COMBINING INTERIOR POINT OPF AND MONTE CARLO SIMULATION IN PROBABILISTIC ANALYSIS OF POWER SYSTEMS

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Abstract: The performance evaluation of a power system can be carried out through deterministic and probabilistic approaches. Deterministic methods presents the basic limitation of not considering the essentially stochastic nature of power systems behavior mainly due to random equipment outages and load. Therefore, the adequacy analysis of a power system should be performed by using stochastic methods such as probabilistic power flow and composite generation/transmission reliability.

The performance indicators are based on the analysis of several possible system operating states, including combination of generator, transformer and line outages, load fluctuations, etc. To achieve a reasonable accuracy in estimating the probabilistic indices we may have to perform a great number of system contingencies. Therefore, in the contingency analysis process, specially in dealing with heavily stressed systems, there may be situations where the Newton-Raphson algorithm does not converge to a solution. These system solvability problems may be alleviated by calculating the minimum load shedding in order to bring solvability to an otherwise unsolvable power flow. In the considered approach, the process of computing the minimum load shedding is carried out by an OPF solved by a direct interior point (IP) method based on the primal-dual logarithm barrier algorithm.

This paper describes the adopted approach to calculate probabilistic indicators of power system performance by combining the nonlinear OPF solved by the IP algorithm and the Monte Carlo simulation method. Applications to a 1600-bus network derived from the Brazilian South/Southeast/Central West system are presented and discussed.

Keywords: probabilistic analysis, reliability evaluation, optimal power flow, interior point methods.

INTRODUCTION

The need for more efficiency in power production and delivery has led to the restructuring of the power sector in several countries. Although the new frameworks are deeply related to the country characteristics, some common features have been emphasized such as industry deverticalization, introduction of competition on energy production and trading, and mandatory access to the transmission network [1-4]. In this context, a concern that has been always raised is associated with the impact of the new regulatory structure on the system performance levels.

The performance evaluation of a power system can be carried out through deterministic and probabilistic approaches. Deterministic methods present the basic limitation of not considering the essentially stochastic nature of power systems behavior mainly due to random equipment outages and load. Therefore, the adequacy analysis of a power system should be performed by using stochastic methods such as probabilistic power flow and composite generation/transmission reliability. The performance indicators are based on the analysis of several possible system operating states, including combination of generator, transformer and line outages, load fluctuations, etc. They can be expressed in terms of probability distributions of selected variables (e.g, power flows, bus voltages, area interchanges, etc.) or through their mean values.

To achieve a reasonable accuracy in estimating the probabilistic indices we may have to perform a great number of system contingencies. Therefore, in the contingency analysis process, specially in dealing with heavily stressed systems, there may be situations where the Newton-Raphson algorithm does not converge to a solution, for a given set of active and reactive power loads. This may occur due to poor starting points, ill-conditioning problems or because the power flow equations have *no real* solution.

In our approach, system solvability problems are alleviated by calculating the minimum load shedding in order to bring solvability to an otherwise unsolvable power flow. In the process of computing the minimum load shedding, an OPF is solved by a direct interior point (IP) method based on the primal-dual logarithm barrier algorithm [23].

In the application of interior point methods to OPF two basic strategies are generally reported in the literature. The first one is based on a load flow-optimization scheme where the interior point algorithm is applied to the resulting linear or quadratic programming problem obtained from the linearization of the power flow equations at the solution of the load flow algorithm [26-27]. The second strategy, called *direct interior point* method, consists in applying the interior point method to the original nonlinear programming problem which is the OPF [23]. This second strategy, which will be adopted here, is more adequate for our purpose because it does not depend on the convergence of any load flow algorithm - in its iterative scheme the power flow equations are only required to be attained at the optimal solution. Also, numerical experiences have shown that direct interior point methods are very effective in dealing with large scale illconditioned and voltage problem networks [14,23].

Also, in this OPF formulation is possible to define a set of objective functions which are tremendously import in a competitive environment, such as:

- minimum load shedding;
- minimum active generation costs;
- minimum reactive power injection;
- maximum active power injection;
- maximum simultaneous transfer capability (bus to bus, bus to area, area to bus, area to area);
- maximum wheeling transaction;
- maximum system loadability.

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All these objective functions can be used inside a probabilistic framework, using both successive enumeration or Monte Carlo simulation. In this case, each selected contingency is analyzed by the IP OPF using the selected objective function. To improve computational efficiency, variance reduction techniques can be used in the Monte Carlo simulation scheme.

Finally, sensitivities of the probabilistic indicators with respect to system reinforcement as well as with respect to equipment failure parameters can also be obtained.

All these features were incorporated in a computational program, which was developed by CEPEL in close cooperation with the Brazilian utilities through the Multi-Utility Reliability Working Group (SGC/GCPS). It has been the official program of the 10-year Transmission Planning Working Groups (GTPD/GCPS).

This paper describes the adopted approach to calculate probabilistic indicators of power system performance by combining the nonlinear OPF solved by the IP algorithm and the Monte Carlo simulation method. Applications of the approach to a 1600-bus network derived from the Brazilian South/Southeast/Central West system Brazilian system are also presented and discussed.

OUTLINE OF THE INTERIOR POINT OPF

To achieve a reasonable accuracy in estimating the probabilistic indices we may have to perform a great number of system states, including combination of generator and circuit outages, and load uncertainty. Therefore, in the contingency analysis process, specially in dealing with heavily stressed systems, there may be situations where the Newton-Raphson algorithm does not converge to a solution, for a given set of active and reactive power loads. This may occur due to poor starting points, ill-conditioning problems or because the power flow equations have *no real* solution.

Mathematical Formulation

In the adopted approach, system solvability problems are alleviated by calculating the minimum load shedding in order to bring solvability to an otherwise unsolvable power flow. In the process of computing the minimum load shedding, an OPF is solved by a direct interior point (IP) method. The mathematical formulation of the minimum load shedding problem is:

$$Min P^{t} \theta$$
(1)

s.t.

$$(1-\theta_i)P_i - P_i(x) = 0, i = 1, ..., N$$
 (1.1)

$$(1-\theta_i)Q_i - Q_i(x) = 0, i = 1, ..., N$$
 (1.2)

$$\mathbf{a} \le (\mathbf{\theta}, \mathbf{x}) \le \mathbf{b} \tag{1.3}$$

where:

 P_i , Q_i are active and reactive loads at bus i, i =1, ..., N;

- N is the number of buses;
- θ is a vector which represents the fraction of load curtailed in each bus;
- P is the vector of active power loads;
- x is a vector which represents the power flow control and state variables.

In problem (1), equations (1.1), (1.2) represent the active and reactive power flow balance equations at bus i, i = 1, ..., N, and

(1.3) bounds on variables. For instance, each component of θ should be greater or equal to zero and less or equal to one. Note that if for a given vector of active/reactive loads the power flow is solvable, in the optimal solution of problem (1) $\theta_i = 0, i = 1, \ldots N$. All control variables may be fixed in the optimization but if control optimization is allowed the corresponding variables should be within bounds. Problem (1) neglects voltage limits for load buses, circuit flow limits or any other operating constraint.

Solution Algorithm

Letting $z=(\theta, x, y)$, problem (1) can be stated in a more general form as:

$$Min g(z) (2)$$

s.t.

$$h(z) = 0$$
 (2.1)

$$a \le z \le b \tag{2.2}$$

where:

$$g(z) = P^{t}\theta;$$

h(z) = 0 represents constraints (1.1), (1.2) of problem (1);

 $g(z) = P^t \theta$ in (1).

The general problem (2) will be solved by an interior point method based on the primal-dual logarithm barrier algorithm [23]. In the application of interior point methods to OPF two basic strategies are generally reported in the literature. The first one is based on a load flow-optimization scheme where the interior point algorithm is applied to the resulting linear or quadratic programming problem obtained from the linearization of the power flow equations at the solution of the load flow algorithm [26-27]. The second strategy, called direct interior point method, consists in applying the interior point method to the original nonlinear programming problem which is the OPF [23]. This second strategy, which will be adopted here, is more adequate for our purpose because it does not depend on the convergence of any load flow algorithm - in its iterative scheme the power flow equations are only required to be attained at the optimal solution. Also, numerical experiences have shown that direct interior point methods are very effective in dealing with large scale illconditioned and voltage problem networks [14,23].

The first step in the application of the primal-dual algorithm to problem (2) is to incorporate constraints (2.2) to a logarithmic barrier function:

$$Min \quad \left\{ g(z) - \mu \sum_{j} \log(z_{j} - a_{j}) - \mu \sum_{j} \log(b_{j} - z_{j}) \right\}$$
(3)

s.t.

$$h(z) = 0$$
 (3.1)

where $\boldsymbol{\mu}$ is the barrier parameter.

The basic idea of the algorithm is to solve approximately problem (3) for each value of μ and force μ go to zero; at the limit, the optimal solution of problem (2) is obtained. For each value of μ one iteration of the Newton-Raphson algorithm is applied to the nonlinear system of equations derived from the optimality conditions of problem (3). A crucial point in the method is the control of the primal and dual variables in its iterative process.

In the next sections we will describe the use of this Interior Point based OPF in reliability e simultaneous transfer capability analysis.

RELIABILITY ASSESMENT

As we have seen, after a contingency occurrence the system can present solvability problems. However, after restoring system solvability, it may remain some operating constraints violations such as bus voltage deviations and circuit overloads. In this case, the adequacy analysis of each selected system state can be carried out in two steps. In the first, the previous OPF formulation is used to compute the minimum load shedding to restore system solvability, neglecting operational constraints such as bus voltage levels and circuit power flows. In the second one, the additional minimum load curtailment to alleviate any operating limit violations is calculated also using the IP algorithm, and related reliability indices are evaluated. Observe that this approach can be used in both enumeration and Monte Carlo simulation methods.

Calculation of the Minimum Load Shedding Due to Operational Constraints

The minimum additional load shedding to restore system feasibility is a standard OPF problem and can be stated as:

Min
$$P^{at}\theta$$
 (4)

s.t.

$$(1 - \theta_i)P_i^a - P_i(x) = 0, i = 1, ...N$$
 (4.1)

$$(1 - \theta_i)Q_i^a - Q_i(x) = 0, i = 1, ...N$$
 (4.2)

$$\mathbf{f}(\mathbf{x}) \le \mathbf{0} \tag{4.3}$$

$$\mathbf{a} \le (\mathbf{\theta}, \mathbf{x}) \le \mathbf{b} \tag{4.4}$$

where:

- P_i^a , Q_i^a are active and reactive remaining loads (at the optimal solution of problem (1)) at bus i, i = 1, ..., N;
- f(x) represents functional constraints (line flow limits or any other operating constraints);

In addition to the constraints in (1), problem (4) takes into account voltage limits for all load buses, line flow limits and any other operating constraints.

Conceptual Definition of Reliability Indices

The evaluation of probabilistic indices is equivalent to calculating the expected value of a given test function [28]:

$$E(F) = \sum_{x \in X} F(x) P(x)$$
(5)

where:

- x vector representing the *system state*; each component in x represents the state of a system element (e.g. generators, circuits or loads);
- *X* state space, i.e. the set of all possible states *x* arising from combinations of component states;
- P(x) probability of state x;
- F(x) *test function*; its objective is to verify whether the operating point resulting from that specific configuration of generators,

circuits and loads belongs to the unsolvable, infeasible or feasible regions.

As we will see in the next sections, different test functions will result in different indices.

Voltage Collapse Reliability Indices

As stated before, voltage collapse problems are closely related with system solvability [17-22]. Thus, we can define a set of probabilistic indices associated with solvability analysis. The first one is the *probability of unsolvable cases* (PUC), related to those contingencies where the *traditional* load flow algorithm does not converge. For those cases, the test function F(x) in (5) is equal to one; otherwise it is equal to zero.

Using the IP model, a subset of these contingencies has solvability restored without load shedding, whereas for the other subset load curtailment is needed. Therefore we can define three other indices, the *probability of load curtailment to restore solvability* (PLCRS), the *frequency of load curtailment to restore solvability* (FLCRS) and the *expected load curtailment to restore solvability* (ELCRS). For the PLCRS index, the test function F(x) is equal to one for those cases where there is load curtailment; otherwise, F(x) is equal to zero. In turn, for the FLCRS and ELCRS indices, F(x) is respectively the incremental transition rate [29] and the amount of load curtailed to restore solvability associated with the state x. Observe that the difference between the PUC and PLCRS indices gives a measure of the effectiveness of the IP formulation in restoring system solvability retaining total load, including possible control actions.

Additionally, we can compute the probability distribution of such load shedding, from which we can estimate, for example, the probability of having a load curtailment greater or equal to a specific value.

Adequacy Reliability Indices

The primary objective of an adequacy analysis is to quantify, after a contingency occurrence, the *total* amount of load shedding required to move a system state from an unsolvable or infeasible region to a feasible region. In this sense, a reliability evaluation program usually produces the following basic indices: the loss of load probability (LOLP), the expected power (or energy) not supplied (EPNS) and the loss of load frequency (LOLF). The LOLP index corresponds to the expected value of an *indicator function* F(x), where F(x) = 1 if x is a *failure state* (i.e. if there is load curtailment due to solvability problems, islanding or operating violations in that state); otherwise, F(x) = 0 [28]. In turn, for the LOLF and EPNS indices, F(x) is respectively the incremental transition rate and the total load curtailment associated with the state x.

Observe that this total amount of load shedding can be originated by islanding, voltage collapse and operational constraint problems. Therefore we can additionally calculate the *expected load curtailment due to islanding problems* (ELCIP) and the *expected load curtailment due to operational constraints* (ELCOC). In other words, the EPNS (or EENS) index can be expressed as:

$$EPNS = ELCIP + ELCRS + ELCOC$$
(6)

The system problems probability (SPP), the loss of load expectation (LOLE) and the expected energy not supplied (EENS) can be directly obtained from the previous expressions.

Basic Reliability Algorithm

The proposed algorithm will provide the calculation of annualized indices, i.e., conditioned to each load *scenario*, or annual indices, i.e., integrated over the load scenarios. One *scenario* is

characterized by the base system configuration, including the following elements: system topology, equipments and load level. Associated to each scenario, there is a set of generation dispatches which, together with the voltage profile, define a set of *operating points*. Note that, in contrast to thermal-dominated systems, in which the operating point is associated to the economic fuel dispatch, there may be several hourly dispatches in a hydro-dominated system. Therefore, a *system base case*, which corresponds to an adjusted power flow solution, should be associated with each scenario.

The basic algorithm of the proposed model is composed of the following steps:

- 1. Set up the scenarios and associated base cases, i.e., system configurations and load levels.
- 2. Select one scenario (and base case), by either successive enumeration or stratified sampling.
- 3. Select a system state, i.e., define equipment availability. The selection is carried out either by successive enumeration of system states, based on their severity/likelihood, or by Monte Carlo sampling of equipment availability from their respective probability distributions.
- 4. Implement the set of changes and adjustments associated with the selected contingency, including: network reconfiguration (switching); identification of electric disconnection in the transmission network (islanding); adjustment for islanding, i.e., definition of new slack buses, removal of isolated buses, generation/demand balance per island; automatic generation control; and load shedding due to insufficiency of generation. Update the estimate of the ELCIP index.
- 5. Run a *traditional* Newton-Raphson AC power flow for the selected state. If the algorithm converges to a solution, go to step (7); otherwise, update the PUC index estimate and go to step (6).
- 6. Apply the described IP algorithm to restore solvability, using the base case condition as the starting operating point. If the system solvability is restored *only with* load shedding, update the estimates of the PLCRS, FLCRS and ELCRS indices.
- 7. Check the feasibility of the selected state, i.e., verify operating limit violations in the system, such as circuit overloads and bus voltage violations, based on pre-specified criteria. In case of violations go to step (8); otherwise go to step (9).
- 8. Apply again the described IP algorithm to achieve a feasible operating point by including the system operational constraints. If necessary take control actions such as generation rescheduling, bus voltage corrections, LTC tap changing and, as a last resort, load curtailment. Calculate the load curtailment due to operational constraints and update the ELCOC estimate.
- 9. Update the estimates of scenario adequacy reliability indices, such as LOLP, EPNS and F&D, based on the *total* load curtailment performed (computed in steps 4, 6 and 8). If the accuracy of all estimates is acceptable or the pre-specified sample size is reached (Monte Carlo option) or the pre-specified set of contingencies is exhausted (enumeration option), go to the next step; otherwise, go to step (3).
- 10 If the set of scenarios is not exhausted, go to step (2); otherwise, if the enumeration approach was selected, stop; if the Monte Carlo approach was selected, go to step (11).
- 11. Compute estimates and verify the accuracy of annual indices; if acceptable, stop; otherwise, if allowed by the sampling

budget, set up additional samplings for the scenarios according to a sampling plan, calculated by the model, and go to step (2).

In step 6, the IP formulation allows to observe the impact of each control optimization in system solvability. However, when all controls except the load curtailment are fixed, we can identify the false indications of unsolvable load flow cases. This occurs when using the IP model, a contingency has solvability restored *without* load curtailment, meaning that the power flow does have a solution, but the traditional load flow algorithm was not able to find it due to ill-conditioning in the Jacobian matrix.

Application to the BSSW System

The main features of the proposed approach will be illustrated in case studies with a configuration of the Brazilian South/Southeast/Central West (BSSW) system, planned for 1996. This a 1629-bus, 2597-circuit, 297-generator system, with an installed capacity is 46,268 MW for a peak load of 34,202 MW.

The composite reliability indices, including voltage collapse problems, were estimated through non-sequential Monte Carlo simulation, taking into account generation and transmission outages, and load uncertainty. In each study, it was chosen a sample size so as to obtain a coefficient of variation [28] of 4% for the EENS index. Also, the generator reactive limits were taken into account.

Peak Load Analysis

Initially, it was carried out a composite reliability assessment for the peak load condition, without taking into account load uncertainties, i.e., only generator and circuit outages were considered. It was used a sample size of 5,000 observations. In the solvability phase of the algorithm (step 6), all the controls in the IP algorithm, except the load shedding, were considered as fixed, whereas in the feasibility phase (step 8), all controls were allowed to be optimized. The annualized adequacy and voltage collapse reliability indices for this system are presented in Table 1, for the *no uncertainty* column.

Table 1 -	Peak	Load	Case	Study
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Index	No Uncertainty	Uncertainty
SPP (%)	44.60	45.4
PUC (%)	1.12	1.64
LOLP(%)	26.30	26.50
PLCRS (%)	0.98	1.26
LOLF (occ./year)	143.8	146.6
FLCRS (occ./year)	38.5	45.3
EENS (GWh/year)	70.74	76.42
ELCIP (GWh/year)	59.73	60.09
ELCRS (GWh/year)	1.75	6.74
ELCOC (GWh/year)	9.26	9.60

From this Table, we see that the impact of the PUC and PLCRS indices in the SPP and LOLP indices were relatively low. This is due to the representation of radial subtransmission networks in the BSSW system. In this way, the major impact in the adequacy indices comes from islanding problems, contributing for more than 85% of the SPP and LOLP indices. On the other hand, the voltage collapse frequency index (FLCRS) represents 27% of the adequacy frequency of failures (LOLF).

Now comparing the expected amount of load curtailments, we see that the islanding is still the major contribution (84%), followed by operational constraints (13%) and voltage collapse (3%) problems.

Comparing the SPP and LOLP indices we can see that the effectiveness of the remedial actions was 41% in this case.

As in this study the solvability problems were mitigated without control optimization, we can identify the false indications of voltage collapse problems, by comparing the PUC and PLCRS indices. From the initial probability of 1.12% of nonconvergent load flow cases, only in 0.98% of the cases load curtailment was required to restore system solvability. Therefore, in 0.14% of the cases, the power flow equations did have a solution, but the traditional load flow algorithm was not able to find it due to ill-conditioning in the Jacobian matrix. On the other hand, with the employment of a robust optimization method, as the IP algorithm with its augmented matrix, these false unsolvability indications were cleaned up.

This case study was carried out on a Digital Alpha Server 1000 Workstation. The execution times were 37 minutes. From this total time, 54% was spent in 760 contingencies solved by the IP algorithm. The average execution time per IP solution was 1.65 seconds.

The Effects of Load Uncertainty

The probabilistic indices were also calculated considering an uncertainty of 1.5%, normally distributed around the mean value of the system peak load. Again, a sample size of 5,000 system states was used, arising from combinations of generator and circuit outages and load uncertainty. In the solvability phase, the only control action allowed to the IP algorithm was the load curtailment. The results are also listed in Table 1, for the *uncertainty* column.

As we could expect, with the introduction of load uncertainties, the reliability indices have increased. However, the variation on the adequacy indices was provoked by increasing of the voltage collapse related indices. Hence, the growth of the SPP, LOLP and LOLF indices were less than 2% while of the PUC, PLCRS and FLCRS ranged from 18% to 46%. Comparing the PUC and PLCRS indices in each study, we see that the number of cases with false indications of voltage collapses increased from 0.14% to 0.38%.

The greatest change is related to the ELCRS index, which increased almost 4 times, from 1.75 to 6.74 GWh/year. This bulk sensitivity to load variations may indicate that there are areas in this system operating near to their loadability limits. The proposed algorithm can also identify these critical areas.

As we have seen, the BSSW system is islanding problems dominated. However it is interesting to further observe the severity of the load shedding when it occurs. With this objective, Figure 1 shows the probability distribution of the load curtailment in MW due to islanding, voltage collapse and operational constraints problems, *conditioned* to those cases where load shedding was implemented. From this Figure, we note that, contrasting with the unconditional case, the conditioned ELCIP index is no longer the largest one, because the range of load curtailment values due to islanding is comparatively tight. Conversely, the amount of load curtailments associated to collapse problems tends to be spreaded and quite larger than the other two failure modes. As a consequence, its conditioned mean value is the largest one. However, because of their probability of occurrence is relatively low, the unconditioned ELCRS is the smallest.



Figure 1 - Probability Distribution of Load Curtailments

MAXIMUM SIMULTANEOUS TRANSFER CAPABILITY ASSESSMENT

Again, due to possibility of solvability and operational constraints problems in the contingency analysis process, the STC assessment is carried out in two steps, for each selected system state. The first step is related with the alleviation of the potential operational violations in order to achieve a feasible operating point. Further, the STC is maximized starting from the feasible operating point obtained in the previous step. In both cases, the direct interior point OPF is used. Figure 2 illustrates the adopted procedure.

Calculation of Maximum STC

The simultaneous transfer capability problem consists in maximizing the net active power transfer from a given set of areas in the network to other areas. The problem can be formulated as:

$$\operatorname{Max}\sum_{(i,j) \in \Omega} P_{ij} \tag{7}$$

s.t.

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$$PL_{i} - P_{i}(x) = 0, i = 1, ..., N$$

$$QL_{i} - Q_{i}(x) = 0, i = 1, ..., N$$

$$(7.1)$$

$$(7.2)$$

$$\mathbf{x}) \le \mathbf{0} \tag{7.3}$$

$$\leq x \leq b$$
 (7.4)

where:

- PL_i, QL_i are active and reactive loads at bus i, i =1, ..., N and constraints (7.1), (7.2) represent active and reactive power balance in each bus;
- N is the number of buses;
- P_{ii} is the vector of active power flow in circuit (i,j);
- x is a vector which represents the power flow control and state variables;
- Ω is the set of tie lines connecting the areas from which net active power transfer is to be maximized to other areas;
- f(x) represents functional constraints (line flow limits or any other operating constraints).

When formulating problem above we assume that there is at least one operating point with no constraint violations and we do not allow any load shedding in order to maximize active power transfer.



Figure 2 - Proposed Approach

Probabilistic Maximum STC Approach

The feasibility and STC steps described previously, using the proposed Interior Points algorithm, can be recursively applied for each sampled contingency, to calculate the associated maximum STC. Therefore it can be defined a probabilistic index, the expected simultaneous transfer capability (ESTC), which corresponds the average STC over the system states. Clearly, for the ESTC index, the test function F(x) is equal to maximum STC associated with the state x. Moreover, it is possible to calculate the complete probability distribution of random variables such as STC, transmission margins, etc. This information is important to estimate the probability of having a STC greater or equal to a specific value. Also, as noted before in the optimal solution of the IP optimization model the dual variables provide important sensitivity information for planning and operation purposes. Again, we can compute expected values of these variables from the several analyzed system states.

Application to the BSSW System

The proposed approach was also applied to the Brazilian South/Southeast (BSSW) system, planned for 1996. Table 2 summarizes basic information for each system area.

Table 2 - Basic Information of BSSW System

System Areas	Peak Load Allocation (MW)	Installed Capacity (MW)
Furnas	52	7989
Itaipu	-	12,600
Central	1771	711
Minas	4525	5206
Rio	5225	1017
Sao Paulo	13,944	11,392
South	6401	7884
Total	31,920	46,799

Probabilistic MSTC from Furnas for Peak Load

The aim of probabilistic MSTC evaluation was to estimate the net export interchange from Furnas area, considering various system operating points, imposed by generator and circuit outages as well as load uncertainties. The estimates and distributions related to MSTC can be fairly computed through the Monte Carlo simulation scheme. Moreover, due to the number of system components (297 generating units plus 2597 circuits) it is only possible to obtain accurate estimates through this simulation technique.

The probabilistic evaluation of MSTC from Furnas on peak load conditions was achieved considering an uncertainty of 2% normally distributed around peak value. It was used a sample size of 1,000 observations, and the expected MSTC from Furnas area was 7171 MW with an uncertainty $\beta = 0.21$ %. Table 3 presents the minimum, maximum and mean of resulting interchanges with neighbor areas from probabilistic assessment of MSTC from Furnas.

From Furnas to	Interchanges (MW)		
other Areas	Min.	Mean	Max.
Itaipu	-10,008	-9212	-5745
Central	1095	1264	1572
Minas	595	2612	3035
Rio	4079	4485	4821
Sao Paulo	2651	5631	7821
South	-1105	1201	1667

Table 4 provides the basic statistics calculated for probabilistic MSTC study. Figure 3 shows the probability distribution (cumulative and individual) of MSTC using the proposed optimization model. The resulting information from probability distribution provides a general framework for probabilistic analysis of MSTC.

Table 4 - Basic Statistics of MSTC Distribution

Index	Value (MW)	
Mean	7171	
Median	7276	
Mode	7143	
Standard Deviation	472	
Minimum	3196	
Maximum	7728	
Lower Quartile	7042	
Upper Quartile	7440	



Figure 3 - Probability Distribution of MSTC from Furnas

CONCLUSIONS

This paper described an approach to calculate probabilistic indicators of power system performance by combining the non linear OPF solved by the IP algorithm and the Monte Carlo simulation method.

The system solvability problems, associated to the contingency analysis process, were alleviated by calculating the minimum load shedding in order to bring solvability to an otherwise unsolvable power flow. In the considered approach, the process of computing the minimum load shedding was carried out by an OPF solved by a direct interior point (IP) method based on the primal-dual logarithm barrier algorithm.

In this OPF formulation is possible to define a set of objective functions which are tremendously import in a competitive environment, such as: minimum load shedding, minimum active generation costs, minimum reactive power injection, maximum active power injection; maximum simultaneous transfer capability (bus to bus, bus to area, area to bus, area to area), maximum wheeling transaction, maximum system loadability, etc. All these objective functions can be used inside a probabilistic framework, using both successive enumeration or Monte Carlo simulation.

Applications to a 1600-bus network derived from the Brazilian South/Southeast/Central West system Brazilian system were presented and discussed.

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