Implicit Economic Slack Bus in the Calculation of Transmission Tariffs for the LRMC Methodology

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1. Objective

This technical report aims at discussing possible doubts related to the concept of an *implicit slack bus* in the calculation of transmission tariffs according to the Long Run Marginal Cost (LRMC) methodology.

2. LRMC Methodology

Transmission tariffs calculated according to the LRMC methodology are based on the variation of incremental flows in each circuit of the transmission system with respect to an incremental injection of power in each bus. This variation is calculated based on the *Sensitivity Matrix* β, which is obtained from the modeling of a lossless linear power flow.

2.1. Electrical Slack Bus

The need of an electrical slack to perform load flow calculations derives from the fact that the problem of the lossless linear power flow is formulated through a set of linear equations, which are linear dependent. Due to this limitation, this problem can be only solved when a variable value is predetermined (voltage angle) and an equation is excluded from the problem, thus allowing the calculation oh the others variables. This predetermined variable is related to the slack bus of the system. In the context of the power flow solution, this slack bus is used as a reference angle to the others buses in the system, besides being responsible for the mismatch between total generation and total demand. In other words, the slack bus has its generation value calculated in order to allow the balance between generation and demand. In this context, this bus will be referred as "electrical slack bus".

Once a slack bus is defined, the *Sensitivity Matrix* β can be calculated. This matrix has the dimension of number of circuits versus number of buses, and each element β_{ki} represents the variation of the power flow in circuit ***k*** when 1 MW is injected in bus *i*. Given that β implicitly represents a lossless linear power flow, the injection of 1 MW in each bus of the system must supply 1 MW of demand, which is located at the electrical slack bus. In other words, the elements in the matrix β are directly dependent of the chosen electrical slack bus. It can also be notedthat the column of the matrix corresponding to the electrical slack bus has only zero values: this is due to the fact that the injection of 1 MW in this bus does not cause any variations in the transmission system flows, once the compensation of this 1 MW is done in the same bus.

In practical terms, to choose the electrical slack bus implies in choosing where the power injections will be compensated and in the implicitly determination of the incremental power flows in each circuit, which are responsible for the LRMC calculations. In radial systems where a concentration of demand can be easily identified in the transmission system, the chose of the electrical slack bus becomes simple. However, in real systems, where the circuits are usually meshed and many "load centers" may exist, this choice may be impartial and prejudicial to agents distant from the electrical slack bus.

2.2. LRMC Transmission Tariff

The calculation of transmission tariffs following the LRMC method can be carried out through the following procedure:

- Calculation of Matrix β^r as a function of an arbitrarily chosen electrical slack bus *r*;
- Calculation, for each bus *i*, of the incremental power flow in each circuit ***k***, as the difference between the coefficients related to the bus *i* and the slack bus *r*, i.e.:

$$f_k^{ir} = \beta_{ki} - \beta_{kr} \tag{2.1}$$

Where:

f_k^{ir} Incremental power flow in the circuit ***k*** due to an injection of 1 MW in the bus *i* and its compensation in the slack bus *r*.

In other words, the LRMC Tariff, as function of an electrical slack bus, is given by:

$$\tilde{\pi}_i = \sum_{k=1}^{NLin} c_k \cdot f_k^{ir} = \sum_{k=1}^{NLin} c_k \cdot (\beta_{ki} - \beta_{kr}) \tag{2.2}$$

2.3. CHANGING THE ELECTRICAL SLACK BUS

An analysis of equation 2.2 allows us to note that a changing of the electrical slack bus has the same effect of the addition of a constant value to all tariffs. For instance, if a slack bus ***q*** is chosen, the incremental added value would be:

$$\Delta_{rq} = \sum_{k=1}^{NLin} c_k \cdot (\beta_{kr} - \beta_{kq}) \tag{2.3}$$

Where Δ_{rq} represents the incremental constant value to be added to the tariffs to change the slack bus from *r* to ***q***.

It is important to remember that LRMC tariffs are symmetrical, i.e., in each bus, generation and demand have tariffs with same values and opposite signals ($\pi_i^d = -\pi_i^g$), and this behavior is independent of the electrical slack bus to be chosen.

It is important to note that this added value to the tariffs does not imply that the total network costs are recovered by the methodology. In other words, the electrical slack bus has no interference in the total amount recovered ($) by the methodology. This can be easily explained due to the symmetrical behavior of LRMC tariffs: the same values that are added to the generation tariffs, are subtracted in the case of demand tariffs and, given that the total generation is equal to the total demand, the global amount recovered by the methodology remains unaltered. This will be shown in the next section.

In this way, it can be concluded that the changing of the electrical slack bus has the effect of altering the balance of total payments between generator and consumers.

2.4. TOTAL AMOUNT RECOVERED BY LRMC

Once an electrical slack bus *r* is chosen and the tariffs are calculated, the total amount recovered by LRMC methodology is given by:

$$TR_r = TR_r^g + TR_r^d = \sum_{i=1}^{NBar} \tilde{\pi}_i \cdot g_i + \sum_{i=1}^{NBar} -\tilde{\pi}_i \cdot d_i = \sum_{i=1}^{NBar} \tilde{\pi}_i \cdot (g_i - d_i) \tag{2.4}$$

Where:

TR_r Total amount recovered by LRMC methodology when an electrical slack bus *r* is adopted;

TR_r^g total amount recovered by generation payments when an electrical slack bus *r* is adopted;

TR_r^d	total amount recovered by demand payments when an electrical slack bus *r* is adopted.

When the slack bus is changed to bus ***q***, the total recovery is given by:

$$TR_q = \sum_{i=1}^{NBar}\left(\tilde{\pi}_i + \Delta_{rq}\right)\cdot\left(g_i - d_i\right) = \sum_{i=1}^{NBar}\tilde{\pi}_i\cdot\left(g_i - d_i\right) + \Delta_{rq}\sum_{i=1}^{NBar}\left(g_i - d_i\right) \qquad (2.5)$$

Given that the total generation is equal to the total demand, the second parcel of the expression 2.5 is null, which results in:

$$TR_q = \sum_{i=1}^{NBar}\tilde{\pi}_i\cdot\left(g_i - d_i\right) = TR_r \qquad (2.6)$$

However:

$$\begin{aligned} TR_r^g &\neq TR_q^g \\ TR_r^d &\neq TR_q^d \end{aligned} \qquad (2.7)$$

Therefore, the total amount recovered by LRMC methodology does not depend on the electrical slack bus, and the changing of this slack bus affect only the absolute values of the tariffs. However, the *relative difference* between them is preserved.

2.5. <u>COST ALLOCATION BETWEEN GENERATION AND DEMAND</u>

As observed in the previous section, the total revenue recovered by the methodology does not depend on the electrical slack bus to be chosen. However, the changing of the slack bus implies in the addition of a constant value to all tariffs and, consequently, in the alteration of the proportional payments between generation and consumers. As a total cost allocation criteria, countries such as Brazil and United Kingdom, have adopted a 50% / 50% proportion between generation and consumers. On the other hand, Colombia, for instance, has adopted an 80% proportion for generators total payments.

One possible procedure to obtain this desired proportional allocation would be to calculate transmission tariffs for different slack buses until the allocation criteria is obtained.

However, this procedure is not adopted, even because a physical electrical bus that allows the exact desired proportional allocation may not exist. In practice, this problem can be solved with the addition of a constant value α to all tariffs in order to obtain the desired proportion. Observe that this procedure is similar to *changing* the electrical slack bus, creating the concept of a *virtual* or *economic* slack bus (or *implicit*).

The development next demonstrates how the value α can be calculated. Assuming that σ is the proportional payment factor for generation (ex. $\sigma = 50\%$), we have that the total amount recovered by generation payments is given by:

$$TR_r^g = \sigma\cdot\left(TR_r^g + TR_r^d\right) \qquad (2.8)$$

By rearranging terms, we have:

$$TR_r^g = \frac{\sigma}{(1-\sigma)} \cdot TR_r^d = \kappa \cdot TR_r^d \tag{2.9}$$

Substituting the terms, we have:

$$\sum_{i=1}^{NBar}\left(\tilde{\pi}_i + \alpha\right)\cdot g_i = \kappa \cdot \sum_{i=1}^{NBar} -\left(\tilde{\pi}_i + \alpha\right)\cdot d_i \tag{2.10}$$

$$\alpha \cdot \left(\sum_{i=1}^{NBar} g_i + \kappa \sum_{i=1}^{NBar} d_i\right) = \sum_{i=1}^{NBar} \tilde{\pi}_i \cdot g_i - \kappa \cdot \sum_{i=1}^{NBar} \tilde{\pi}_i \cdot d_i \tag{2.11}$$

$$\alpha = -\frac{\sum_{i=1}^{NBar} \tilde{\pi}_i \cdot g_i - \kappa \cdot \sum_{i=1}^{NBar} \tilde{\pi}_i \cdot d_i}{\sum_{i=1}^{NBar} g_i + \kappa \sum_{i=1}^{NBar} d_i} \tag{2.12}$$

Where:

σ Proportional payment factor for generation (ex. $\sigma = 50\%$);

κ ratio between generation and demand payments, $\kappa = P^g / P^d = \sigma/(1-\sigma)$;

α Constant value related to the "displacement" of the original electrical slack bus to the virtual slack bus that must be added to tariffs in order to guarantee the proportional payment factor of σ.

Is important to realize that, even with the addition of the constant value α, the LRMC tariffs remain symmetrical. Thus, the final tariff in each bus is given by:

$$\bar{\pi}_i = \tilde{\pi}_i + \alpha \tag{2.13}$$

These tariffs take into account the implicit slack bus, which represents the economical "center of gravity" of the transmission system, i.e., the region where the total generators payment is equal to the total consumers' payment (for a 50% / 50% allocation criteria). Given that α is a mathematical artifice, it is probable that the desired proportional allocation may be not obtained by choosing an *explicit* physical bus to serve as the electrical slack bus.

Therefore, this procedure assures that the resulting final tariffs can respect the desired criteria for proportional allocation between generators and consumers.