraints;
- Optional project constraints;
- Mutually exclusive projects;
- Associated projects;
- Minimum/Maximum installed capacity in each year;
- Firm energy per system;
- Firm capacity per system.

For notational simplicity, and without loss of generality, we make the following assumptions:

- All hydro plants have reservoirs (run of river plants have a reservoir with zero storage capacity);

- Load is constant in each time stage (the actual moof allows up to five load levels in each stage);
- Only one inflow scenario is used (the actual moof allows the representation of several inflow scenarios, with user-defined probabilities of occurrence).

The operating decision variables refer to the dispatch of generators, and are basically: $g(t,i)$ generation of thermal plant i in stage t; $v(t,i)$ stored volume at the end of stage t in reservoir i; $q(t,i)$ turbined outflow volume of hydro plant i in stage t; $s(t,i)$ spilled outflow volume of hydro plant i in stage t; and, finally, $f(t,k)$ energy flow through interconnection k in stage t. Because each regional interconnection k may have different power flow limits and/or loss factors in each direction, the power flow is assumed to go from an origin region $n_o(k)$ to a destination region $n_{\mathrm{d}}(k)$.

These operating decisions are subject to the following set of constraints:

- Water balance in each hydro plant;
- Power balance in each region;
- Emission constraints: NO_x, CO_2 and SO_2;
- Fuel consumption constraints;
- Operating limits.

2.3 Dictionary of variables

2.3.1 Sets

T	set of stages in the study period
N	set of subsystems or regions
J_1	set of mandatory projects
J_2	set of optional projects
J	set of projects, $J = J_1 \cup J_2$
$J_3(l)$	set of projects in the l-th constraint of mutually exclusive projects
$J_4(l)$	set of projects in the l-th constraint of associated projects
$J_5(l)$	set of projects in the l-th constraint of minimum/maximum installed capacity
$J_6(l)$	set of projects in the l-th constraint of precedence in construction. The precedence between projects is given by the order in which they appear in the constraint
I	set of thermal plants
I_n	set of thermal plants in subsystem n
H	set of hydro plants
H_n	set of hydro plants in subsystem n
G_n	set of existent hydro and thermal plants in subsystem n
J_n	set of hydro and thermal projects in subsystem n
K	set of transmission lines among subsystems
$E_1(l)$	set of thermal plants in the l-th NO_x emission constraint
$E_2(l)$	set of thermal plants in the l-th SO_2 emission constraint
$E_3(l)$	set of thermal plants in the l-th CO_2 emission constraint
$F(l)$	set thermal plants in the l-th fuel availability constraint
$MT(i)$	set of hydro plants immediately upstream (for turbining) of plant i

MV(i)	set of hydro plants immediately upstream (for spillage) of plant i

2.3.2 Indexes

t	indexes stages, set T
j	indexes candidate projects, set J
i	indexes plants, sets I or H
k	indexes interconnections, set K
n	indexes subsystems or regions, set N
f	indexes fuels, set F
l	indexes constraints in general

2.3.3 Constants

N_3	number of constraints with mutually exclusive projects
N_4	number of constraints with associated projects
N_5	number of constraints of minimum/maximum installed capacity
N_6	number of precedence constraints
NE_1	number of NO_x emission constraints
NE_2	number of SO_2 emission constraints
NE_3	number of CO_2 emission constraints
NF	number of fuel consumption limit constraints
$c(t,j)$	present value of the investment cost of project j in stage t
$\underline{t}(j)$	earliest date for deciding to invest in project j
$\overline{t}(j)$	latest date for deciding to invest in project j
$\underline{\tau}(l)$	initial stage for the l-th minimum installed capacity constraint
$\overline{\tau}(l)$	final stage for the l-th minimum installed capacity constraint
$w(j)$	installed capacity of project j
$\underline{w}(l)$	value of the l-th minimum installed capacity constraint
$\overline{w}(l)$	value of the l-th maximum installed capacity constraint
$d(t,i)$	present value of the operating cost of plant i in stage t
$a(t,i)$	lateral inflow to reservoir i in stage t
$\rho(i)$	average production factor of hydro plant i
$n_o(k)$	"from" node of interconnection k
$n_d(k)$	"to" node of interconnection k
$p(k)$	loss factor interconnection k
$\overline{f}(k)$	power flow limit of interconnection k in stage t
$\overline{g}(i)$	generation capacity of thermal plant i in stage t
$\overline{v}(i)$	maximum storage capacity of reservoir i in stage t
$\overline{q}(i)$	maximum turbining capacity of reservoir i in stage t
$v_0(i)$	volume stored in reservoir i in the beginning of stage t
$D(t,n)$	demand of subsystem n in stage t
HR(i)	heat rate of thermal plant i
$e(i,1)$	NO_x emission factor of thermal plant i
$e(i,2)$	SO_2 emission factor of thermal plant i
$e(i,3)$	CO_2 emission factor of thermal plant i

fuel(*i*)	indexes the fuel of thermal plant *i*
ECnt(*f*)	energy content of fuel *f*
$E_1(t,l)$	emission limit in the *l*-th NO_x emission constraint in stage *t*
$E_2(t,l)$	emission limit in the *l*-th SO_2 emission constraint in stage *t*
$E_3(t,l)$	emission limit in the *l*-th CO_2 emission constraint in stage *t*
F(*t*,*l*)	fuel availability limit in the *l*-th fuel limit constraint in stage *t*
EF(*t*,*i*)	firm energy of generator *i* in stage *t*
FEF(*t*,*n*)	reserve factor of firm energy demand of subsystem *n* in stage *t*
PF(*t*,*i*)	firm capacity of generator *i* in stage *t*
FPF(*t*,*n*)	reserve factor of firm capacity demand of subsystem *n* in stage *t*

2.3.4 Variables

$x(t,j)$	binary variable associated to the decision of building project *j* in stage *t*
$g(t,i)$	generation of plant *i* in stage *t*
$f(t,k)$	energy flow across line *k* in stage *t*
$v(t,i)$	stored volume of reservoir *i* at the end of stage *t*
$q(t,i)$	turbined volume of hydro plant *i* in stage *t*
$s(t,i)$	spilled volume of reservoir *i* in stage *t*

2.3.5 Notation

$y(j)$	sum of decision variables associated to project *j* along the allowed decision interval $[\underline{t}(j),\overline{t}(j)]$	$y(j) = \sum_{t=\underline{t}(j)}^{\overline{t}(j)} x(t,j)$
$y(t,j)$	sum of decision variables associated to project *j* until stage *t*	$y(t,j) = \sum_{\tau=\underline{t}(j)}^{t} x(\tau,j),\ t \in T$

2.4 Formulation

The least-cost expansion planning of generation and transmission is formulated as a mathematical programming problem with the following objective function:

$$\text{Min} \quad \sum_{t\in T}\sum_{j\in J} c(t,j) \times x(t,j) + \sum_{t\in T}\sum_{i\in I} d(t,i) \times g(t,i), \tag{2.1}$$

subject to the following constraints:

Earliest and latest decision dates:

$$x(t,j) = 0, \qquad \forall\ t \notin [\underline{t}(j),\overline{t}(j)]$$

Mandatory projects constraints:

$$\sum_{t=\underline{t}(j)}^{\overline{t}(j)} x(t,j) = 1, \qquad \forall\, j \in \mathrm{J}_1$$

Optional projects constraints:

$$\sum_{t=\underline{t}(j)}^{\overline{t}(j)} x(t,j) \le 1, \qquad \forall\, j \in \mathrm{J}_2$$

Mutually exclusive projects constraints:

$$\sum_{j\in \mathrm{J}_3(l)} y(j) \le 1, \qquad l = 1, ..., \mathrm{N}_3$$

Associated projects constraints:

$$y(j_1) = y(j_2), \qquad \forall\, j_1, j_2 \in \mathrm{J}_4(l),\ l = 1, ..., \mathrm{N}_4$$

Minimum installed capacity constraints:

$$\sum_{t=\underline{\tau}(l)}^{\overline{\tau}(l)} \sum_{j\in \mathrm{J}_5(l)} w(j) \times x(t,j) \ge \underline{w}(k), \qquad l = 1, ..., \mathrm{N}_5$$

Maximum installed capacity constraints:

$$\sum_{t=\underline{\tau}(l)}^{\overline{\tau}(l)} \sum_{j\in \mathrm{J}_5(l)} w(j) \times x(t,j) \le \overline{w}(k), \qquad l = 1, ..., \mathrm{N}_5$$

Project construction precedence constraints:

$$y(t,j_{i+1}) - y(t,j_i) \ge 0, \qquad \forall\, j_i \in \mathrm{J}_6(l),\ \forall\, t \in \mathrm{T},\ l = 1, ..., \mathrm{N}_6$$

Firm energy per system:

$$\sum_{i\in \mathrm{G}_\mathrm{n}} \mathrm{EF}(t,i) + \sum_{j\in \mathrm{J}_\mathrm{n}} \mathrm{EF}(t,j) \times y(t,j) \ge \mathrm{FEF}(t,n) \times \mathrm{D}(t,n), \qquad \forall\, t \in \mathrm{T},\ \forall\, n \in \mathrm{N}$$

Firm capacity per system:

$$\sum_{i \in \mathrm{G_n}} \mathrm{PF}(t,i) + \sum_{j \in \mathrm{J_n}} \mathrm{PF}(t,j) \times y(t,j) \geq \mathrm{FPF}(t,n) \times \mathrm{D}(t,n), \qquad \forall\, t \in \mathrm{T}, \forall\, n \in \mathrm{N}$$

Water balance equations:

$$v(t,i) - v(t-1,i) + q(t,i) + s(t,i) - \sum_{j \in \mathrm{MT}(i)} q(t,j) - \sum_{j \in \mathrm{MV}(i)} s(t,j) = a(t,i),$$

$$\forall\, t \in \mathrm{T}, \forall\, i \in \mathrm{H}$$

Load supply constraints:

$$\sum_{i \in \mathrm{I_n}} g(t,i) + \sum_{i \in \mathrm{H_n}} \rho(i) \times q(t,i) - \sum_{k \in \mathrm{K} | n_o(k)=n} f(t,k) + \sum_{k \in \mathrm{K} | n_d(k)=n} (1 - p(k)) \times f(t,k) = D(t,n),$$

$$\forall\, t \in \mathrm{T}, \forall\, n \in \mathrm{N}$$

Emission constraints:

$$\sum_{i \in \mathrm{E_1}(l)} e(i,1) \times g(t,i) \leq \mathrm{E_1}(t,l), \qquad \forall\, t \in \mathrm{T}, l = 1, \ldots, \mathrm{NE_1}$$

$$\sum_{i \in \mathrm{E_2}(l)} e(i,2) \times g(t,i) \leq \mathrm{E_2}(t,l), \qquad \forall\, t \in \mathrm{T}, l = 1, \ldots, \mathrm{NE_2}$$

$$\sum_{i \in \mathrm{E_3}(l)} e(i,3) \times g(t,i) \leq \mathrm{E_3}(t,l), \qquad \forall\, t \in \mathrm{T}, l = 1, \ldots, \mathrm{NE_3}$$

Fuel availability constraints:

$$\sum_{i \in \mathrm{F}(l)} \frac{\mathrm{HR}(i) \times \mathrm{g}(t,i)}{\mathrm{ECnt}(\mathrm{fuel}(i))} \leq \mathrm{F}(t,l), \qquad \forall\, t \in \mathrm{T}, l = 1, \ldots, \mathrm{NF}$$

Operating limits:

$$g(t,i) \leq \overline{g}(i), \qquad \forall\, i \in \mathrm{I} \mid i \notin \mathrm{J}$$

$$g(t,i) - \overline{g}(i) \times y(t,i) \leq 0, \qquad \forall\, i \in \mathrm{I} \mid i \in \mathrm{J}$$

$$v(t,i) \leq \overline{v}(i), \qquad \forall\, i \in \mathrm{H} \mid i \notin \mathrm{J}$$

$$v(t,i) - \overline{v}(i) \times y(t,i) \leq 0, \qquad \forall\, i \in \mathrm{H} \mid i \in \mathrm{J}$$

$$q(t,i) \leq \overline{q}(i), \qquad \forall\, i \in \mathrm{H} \mid i \notin \mathrm{J}$$

$$q(t,i) - \overline{q}(i) \times y(t,i) \leq 0, \qquad \forall\, i \in \mathrm{H} \mid i \in \mathrm{J}$$

$$f(t,i) \leq \overline{f}(i), \qquad \forall\, i \in \mathrm{K} \mid i \notin \mathrm{J}$$

$$f(t,i) - \overline{f}(i) \times y(t,i) \leq 0, \qquad \forall\, i \in \mathrm{K} \mid i \in \mathrm{J}$$

Initial storage constraints:

$$v(0,i) = v_0(i), \qquad \forall\, i \in \mathrm{H}$$

Decision variables integrality constraints:

$$x(t,j) \in \{0,1\}, \qquad \forall\, t \in \mathrm{T}, \forall\, j \in \mathrm{J}$$

We see that problem (2.1) is a large-scale mixed integer programming model. The number of integer variables depends on the number of candidate projects and on the study horizon. The number of continuous variables and constraints may be very large, depending on the number of subsystems or regions, plus number of hydro plants, thermal plants and interconnections. The number of variables and constraints increase further when load blocks are represented.

We also note that the problem has a block structure:

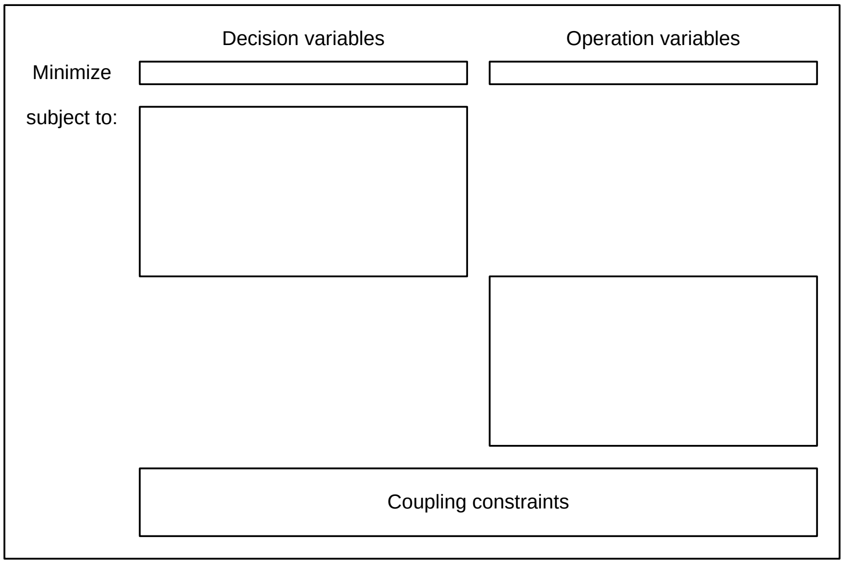


Figure 2.1 – Block structure of the expansion planning problem

This structure is suitable for the application of decomposition techniques. In OptGen, we apply the Benders decomposition methodology, described next.

3 DECOMPOSITION METHODOLOGY

3.1 Deterministic problem formulation

The expansion planning problem (2.1) can be rewritten as:

$$\begin{aligned} &\text{Min} \quad z(x) = c(x) + w(x) \\ &\text{s a} \quad x \in X \end{aligned} \tag{3.1}$$

The matrix of binary variables $x = (x(t,j))$ represents the investment decisions. As defined in the previous chapter, $x(t,j) = 1$ indicates that project j will be built in stage t.

The set X represents the feasible investment decisions, that is, those meeting the constraints of minimum installed capacity, associated and mutually exclusive projects, and so on.

Finally, the functions $c(x)$ and $w(x)$ represent respectively the investment and operation costs of the candidate expansion plan x, as shown in Figure 3.1.

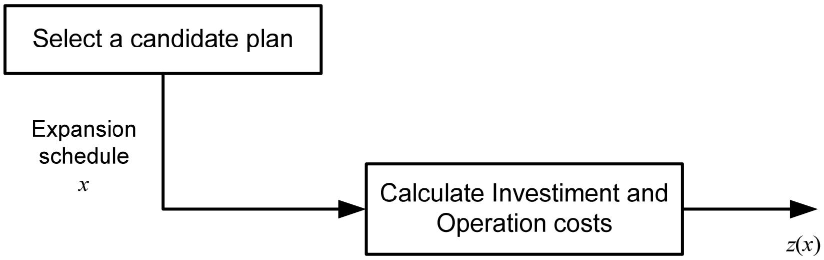


Figure 3.1 - Planning process - One scenario

Figure 3.2 illustrates the optimization process.

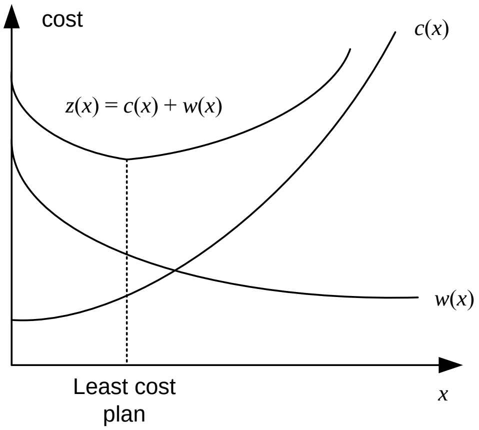


Figure 3.2 - Optimization process

Note that the investment cost function $c(x)$ is known, whereas the operation cost function $w(x)$ is represented implicitly through the solution of the following operating problem:

$$\begin{aligned} w(x) = \text{Min} \quad & dy \\ \text{s a} \quad & F\,y \geq h - E\,x \\ & y \geq 0 \end{aligned} \tag{3.2}$$

where vector y represents the operating variables (stored volumes, turbined and spilled outflows, thermal plant generation, load curtailment etc.) The constraints $Fy \geq h - Ex$ represent the operative equations (water balance, load supply, generation limits and maximum energy interchange capacity).

The solution methodology adopted in OptGen, known as Benders decomposition, builds the function $w(x)$ from the repeated solution of an approximation to the expansion problem (3.1) and of the operation problem (3.2).

3.2 Characteristics of function *w*(*x*)

Problem (3.2) is a linear programming (LP) optimization model. From LP theory, its Dual is formulated as:

$$\begin{aligned} w(x) = \text{Max} \quad & \pi\,(h - E\,x) \\ \text{s a} \quad & \pi\,F \leq d \\ & \pi \geq 0 \end{aligned} \tag{3.3}$$

From LP theory, we know that the optimal solution values of the dual problem (3.3) and the operation problem (3.2) (known as the primal) coincide. In addition, the dual variables π are the vector of simplex multipliers associated to the constraints of the primal problem (3.2) in the optimal solution.

Let $\Pi = \{\pi^i, i = 1, ..., r\}$ be the set of basic feasible solutions of the dual problem (3.3). Note that this set does not depend on the investment decision x. Therefore, the optimal solution could in principle be obtained by enumeration:

$$w(x) = \text{Max}\ \{\pi^i (h - Ex),\ \pi^i \in \Pi\} \tag{3.4}$$

Problem (3.4) can be rewritten in the following equivalent manner:

$$\begin{aligned} w(x) = \text{Min} \quad & \alpha \\ \text{s a} \quad & \alpha \geq \pi^1\,(h - E\,x) \\ & \alpha \geq \pi^2\,(h - E\,x) \\ & \dots \\ & \alpha \geq \pi^r\,(h - E\,x) \end{aligned} \tag{3.5}$$

where α is an unconstrained scalar variable (positive or negative values). Because α should be greater than or equal to each of the constraints $\alpha \geq \pi^i\,(h - Ex)$, if follows that it should be greater than or equal to the maximum of these constraints, $\mathrm{Max}\{\pi^i\,(h - Ex)\}$. Finally, because the objective is to minimize α, we conclude that it will be equal to $\mathrm{Max}\{\pi^i\,(h - Ex)\}$, which is expression (3.4).

The advantage of formulation (3.5) is that it clearly shows that $w(x)$ is a piecewise linear function, as shown in Figure 3.3:

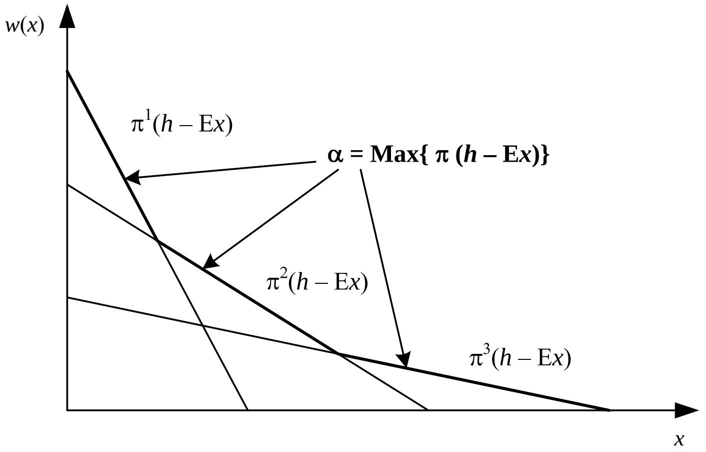


Figure 3.3 - Graph of function *w*(*x*)

3.3 The approximate investment problem

Replacing expression (3.5) into the expansion problem (3.1), we obtain:

$$\begin{aligned} \text{Min} \quad & c(x) + \alpha && && (3.6) \\ \text{s a} \quad & \alpha \geq \pi^i\,(h - E\,x), && i = 1, ..., r \\ & x \in X \end{aligned}$$

The number of constraints $\alpha \geq \pi^i\,(h - Ex)$ in problem (3.6) may be very high. However, only a few of those constraints will be binding (i.e. met in equality) at the optimal solution; this means that the remainder can be relaxed with no loss of optimality.

The Benders decomposition algorithm, presented next, is based on the relaxation of problem (3.6) and in the iterative generation of the “right” (i.e. binding) constraints $\alpha \geq \pi^i\,(h - Ex)$ from the solution of the operation problem (3.2).

3.4 Decomposition algorithm

1. Initialize: number of iteration $\nu = 0$; upper bound $\overline{z} = +\infty$; tolerance for convergence ξ (input data)
2. Increment the number of iterations $\nu = \nu + 1$ and solve the approximate investment problem:

$$\begin{aligned} z = \quad & \text{Min} \quad c\,x + \alpha \\ & \text{s a} \quad \alpha \geq \pi^{\mu}\,(h - E\,x),\ \mu = 1, ..., \nu{-}1 \\ & \qquad\ x \in X \end{aligned} \tag{3.7}$$

3. Let $\{x^{\nu},\alpha^{\nu}\}$ be the optimal solution of (3.7). Because this problem is a relaxation of the original problem (3.6), its optimal value is a lower bound to the optimal solution value of the original problem. Set the lower bound $\underline{z}$ to:

$$\underline{z} = c\,x^{\nu} + \alpha^{\nu} \tag{3.8}$$

4. Solve the operation problem:

$$\begin{aligned} w(x^{\nu}) = & \text{Min} \quad dy \\ & \text{s a} \quad F\,y \geq h - E\,x^{\nu} \\ & \qquad\ y \geq 0 \end{aligned} \tag{3.9}$$

5. Let y^{ν} be the optimal solution of (3.9). The set (x^{ν},y^{ν}) is a feasible solution of the original problem (3.6), but not necessarily the optimal solution. Because the cost of a feasible solution is by definition higher than or equal to the optimal value, we have an upper bound:

$$\overline{z} = \text{Min}\ \{\overline{z},\, c\,x^{\nu} + d\,y^{\nu}\} \tag{3.10}$$

6. If $\overline{z} - \underline{z} \leq \xi$, the problem is solved; the solution associated to $\overline{z}$ is a ξ-optimal solution to the original problem. Otherwise, generate the following linear constraint, known as a Benders cut:

$$\alpha \geq \pi^{\nu}\,(h - E\,x) \tag{3.11}$$

where π^{ν} is the vector of simplex multipliers associated to the constraints of operation problem (3.9), y return to step 2.

3.5 Geometric interpretation of the algorithm

We can write the Benders cuts in an alternative way, starting from the equality of the primal and dual solutions of the operation problem (3.3):

$$w(x^{*}) = \pi^{*}\,(h - E\,x^{*}) \tag{3.12}$$

From (3.12), we have an expression for $\pi^{*}\,h$:

$$\pi^{*}\,h = w(x^{*}) + \pi^{*}\,E\,x^{*} \tag{3.13}$$

Replacing (3.13) in the Benders cut expression $\alpha \geq \pi^* (h - Ex)$ results in:

$$\alpha \geq w(x^*) - \pi^* E (x - x^*) \tag{3.14}$$

The alternative expression (3.14) can be derived in a different way. Let the function:

$$\mathrm{H}(x) = h - E\,x \tag{3.15}$$

represent the right hand side of the constraints of operation problem (3.3). If π^* is the vector of dual variables associated to the optimal solution of this problem, we know that:

$$\left.\frac{\partial w(x)}{\partial \mathrm{H}(x)}\right|_{x=x^*} = \pi^* \tag{3.16}$$

Using the chain rule, we can deduce the derivative of $w(x)$ with respect to x:

$$\left.\frac{\partial w(x)}{\partial x}\right|_{x=x^*} = \frac{\partial w(x)}{\partial \mathrm{H}(x)} \times \left.\frac{\partial \mathrm{H}(x)}{\partial x}\right|_{x=x^*} = -\,\pi^* E \tag{3.17}$$

Because $w(x)$ is a piecewise linear function, expression (3.17) corresponds to a subgradient of $w(x)$ around the point $x = x^*$. Therefore, we can conclude that:

$$w(x) \geq w(x^*) + \left.\frac{\partial w(x)}{\partial x}\right|_{x=x^*} (x - x^*) \tag{3.18}$$

Defining $\alpha = w(x)$, we obtain expression (3.14).

We see in (3.18) that the Benders cut can be interpreted as a first-order approximation of the operating cost function $w(x)$ around the investment decision vector x^* produced by the investment problem (3.7).

3.6 Alternative expression for the approximate investment problem

Substituting (3.14) into the approximate expansion problem (3.7), we have:

$$\begin{aligned} z = \quad & \text{Min} \quad c\,x + \alpha \\ & \text{s a} \quad \alpha \geq w(x^\mu) + \lambda(x^\mu)(x - x^\mu),\ \mu = 1, \ldots, \nu \\ & \qquad\;\; x \in X \end{aligned} \tag{3.19}$$

where $\lambda(x^\mu) = -\,\pi^\mu \mathrm{E}$, $\mu = 1, \ldots, \nu$.

Figure 3.4 illustrates the decomposition process:

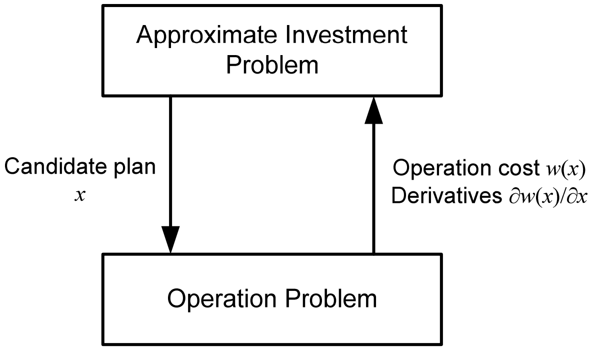


Figure 3.4 - Decomposition process

4 APPLICATION TO THE EXPANSION PLANNING PROBLEM

4.1 Application of the decomposition methodology

In this chapter, we illustrate the application of the Benders decomposition methodology to the expansion planning problem (2.1).

As discussed in the previous chapter, the operation problem is solved for each candidate investment plan. The Benders cut is constructed from the optimal cost and the vector of simplex multipliers of the operation problem.

This cut is added as a new linear constraint to the approximate investment problem, which is then re-solved and produces a new candidate expansion plan.

4.1.1 Operation problem

Given an expansion plan x^v, the operation problem is formulated as the minimization of operating cost, subject to the water balance and load supply equations, operating limits and emission constraints:

$$w(x^v) = \text{Min} \sum_{t \in T} \sum_{i \in I} d(t,i) \times g(t,i) \tag{4.1}$$

subject to:

Water balance equations:

$$v(t,i) - v(t-1,i) + q(t,i) + s(t,i) - \sum_{j \in MT(i)} q(t,j) - \sum_{j \in MV(i)} s(t,j) = a(t,i)$$

$$\forall\, t \in T, \forall\, i \in H$$

Load supply constraints:

$$\sum_{i \in I_n} g(t,i) + \sum_{i \in H_n} \rho(i)\, q(t,i) - \sum_{k \in K | n_o(k)=n} f(t,k) + \sum_{k \in K | n_d(k)=n} (1 - p(k))\, f(t,k) = D(t,n)$$

$$\forall\, t \in T, n \in N$$

Emission constraints:

$$\sum_{i \in E_1(l)} e(i,1) \times g(t,i) \le E_1(t,l), \qquad \forall\, t \in T, l = 1, \ldots, NE_1$$

$$\sum_{i \in E_2(l)} e(i,2) \times g(t,i) \le E_2(t,l), \qquad \forall\, t \in T,\ l = 1, \ldots, NE_2$$

$$\sum_{i \in E_3(l)} e(i,3) \times g(t,i) \le E_3(t,l), \qquad \forall\, t \in T,\ l = 1, \ldots, NE_3$$

Initial storage constraints:

$$v(0,i) = v_0(i), \qquad \forall\, i \in H$$

Operating limits:

$$g(t,i) \le \overline{g}(i), \qquad \forall\, i \in I \mid i \notin J$$

$$g(t,i) - \overline{g}(i) \times y(t,i) \le 0, \qquad \forall\, i \in I \mid i \in J$$

$$v(t,i) \le \overline{v}(i), \qquad \forall\, i \in H \mid i \notin J$$

$$v(t,i) - \overline{v}(i) \times y(t,i) \le 0, \qquad \forall\, i \in H \mid i \in J$$

$$q(t,i) \le \overline{q}(i), \qquad \forall\, i \in H \mid i \notin J$$

$$q(t,i) - \overline{q}(i) \times y(t,i) \le 0, \qquad \forall\, i \in H \mid i \in J$$

$$f(t,i) \le \overline{f}(i), \qquad \forall\, i \in K \mid i \notin J$$

$$f(t,i) - \overline{f}(i) \times y(t,i) \le 0, \qquad \forall\, i \in K \mid i \in J$$

where $y(t,j)$, defined accordingly to the notation in 2.3.5, is:

$$y(t,j) = \sum_{\tau=\underline{t}(j)}^{t} x(\tau,j), \qquad \forall\, t \in T$$

4.1.2 Evaluation of Benders Cut

In the operation problem (4.1) only the generation limits for thermal plant projects; storage and turbining limits for hydro projects; and energy interchange limits for interconnection projects depend on the investment decisions x^v. For notational ease, we rewrite these constraints in the standard linear programming format:

Dual variables

$$- g(t,i) \ge - \overline{g}(i) \times y^v(t,j), \qquad \forall\, i \in I \mid j \in J,\ t \in T \qquad \pi^g(t,i) \tag{4.2}$$

$$- v(t,i) \ge - \overline{v}(i) \times y^v(t,j), \qquad \forall\, i \in H \mid j \in J,\ t \in T \qquad \pi^v(t,i)$$

$$- q(t,i) \ge - \overline{q}(i) \times y^v(t,j), \qquad \forall\, i \in H \mid j \in J,\ t \in T \qquad \pi^q(t,i)$$

$$- f(t,k) \ge - \overline{f}(k) \times y^v(t,j), \qquad \forall\, k \in K \mid j \in J,\ t \in T \qquad \pi^f(t,i)$$

where $\pi^g(t,i)$, $\pi^v(t,i)$, $\pi^q(t,i)$ y $\pi^f(t,i)$ are the dual variables associated to the constraints in the optimal solution.

Applying the chain rule to calculate the derivative associated to a thermal plant investment decision $x(t,i)$ results in:

$$\left.\frac{\partial w(x)}{\partial x(t,j)}\right|_{x=x^v} = \sum_{\tau=t}^{T} \pi^g(\tau,i)\,(-\bar{g}(i)) = -\,\bar{g}(i) \sum_{\tau=t}^{T} \pi^g(\tau,i) \tag{4.3}$$

The derivative of w with respect to an investment decision for a hydro project i, $x(t,i)$, is:

$$\left.\frac{\partial w(x)}{\partial x(t,j)}\right|_{x=x^v} = \sum_{\tau=t}^{T} \pi^v(\tau,i)\,(-\bar{v}(i)) + \sum_{\tau=t}^{T} \pi^q(\tau,i)\,(-\bar{q}(i)) = -\,(\bar{v}(i) \sum_{\tau=t}^{T} \pi^v(\tau,i) + \bar{q}(i) \sum_{\tau=t}^{T} \pi^q(\tau,i)) \tag{4.4}$$

Finally, the derivative of w with respect to an interconnection investment decision $x(t,i)$ is:

$$\left.\frac{\partial w(x)}{\partial x(t,j)}\right|_{x=x^v} = \sum_{\tau=t}^{T} \pi^f(\tau,k)\,(-\bar{f}(k)) = -\,\bar{f}(k) \sum_{\tau=t}^{T} \pi^f(\tau,k) \tag{4.5}$$

Defining:

$$\lambda^v(t,j) = \left.\frac{\partial w(x)}{\partial x(t,j)}\right|_{x=x^v}$$

The Benders cut is calculated as:

$$\alpha \geq w(x^v) + \sum_{t \in T} \sum_{j \in J} \lambda^v(t,j)\,(x(t,j) - x^v(t,j)) \tag{4.6}$$

Aggregating the known values as:

$$r^v = w(x^v) - \sum_{t \in T} \sum_{j \in J} \lambda^v(t,j)\, x^v(t,j) \tag{4.7}$$

and passing the variables to the left hand side results in:

$$\alpha - \sum_{t \in T} \sum_{j \in J} \lambda^v(t,j)\, x(t,j) \geq r^v \tag{4.8}$$

4.1.3 Approximate expansion problem

The objective of the approximate expansion problem is to minimize the sum of the present value of investment costs of new projects (hydro, thermal and interconnection) plus an approximation to the operating cost (α), subject to project constraints (integrality, unicity and earliest/latest dates for decision) and other constraints. In each iteration of the decomposition algorithm, a new constraint is added to this problem, calculated from the results of the operation problem.

The approximate expansion problem at the ν-th iteration is formulated as:

$$\text{Min} \quad \sum_{t \in \mathrm{T}} \sum_{j \in \mathrm{J}} c(t,j) \times x(t,j) + \alpha \tag{4.9}$$

subject to

Earliest and latest decision dates:

$$x(t,j) = 0, \qquad \forall\, t \notin [\underline{t}(j), \overline{t}(j)]$$

Mandatory projects constraints:

$$\sum_{t=\underline{t}(j)}^{\overline{t}(j)} x(t,j) = 1, \qquad \forall\, j \in \mathrm{J}_1$$

Optional projects constraints:

$$\sum_{t=\underline{t}(j)}^{\overline{t}(j)} x(t,j) \leq 1, \qquad \forall\, j \in \mathrm{J}_2$$

Mutually exclusive projects constraints:

$$\sum_{j \in \mathrm{J}_3(l)} y(j) \leq 1, \qquad l = 1, ..., \mathrm{N}_3$$

Associated projects constraints:

$$y(j_1) = y(j_2), \qquad \forall\, j_1, j_2 \in \mathrm{J}_4(l),\ l = 1, ..., \mathrm{N}_4$$

Minimum installed capacity constraints:

$$\sum_{t=\underline{\tau}(l)}^{\overline{\tau}(l)} \sum_{j \in \mathrm{J}_5(l)} w(j) \times x(t,j) \geq \underline{w}(k), \qquad l = 1, ..., \mathrm{N}_5$$

Maximum installed capacity constraints:

$$\sum_{t=\underline{\tau}(l)}^{\overline{\tau}(l)} \sum_{j \in J_5(l)} w(j) \times x(t,j) \leq \overline{w}(k), \qquad l = 1, ..., N_5$$

Project construction precedence constraints:

$$y(t,j_{i+1}) - y(t,j_i) \geq 0, \qquad \forall\, j_i \in J_6(l),\ \forall\, t \in T,\ l = 1, ..., N_6$$

Firm energy per system:

$$\sum_{i \in G_n} EF(t,i) + \sum_{j \in J_n} EF(t,j) \times y(t,j) \geq FEF(t,n) \times D(t,n), \qquad \forall\, t \in T,\ \forall\, n \in N$$

Firm capacity per system:

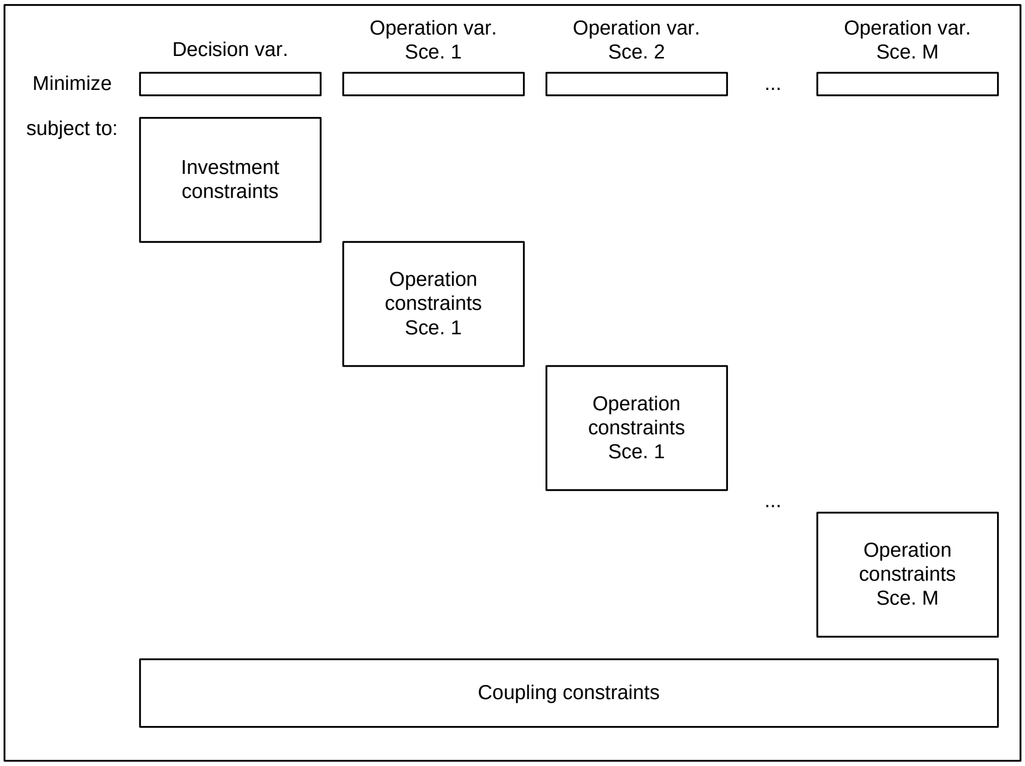
$$\sum_{i \in G_n} PF(t,i) + \sum_{j \in J_n} PF(t,j) \times y(t,j) \geq FPF(t,n) \times D(t,n), \qquad \forall\, t \in T,\ \forall\, n \in N$$

Benders cuts:

$$\alpha - \sum_{t \in T} \sum_{j \in J} \lambda^{\mu}(t,j) \times x(t,j) \geq r^{\mu}, \qquad \mu = 1, ..., \nu$$

5 EXPECTED VALUE CRITERION

The Benders algorithm can also be applied to planning problems with multiple scenarios. The problem block structures is even clearer.



The optimal expansion with an expected value criterion is formulated as:

$$\begin{array}{ll} \text{Min} & c(x) + \bar{w}(x) \\ \text{s a} & x \in X \end{array} \qquad (5.1)$$

where $c(x)$ and $\bar{w}(x)$ are respectively the investment cost and the expected operation cost for the candidate plan x:

$$\bar{w}(x) = \sum_{m=1}^{M} p_m \, w_m(x) \qquad (5.2)$$

where p_m is the probability of scenario m. Given a decision x*, the operation problem for each scenario m, for $m = 1, \ldots, M$, is formulated as the following optimization problem:

$$\begin{array}{llll} w_m(x^*) = & \text{Min} & d_m y_m & \text{Dual variables} \\ & \text{s a} & F_m y_m \geq h_m - E_m x^* & \pi_m^* \\ & & y_m \geq 0 & \end{array} \qquad (5.3)$$

The Benders cut associated to each operation problem is obtained from (5.3):

$$\alpha \geq w_m(x^*) + \lambda_m(x^*)\,(x - x^*), \qquad m = 1, ..., M \tag{5.4}$$

where $\lambda_m(x^*) = -\pi_m^* E_m$. Because the expression (5.2) is linear, the Benders cut is the probability-weighted average of all cuts. Defining:

$$\bar{\lambda}(x^*) = \sum_{m=1}^{M} p_m \lambda_m(x^*) \tag{5.5}$$

We have:

$$\alpha \geq \bar{w}_m(x^*) + \bar{\lambda}(x^*)\,(x - x^*) \tag{5.6}$$

Substituting $\bar{w}(x)$ in (5.1) by the cuts (5.6) generated in each iteration of the decomposition algorithm, we obtain the relaxed expansion problem:

$$\begin{aligned} \text{Min} \quad & c(x) + \alpha \\ \text{s a} \quad & \alpha \geq \bar{w}_m(x^\mu) + \bar{\lambda}(x^\mu)\,(x - x^\mu), \qquad \mu = 1, ..., \nu \\ & x \in X \end{aligned} \tag{5.7}$$

Figure 5.1 illustrates the decomposition process:

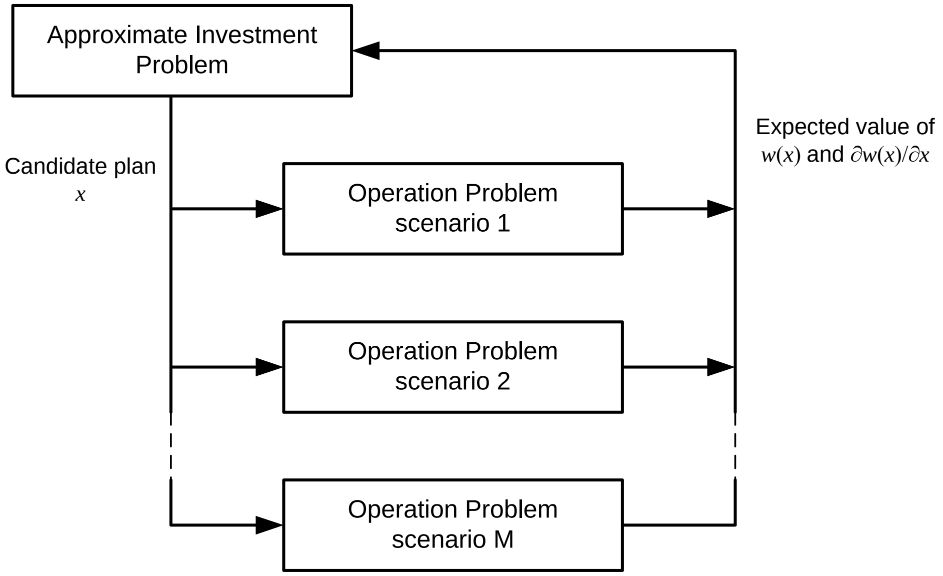


Figure 5.1 - Decomposition scheme for multiple scenarios

6 ADDITIONAL MODELING ASPECTS

6.1 Discount rate and calculation of investment decision costs

The annual discount rate is an input data. If the time stages for investment and/or operation decisions are not annual, they are recalculated internally. For example, let tx^{A} be the annual discount rate. If the operation is carried out on a monthly basis, the monthly interest rate tx^{M} is calculated as follows:

$$\frac{1}{1+tx^{A}} = \frac{1}{(1+tx^{M})^{12}}$$

Therefore, the monthly interest rate is:

$$tx^{M} = (1 + tx^{A})^{1/12} - 1$$

6.1.1 Evaluation of investment decision costs

For each project, the model reads the following data:

Investment cost (M$):	c_1
Cost of connection to the grid ($/KW):	c_2
O&M cost ($/KW):	c_3
Installed capacity (MW)	w
Lifetime (years)	L

Payment schedule:

Number of disbursements	N
Number of years until entrance in operation	n_0
Disbursements (%)	p_n , $n = 1,\ldots,$ N

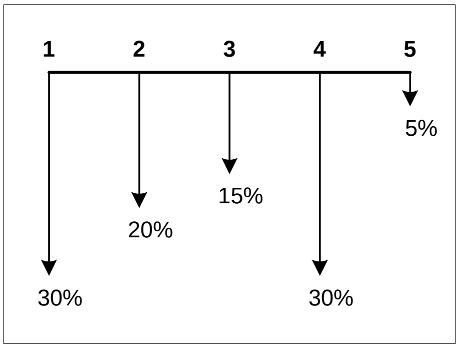


Figure 6.1 – Example of a disbursement schedule

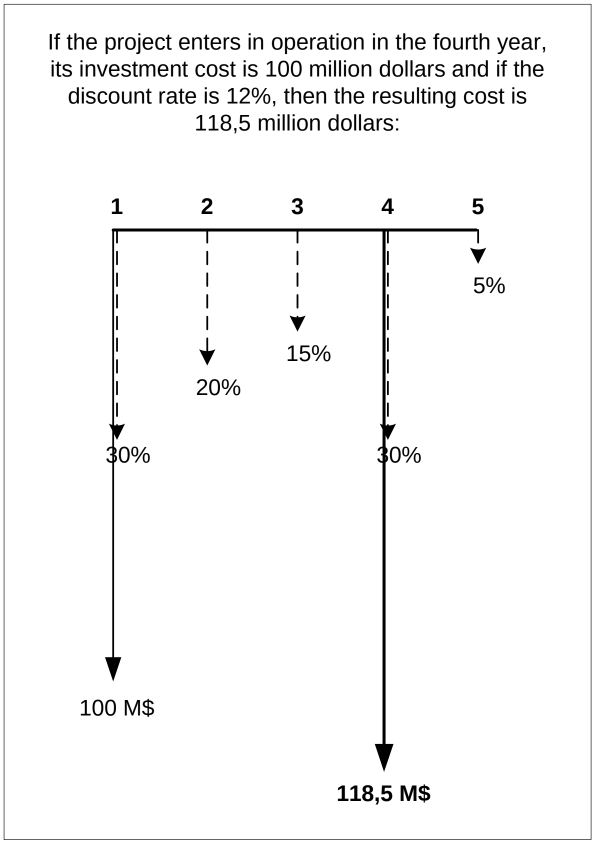
In addition, we have the following information:

Discount rate (%)	tx^A
Study duration (years)	T

The cost of deciding to construct a project in stage *t* is calculated as follows:

Refer the investment cost plus the grid integration cost to the first year of entrance in operation, using the disbursement schedule.

$$c^{(1)} = \left(c_1 + \frac{c_2 \times w}{1000}\right) \times \sum_{n=1}^{N} \frac{p_n}{100} (1+tx^A)^{(n_0 - n)}$$



If the project enters in operation in the fourth year, its investment cost is 100 million dollars and if the discount rate is 12%, then the resulting cost is 118,5 million dollars:

Figure 6.2 – Investment cost referred to the start-operation date

The adjusted investment cost is then annualized over the useful life of the equipment. To this annualized value we add the O&M cost.

$$c^{(2)} = c^{(1)} \times \frac{tx^A(1 + tx^A)^L}{(1 + tx^A)^L - 1} + \frac{c_3 \times w}{1000}$$

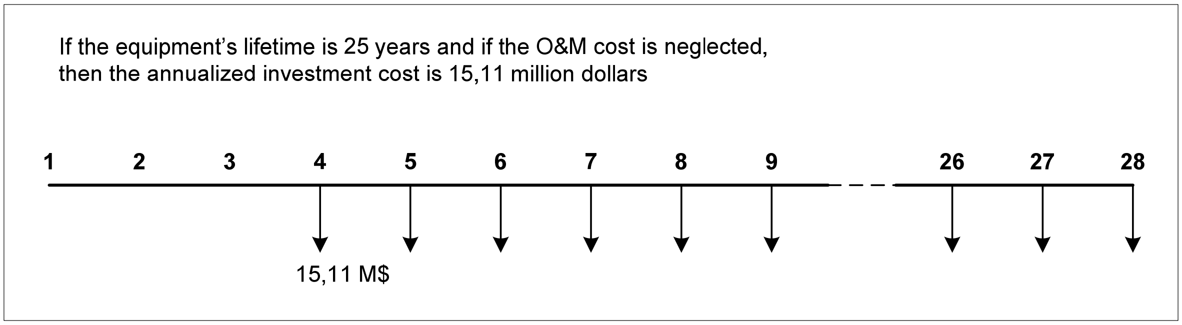


Figure 6.3 – Annualized investment cost

The annualized cost is then referred to the investment decision year:

$$c^{(3)} = c^{(2)} \times \frac{1}{(1 + tx^{A})^{(n0-1)}}$$

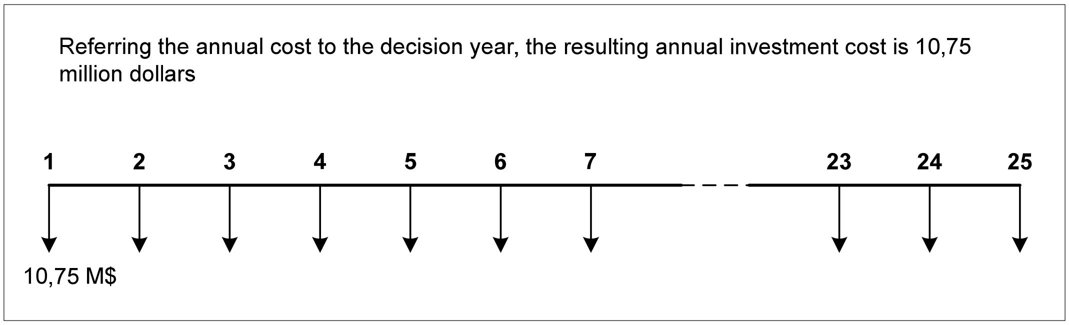


Figure 6.4 – Annualized investment cost referred to the decision year

The calculation of investment cost in stage *t* considers that the annual disbursements:

- start in the year of entrance in operation;
- end at the final study year of at the end of lifetime period;
- are paid at the end of each year.

Therefore, the investment cost in stage *t* corresponds to the present value of a finite disbursement schedule with the following number of installments: $t' = \min\{T - (t + n_0 - 1) + 1, L\}$. The net present value of this cash flow is:

$$c^{(4)} = c^{(3)} \times \frac{(1 + tx^{A})^{t'} - 1}{tx^{A} \times (1 + tx^{A})^{t'}}$$

The investment cost in stage t is obtained by evaluating the present value of a finite cash flow of 10,75 million dollars. It is assumed that each payment is accomplished at the end of each year and the number of disbursements is equal to the number of years since the entrance in operation until the end of the study. For example, if T=15 and t=5, and the project enters in operation in the fourth year from the decision date, then the investment cost in stage t is 53,42 million dollars.

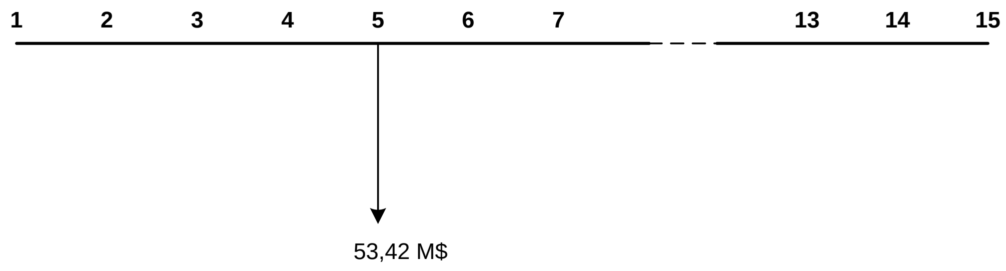


Figure 6.5 – Cost of decision in stage t

In order to compare the different decision alternatives along the study period, all decision costs are referred to the first year of the study.

$$c^{(5)} = c^{(4)} \times \frac{1}{(1 + tx^{A})^{(t-1)}}$$

If the investment cost in stage t=5 is 53,42 million dollars, then the referred cost to the first stage of the study is 33,95 million dollars.

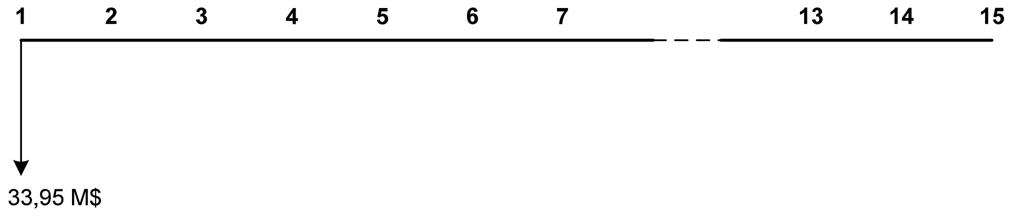


Figure 6.6 – Cost of a decision in stage *t* referred to the first stage

6.1.2 Understanding the file outdisbu.csv

The output file outdisbu.csv shows the disbursements for each project and each decision stage. For example, suppose a planning study with 15 years, three candidate projects and 12% discount rate. The first candidate project has the same characteristics of the example project in the previous item. Suppose that the optimal decision is to construct this project in stage 5, that is, it enter in operation in January 2009. Therefore, there will be a cash flow of $ 15.11 million from the end of that year until the end of the study. As explained in the previous item, the present value of this investment is $ 33.95 million.

The second project has an annual cost of $ 48.25 million and the optimal decision is to enter in operation it in the third year, therefore, its present value is $ 247.08 million.

For the third project, the annual investment cost is $ 4.80 million; in the optimal schedule for this project, it enters in operation in the ninth year. The resulting present value is $ 8.85 million.

Finally, note that the present value of the total investment in the optimal plan is $ 289.88 million. This value corresponds to the investment cost of the last iteration of the Benders decomposition scheme, which appears in the convergence report of file optgen.log.

Stage	**Project 1**	**Project 2**	**Project 3**	**Total**
Jan-02	0.00	0.00	0.00	0.00
Jan-03	0.00	0.00	0.00	0.00
Jan-04	0.00	48.25	0.00	48.25
Jan-05	0.00	48.25	0.00	48.25
Jan-06	0.00	48.25	0.00	48.25
Jan-07	0.00	48.25	0.00	48.25
Jan-08	0.00	48.25	0.00	48.25
Jan-09	15.11	48.25	0.00	63.36
Jan-10	15.11	48.25	4.80	68.16
Jan-11	15.11	48.25	4.80	68.16
Jan-12	15.11	48.25	4.80	68.16
Jan-13	15.11	48.25	4.80	68.16
Jan-14	15.11	48.25	4.80	68.16
Jan-15	15.11	48.25	4.80	68.16
Jan-16	15.11	48.25	4.80	68.16
Present value	33.95	247.08	8.85	289.88

6.2 Associated project constraints

These constraints indicate that a group of projects is subject to a single investment decision, that is, either all projects in the group are constructed, or none. In order to define an associated project constraint, the only information required is the group of projects. The mathematical formulation of these constraints in the investment model is:

$$y(j_1) = y(j_2), \qquad \forall\, j_1, j_2 \in \mathrm{J}_4(l),\ l = 1, ..., \mathrm{N}_4$$

where:

N_4	number of sets of associated projects
$y(t,j)$	sum of the decision variables associated to project j up to stage t $y(t,j) = \sum_{\tau=\underline{t}(j)}^{t} x(\tau,j),\ t \in \mathrm{T}$
$x(t,j)$	binary variable associated to the decision of constructing project j in stage t
$\mathrm{J}_4(l)$	set of projects in the l-th constraint of associated projects

Note that this type of constraint does not determine when the projects are constructed, it only requires that all projects (or none) are constructed at some stage. Therefore, two associated projects may be built at different stages in the optimal solution. If the user wishes to determine an order of entrance for the projects, it is necessary to use the project precedence constraints, described next.

6.3 Precedence constraints

The objective of this type of constraint is to establish a chronological sequence for the entrance of new projects. For example, the construction of new generation capacity in an exporter region could be associated to the construction of a new interconnection, which cannot start operation after the new plants.

In order to define an associated project constraint, the only information required is the group of projects. The mathematical formulation is:

$$y(t,j_{i+1}) - y(t,j_i) \geq 0, \qquad \forall\, j_i \in J_6(l),\ \forall\, t \in T,\ l = 1, ..., N_6$$

where:

N_6	number of sets of precedence projects
$y(t,j)$	sum of the decision variables associated to project j up to stage t
	$y(t,j) = \sum_{\tau=\underline{t}(j)}^{t} x(\tau,j),\ t \in T$
$x(t,j)$	binary variable associated to the decision of constructing project j in stage t
$J_6(l)$	set of projects in the l-th precedence constraint of projects The order in which they appear defines the precedence relation.

6.4 Mutually exclusive projects

The objective in this case is to build only one project (or none) of the projects in a given group. In order to define an associated project constraint, the only information required is the group of projects .The mathematical formulation is:

$$\sum_{j \in J_3(l)} y(j) \leq 1, \qquad l = 1, ..., N_3$$

where,

N_3	number of sets of mutually exclusive projects
$y(t,j)$	sum of the decision variables associated to project j up to stage t
	$y(t,j) = \sum_{\tau=\underline{t}(j)}^{t} x(\tau,j),\ t \in T$
$x(t,j)$	binary variable associated to the decision of constructing project j in stage t
$J_3(l)$	set of projects in the l-th constraint of mutually exclusive projects

6.5 Project reinforcements

Optgen allows the modeling of reinforcements, or upgrades, to existing projects. This is done by defining a new project (existing + upgrades) that replaces the existing project when it starts operating.

Without loss of generality, suppose that there is only one candidate project *j* in the study, which is a reinforcement to an existing plant *i*. The approximate investment problem is:

$$\text{Min} \quad \sum_{t\in T} c(t,j) \times x(t,j) + \alpha$$

$$\text{s.a} \quad \sum_{t\in T} x(t,j) \le 1$$

As mentioned, project *j* replaces the existing plant *i* when it starts operating. The operation problem is:

$$\begin{array}{llll} w(x^v) = \text{Min} & \sum_{t\in T} d(t,i) \times g(t,i) + d(t,j) \times g(t,j) & & \text{Dual variables} \\ \text{s.a} & g(t,i) + g(t,j) = D(t), & \forall\, t \in T & \\ & g(t,i) \le \overline{g}(i) \times (1 - y^v(t,j)), & \forall\, t \in T & \pi^g(t,i) \\ & g(t,j) \le \overline{g}\,(j) \times y^v(t,j), & \forall\, t \in T & \pi^g(t,j) \end{array}$$

where:

$$y^v(t,j) = \sum_{\tau=\underline{t}(j)}^{t} x^v(\tau,j), \qquad t \in \mathrm{T}$$

6.5.1 Evaluation of the Benders cut for reinforcement projects

For notation simplicity, the capacity constraints that depend on the decision variable are rewritten in the standard LP format:

$$\begin{array}{ll} & \text{Dual variables} \\ -\, g(t,i) \ge -\, \overline{g}(i) \times (1 - y^v(t,j)) & \pi^g(t,i) \\ -\, g(t,j) \ge -\, \overline{g}(j) \times y^v(t,j) & \pi^g(t,j) \end{array}$$

where:

$$\left.\frac{\partial w(x)}{\partial x(t,j)}\right|_{x=x^{v}} = \sum_{\tau=t}^{T} \pi^{g}(t,i)\,\overline{g}(i) + \sum_{\tau=t}^{T} \pi^{g}(t,j)\,(-\overline{g}(j))$$

$$= \overline{g}(i) \sum_{\tau=t}^{T} \pi^{g}(t,i) - \overline{g}(j) \sum_{\tau=t}^{T} \pi^{g}(t,j)$$

6.6 Plants with multiple generators

Suppose that a new thermal project j has several units, which enter operation in accordance with a given yearly schedule (in p.u.) $p_1, p_2, \ldots, p_n$. Let f_t, $t = 1, ..., T$, be the cumulative value, in p.u., of the schedule for project j, that is:

$$f_t = \sum_{\tau=1}^{t} p_{\tau}, \qquad \forall\, t \in T$$

For example, and without loss of generality, suppose that the study period has three years. In this case, the generation capacity of project j is:

$$g(1,j) \le \overline{g}(j) \times x(1,j)$$

$$g(2,j) \le \overline{g}(j) \times (x(1,j) + x(2,j))$$

$$g(3,j) \le \overline{g}(j) \times (x(1,j) + x(2,j) + x(3,j))$$

which results in:

$$g(1,j) \le \overline{g}(j) \times f_1\, x(1,j)$$

$$g(2,j) \le \overline{g}(j) \times (f_2\, x(1,j) + f_1\, x(2,j))$$

$$g(3,j) \le \overline{g}(j) \times (f_3\, x(1,j) + f_2\, x(2,j) + f_1\, x(3,j))$$

or, in a more general way:

$$g(t,j) \le \overline{g}(j) \times \sum_{\tau=1}^{t} f_{t-\tau+1}\, x(\tau,j)$$

The example above also applies to new hydro projects and interconnections. The effect of this schedule of entrance in operation is considered in the calculation of derivatives (Benders cuts) in the following way:

$$\frac{\partial w(x)}{\partial x_1} = (\pi_1 f_1 + \pi_2 f_2 + \pi_3 f_3 + \ldots + \pi_T f_T)\,\overline{g} = \sum_{\tau=1}^{T} \pi_\tau f_\tau \overline{g}$$

$$\frac{\partial w(x)}{\partial x_2} = (\pi_2 f_1 + \pi_3 f_2 + \pi_4 f_3 + \ldots + \pi_T f_T)\,\overline{g} = \sum_{\tau=2}^{T} \pi_\tau f_{\tau-1} \overline{g}$$

In general:

$$\frac{\partial w(x)}{\partial x_t} = \sum_{\tau=t}^{T} \pi_\tau f_{\tau-t+1} \overline{g}$$

The actual expressions are a bit more complicated because the time intervals for investments (typically, year) are usually different from those for operation (typically, month):

$g(1,j) \leq \overline{g}(j) \times f_1\, x(1,j)$

$g(2,j) \leq \overline{g}(j) \times f_2\, x(1,j)$

…

$g(12,j) \leq \overline{g}(j) \times f_{12}\, x(1,j)$

$g(13,j) \leq \overline{g}(j) \times (f_{12}\, x(1,j) + f_{13}\, x(2,j))$

etc.

The general expression is:

$$g_t \leq \overline{g} \times \sum_{\tau=1}^{\frac{t-1}{12}+1} f_{t-12(\tau-1)}\, x_\tau$$

6.7 Evaluation of reference costs

Optgen calculates a reference cost ($/MWh) for all projects, as follows:

Hydro projects:

$$\frac{\text{CIA(M\$)}\times 10^6}{\text{P(MW)}\times\text{FCM(pu)}\times 8760\text{h}}$$

Thermal projects:

$$\frac{\text{CIA(M\$)}\times 10^6 + \text{COP(\$/MWh)}\times\text{P(MW)}\times\text{FCM(pu)}\times 8760\text{h}}{\text{P(MW)}\times\text{FCG(pu)}\times 8760\text{ h}}$$

Interconnection projects

$$\frac{\text{CIA(M\$)}\times 10^6}{\text{P(MW)}\times\text{FUT(pu)}\times 8760\text{h}}$$

where:

CIA	Annualized investment cost	M$
COP	Operation cost	$/MWh
P	Capacity	MW
FCM	Average capacity factor	pu
FCG	Firm capacity factor	pu
FUT	Utilization factor	pu

6.8 Evaluation of the unit operating cost of thermal plants

The unit operating cost of thermal plants is calculated as:

$$\text{TCst}(t,i) = \frac{\text{Fcst}(t,\text{fuel}(i))}{\text{ECnt}(\text{fuel}(i))} \times \text{HR}(i) \times 1000$$

where

$\text{TCst}(t,i)$	unit cost of plant i in stage t	$/MWh
$\text{fuel}(i)$	indexes the fuel of thermal plant i	
$\text{Fcst}(t,f)$	unit cost of fuel f in stage t	$/unit
$\text{ECnt}(f)$	energy content of fuel f	kcal/unit
$\text{HR}(i)$	heat rate of thermal plant i	kcal/kWh

6.9 Emission constraints

The objective of these constraints is to impose limits for thermal energy production due to environmental constraints associated to NO_x, CO_2 and SO_2 emissions.

The representation of emission constraints requires information about the emission coefficients of each thermal plant participating in a given constraint and its emission limit.

The mathematical formulation for the NO_x constraints is:

$$\sum_{i \in E_1(l)} e(i,1) \times g(t,i) \le \text{E}_1(t,l), \qquad \forall\, t \in T,\ l = 1, \ldots, \text{NE}_1$$

where,

$E_1(l)$	set of thermal plants in the *l*-th NO_x emission constraint
NE_1	number of NO_x emission constraints
$e(i,1)$	NO_x emission factor of thermal plant *i*

The formulation for SO_2 y CO_2 constraints is analogous.

6.10 Fuel availability constraints

The objective of these constraints is to impose limits on generation of a set of thermal plants which uses the same fuel and whose availability is limited.

The information required to represent a fuel availability constraint comprises: (i) set of plants participating in each fuel constraint; (ii) heat rate of each plant in set (i); (iii) energy content of the fuel; and (iv) fuel availability.

The mathematical formulation is:

$$\sum_{i \in F(l)} \frac{\mathrm{HR}(i) \times g(t,i)}{\mathrm{ECnt}(\mathrm{fuel}(i))} \leq \mathrm{F}(t,l), \qquad \forall\, t \in T,\ l = 1, \ldots, \mathrm{NF}$$

where,

$\mathrm{F}(t,l)$	availability of fuel in the *l*-th constraint, stage *t*
$\mathrm{ECnt}(f)$	energy content of fuel *f*
$\mathrm{fuel}(i)$	fuel of thermal plant *i*
$\mathrm{HR}(i)$	heat rate of thermal plant *i*
NF	number of fuel availability constraints
$\mathrm{g}(t,i)$	generation of plant *i* in stage *t*

7 BIBLIOGRAPHY

J.F.Benders, “Partitioning procedures for solving mixed variables programming problems”, *Numerische Mathematik*, Vol.4, pp. 238-252, 1962.

B.Gorenstin, J.P.Costa, M.V.F.Pereira, N.M.Campodónico, “Power System Expansion Planning Under Uncertainty”, *IEEE Transactions on Power Systems*, Vol.8, No.1, 1993.

J.P.Costa, N.M.Campodónico, B.G.Gorenstin, M.V.F.Pereira, "A Model for Optimal Energy Expansion in Interconnected Hydrosystems", 10th PSCC, Austria, 1990.