APPLICATION OF STOCHASTIC DUAL DP AND EXTENSIONS TO HYDROTHERMAL SCHEDULING

PSRI Technical Report 012/99

M.Pereira N.Campodónico R.Kelman
PSRI

Version 2.0
April 1999

Index

1. INTRODUCTION

This report presents an overview of the stochastic dual DP scheme and its applications to hydrothermal scheduling, including extensions for fuel cost uncertainty and revenue maximization.

2. MODELING OF SYSTEM COMPONENTS

2.1 Thermal plants

In purely thermal systems, the operating cost of each plant depends basically on its fuel cost. Therefore, thermal plants are represented by their unit operating cost $\{c_j, j = 1, \ldots J\}$ ($/MWh) and their generation limits:

$$g_{tj} \le \overline{g}_j \qquad \text{for } j = 1, \ldots, J \tag{2.1}$$

where:

j indexes thermal plants (J number of plants)

g_{tj} energy production of plant j in stage t (MWh)

$\overline{g}_j$ maximum generation capacity of plant j

2.2 Hydro plants

Figure 2.1 shows the schematic diagram of a hydro plant.

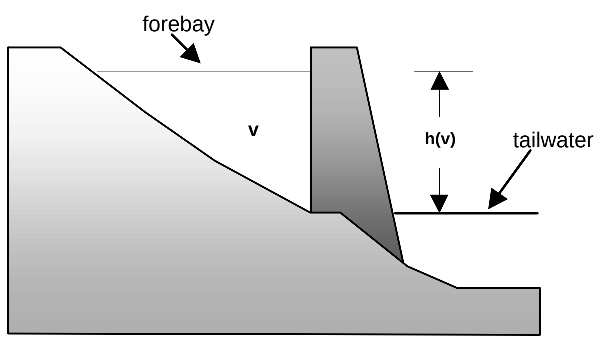


Figure 2.1 - Hydro Plant with Reservoir

Plant operation is modeled through the following equations:

2.2.1 Water Balance

Represents the coupling between successive stages, as illustrated in Figure 2.2: the reservoir storage at the end of stage t (beginning of stage t+1) is equal to initial storage minus outflow volumes (turbined and spilled) plus inflow volumes (lateral inflow plus releases from upstream plants):

$$v_{t+1}(i) = v_t(i) - u_t(i) - s_t(i) + a_t(i) + \sum_{m \in U(i)} [u_t(m) + s_t(m)] \quad \text{for } i = 1,\ldots, I \tag{2.2}$$

where:

i indexes hydro plants (I number of hydro plants)

$v_{t+1}(i)$ stored volume in plant i at the end of stage t (decision variable)

$v_t(i)$ stored volume in plant i at the beginning of stage t (known value)
$a_t(i)$ lateral streamflow arriving at plant i in stage t (known value)
$u_t(i)$ turbined outflow volume in plant i during stage t (decision variable)
$s_t(i)$ spilled outflow during stage t (decision variable)
$m \in U(i)$ set of plants immediately upstream of plant i

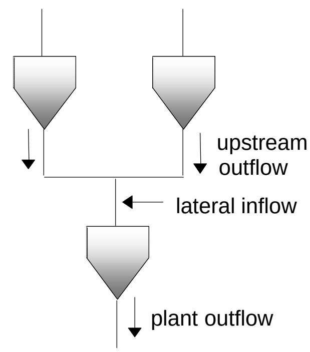


Figure 2.2 - Reservoir Water Balance

2.2.2 Limits on storage and outflow

$$v_t(i) \leq \overline{v}(i) \qquad \text{for } i = 1, ..., I \tag{2.3}$$

$$u_t(i) \leq \overline{u}(i) \qquad \text{for } i = 1, ..., I \tag{2.4}$$

where $\overline{v}(i)$ and $\overline{u}(i)$ are respectively the maximum storage and turbine capacity.

2.2.3 Energy Production

Hydro plants convert the potential energy of stored water into kinetic energy, which is used to rotate turbines coupled to an electric generator. The power production resulting from the release of u m^3 through the turbines is given by:

$$g_h = \rho(v) \times u \tag{2.5}$$

where g_h is the energy generated (MWh) and $\rho(v)$ is the plant *production coefficient* (MWh/m^3):

$$\rho(v) = \eta \times \phi \times \gamma \times h(v) \tag{2.6}$$

where:
η efficiency of the turbine/generator set
ϕ specific mass of water (kg/m^3)
γ gravity factor (m/s^2)
$h(v)$ net head (m) - difference between forebay and tailwater levels - see Figure 2.1

3. THE HYDROTHERMAL DISPATCH PROBLEM

3.1 Problem Characteristics

The objective of hydrothermal scheduling is to determine the sequence of hydro releases which will minimize the expected thermal operation cost (given by fuel cost plus penalties for rationing) along the planning horizon. This problem can be represented as a decision tree, as illustrated in Figure 3.1.

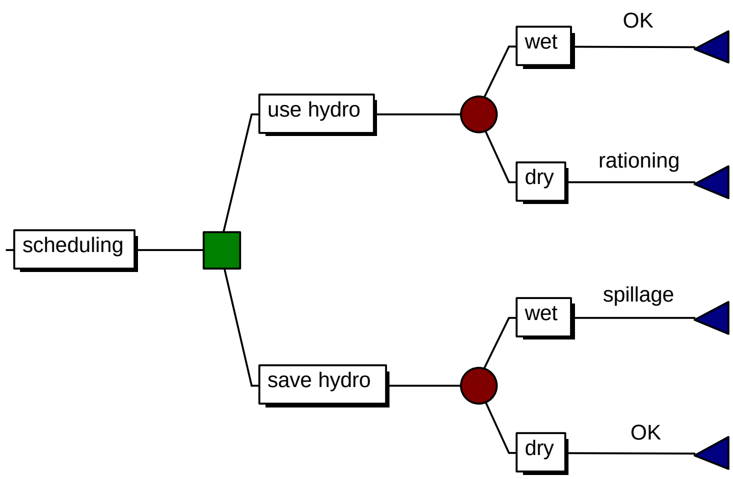


Figure 3.1 - Decision Process for Hydrothermal Systems

As seen in the picture, the operator is faced with the options of using hydro today, and therefore avoiding complementary thermal costs, or storing the hydro energy for use in the next period. If hydro energy is used today, and future inflows are high - thus allowing the recovery of reservoir storage - system operation will result to be efficient.
However, if a drought occurs, it may be necessary to use more expensive thermal generation in the future, or even interrupt load supply.

If, on the other hand, storage levels are kept high through a more intensive use of thermal generation today, and high inflows occur in the future, reservoirs may spill, which is a waste of energy and, therefore, results in increased operation costs. Finally, if a dry period occurs, the storage will be used to displace expensive thermal or rationing in the future.

3.2 Problem Formulation

The stochastic hydrothermal scheduling formulation will be illustrated for one hydro plant and the three-stage inflow tree of Figure 3.1.

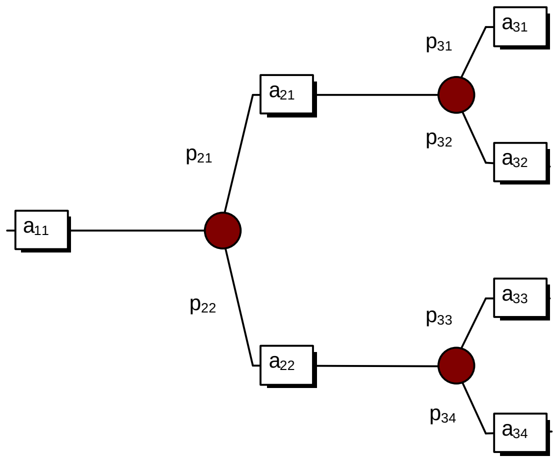


Figure 3.1 - Inflow Scenario Tree

where:

a_{ts} inflow in stage t, scenario s

p_{ts} conditioned probability of inflow scenario $\{t,s\}$

The stochastic scheduling problem is formulated as:

$$\text{Min } c_1(u_{11}) + p_{21}\,[c_2(u_{21}) + p_{31}c_3(u_{31}) + p_{32}c_3(u_{32})] + p_{22}\,[c_2(u_{22}) + p_{33}c_3(u_{33}) + p_{34}c_3(u_{34})]$$

subject to (3.1)

(a) water balance constraints

$$\begin{aligned}
v_{21} &= v_{11} - u_{11} - s_{11} + a_{11}\\
v_{31} &= v_{21} - u_{21} - s_{21} + a_{21}\\
v_{41} &= v_{31} - u_{31} - s_{31} + a_{31}\\
v_{42} &= v_{31} - u_{32} - s_{32} + a_{32}\\
v_{32} &= v_{21} - u_{22} - s_{22} + a_{22}\\
v_{43} &= v_{32} - u_{33} - s_{33} + a_{33}\\
v_{44} &= v_{32} - u_{34} - s_{34} + a_{34}
\end{aligned}$$

(b) constraints on storage and outflow

$v_{t+1,s} \le \overline{v}; u_{ts} \le \overline{u}$ for all stages t; all scenarios s

where:

u_{ts} hydro scheduling decision (turbined outflow) in stage t, scenario s

$c_t(u_{ts})$ thermal generation cost required to complement the hydro scheduling decision

$v_{t+1,s}$ reservoir storage at the end of stage t, scenario s

s_{ts} spilled outflow in stage t, scenario s

The thermal complement function $c_t(u_{ts})$ is implicitly represented as the solution of the following LP problem:

$$c_t(u_{ts}) = \text{Min} \sum_{j=1}^{J} c_j \, g_{tj}$$

$$\text{subject to} \tag{3.2}$$

$$\sum_{j=1}^{J} g_{tj} = d_t - \rho \, u_{ts} \tag{3.2a}$$

$$g_{tj} \leq \overline{g}_j \qquad \text{for } j = 1, \ldots, J \tag{3.2b}$$

where:

j indexes thermal plants (J number of plants)

c_j unit operating cost of plant j

g_{tj} energy production of plant j

d_t load in stage t

ρ hydro plant production coefficient (assumed to be constant in this example)

Problem (3.1) can in principle be solved by linear programming (LP) algorithms. However, the actual scheduling problem involves several hydro plants and, in many cases, a planning horizon of several years. Due to the exponential increase of inflow branches with time, the resulting stochastic optimization problem quickly becomes computationally infeasible. This has motivated the development of solution approaches based on a state-space formulation, described next.

4. STATE SPACE SOLUTION APPROACH

4.1 Immediate and Future Costs

As shown in Figure 4.1, the scheduling problem is decomposed into several one-stage subproblems, where the objective is to minimize the sum of *immediate* and *future* operating costs

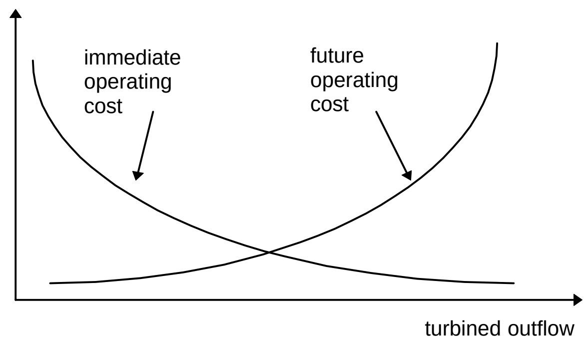


Figure 4.1 - Immediate and Future Costs versus Final Storage

The immediate cost function - ICF - corresponds to the thermal complement function $c_t(u_t)$ discussed in the previous section. We see in the Figure that, as hydro outflow increases, less thermal generation is needed, and the immediate cost decreases. In turn, the future cost function - FCF – reflects the *expected* thermal generation expenses from stage t+1 to the end of the planning period. We see that the FCF increases with the turbined outflow, as less water will be available in the future to displace thermal generation.

4.2 One-Stage Hydrothermal Dispatch

Given the initial storage v_t and the future cost function $\alpha_{t+1}(v_{t+1})$, the one-stage hydrothermal scheduling problem is formulated as:

$$
\begin{aligned}
z_t = \; & \text{Min } c_t(u_t) + \alpha_{t+1}(v_{t+1}) \\
& \text{subject to} \\
& \quad v_{t+1} = v_t - u_t - s_t + a_t \\
& \quad v_{t+1} \le \overline{v} \\
& \quad u_t \le \overline{u}
\end{aligned}
\tag{4.1}
$$

In contrast with the very large stochastic LP problem (3.1), the one-stage problem (4.1) can be easily solved by standard LP schemes.

4.3 Future Cost Function

4.3.1 Enumeration of all future states

The future cost function calculation is naturally the key aspect of the state-space scheme. In theory, $\alpha_{t+1}(v_{t+1})$ could be calculated by *simulating* system operation in the future for different starting values of initial storage and calculating the operating costs, as illustrated in Figure 4.2.

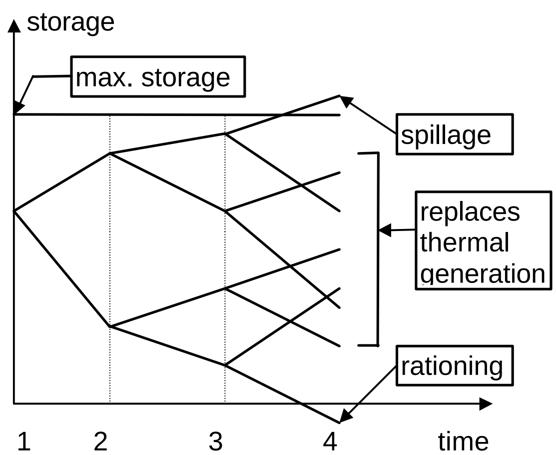


Figure 4.2 - "Brute Force" FCF Calculation

However, this "brute force" approach has the same computational drawbacks as the explicit stochastic formulation. Therefore, the future cost function in each stage is calculated through a more efficient *stochastic dynamic programming* (SDP) recursion:

4.3.1 SDP Recursion

a) for each stage *t* (typically a week or month) define a set of *system states* indexed by *m* = 1, ..., M, for example, reservoir levels at 100%, 90%, etc. until 0%. Figure 4.3 illustrates the system state definition for a single reservoir. Note that the initial state (i.e. storage levels at the beginning of the first stage) is assumed to be known.

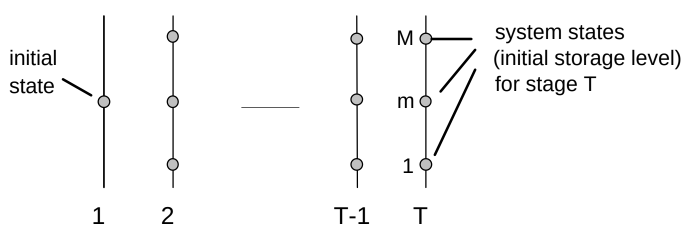


Figure 4.3 - Definition of System States

b) start with the *last* stage, T, and solve the *one-stage hydrothermal dispatch problem* (4.1) assuming that the initial reservoir storage corresponds to the first storage level selected in step (a) - for example, 100%. Because we are at the last stage, assume that the future cost function is zero. Also, because of inflow uncertainty, the hydro scheduling problem is successively solved for K different inflow *scenarios*, i.e. different possible values for inflows in that stage. The procedure is illustrated in Figure 4.4.

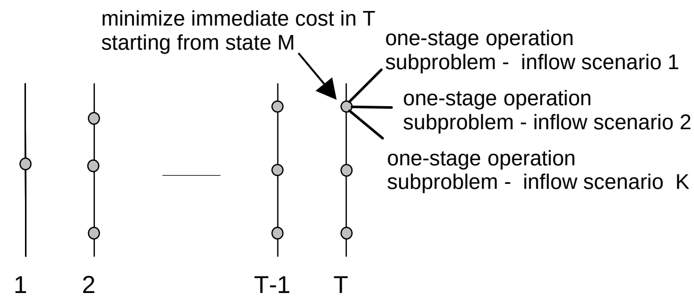


Figure 4.4 - Optimal Strategy Calculation - Last Stage

c) Calculate the expected operation cost associated to storage level 100% as the mean of the K one-stage subproblem costs. This will be the first point of the expected future cost function for stage T-1, i.e. $\alpha_T(v_T)$. Repeat the calculation of expected operation costs for the remaining states in stage T. Interpolate the costs between calculated stages, and produce the FCF $\alpha_T(v_T)$ for stage T-1, as illustrated in Figure 4.5.

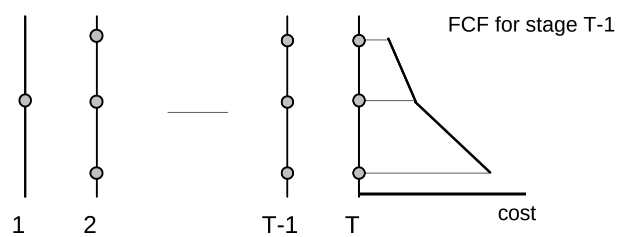


Figure 4.5 - Calculation of the FCF for Stage T-1

d) The process is then repeated for all selected states in stage T-1, T-2 etc. as illustrated in Figure 4.6. Note that the objective is now to minimize the immediate operation cost in stage T-1 plus the expected future cost, given by the previously calculated FCF.

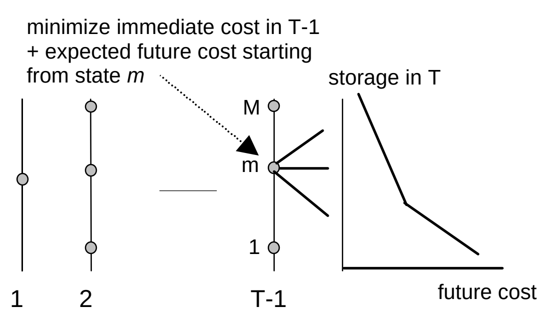


Figure 4.6- Calculation of Operation Costs for Stage T-1 and FCF for stage T-2

4.3.2 Implementation of the SDP Scheme

initialize the end-of-horizon future cost function $\alpha_{T+1}(v_T) \leftarrow 0$
for t = T, T-1, ..., 1

for each storage value $v_t = v_t^1, ..., v_t^m, ..., v_t^M$

for each inflow scenario $a_t = a_t^1, ..., a_t^k, ..., a_t^K$

solve the one-stage problem (4.1) for initial storage v_t^m and inflow a_t^k:

$$\alpha_t^k(v_t^m) = \text{Min } c_t(u_t) + \alpha_{t+1}(v_{t+1}) \quad \text{subject to} \quad v_{t+1} = v_t^m - u_t - s_t + a_t^k, \quad v_{t+1} \le \bar{v}, \quad u_t \le \bar{u} \tag{4.2}$$

next

calculate the expected operation cost over all inflow scenarios:

$$\alpha_t(v_t^m) = \sum_{k=1}^{K} p_k \times \alpha_t^k(v_t^m)$$

next

create a complete future cost function $\alpha_t(v_t)$ for the previous stage by interpolation on the discrete values $\{\alpha_t(v_t^m), m = 1, ..., M\}$

next

4.4 Representation of Serial Correlation by Markov Chains

Most inflow sequences present serial correlation, i.e. if the inflow in the past month was "drier" than average, there is a tendency for the following inflows to be drier as well. This is due to the "capacitor" characteristics of underground aquifers, where the rate of release depends on the amount of water absorbed in the past. This correlation may be represented by a Markov chain, where p_{kl} represents the transition probability from inflow a_t^k in stage t to inflow a_{t+1}^l in stage t+1:

t+1→ t↓	**a_{t+1}^1**	**...**	**a_{t+1}^l**	**...**	**a_{t+1}^L**
a_t^1	p_{11}	…	p_{1l}	…	p_{1L}
...	…	…	…	…	…
a_t^k	p_{k1}	…	p_{kl}	…	p_{kL}
...	…	…		…	
a_t^K	p_{K1}	…	p_{Kl}	…	p_{KL}

The future cost function in the SDP scheme now has two state variables: storage at the beginning of stage t and inflow along the stage. The procedure is implemented as follows:

initialize the end-of-horizon future cost function $\alpha_{T+1}(v_T,a_T) \leftarrow 0$
for t = T, T-1, ..., 1

for each storage value $v_t = v_t^1, ..., v_t^m, ..., v_t^M$

for each inflow scenario $a_t = a_t^1, ..., a_t^k, ..., a_t^K$

solve the one-stage problem for initial storage v_t^m and inflow a_t^k as the minimization of immediate cost plus the *expected value* of future costs over all transitions from a_t^k to $\{a_{t+1}^l\}$ in the next stage:

$$\alpha_t(v_t^m,a_t^k) = \text{Min } c_t(u_t) + \sum_{l=1}^{L} p_{kl} \times \alpha_{t+1}(v_{t+1}^k, a_{t+1}^l)$$

$$\text{subject to} \tag{4.3}$$

$$v_{t+1}^k = v_t^m - u_t - s_t + a_t^k$$

$$v_{t+1}^k \le \overline{v};\; u_t \le \overline{u}$$

next

next

create a complete future cost function $\alpha_t(v_t,a_t)$ for the previous stage by interpolation over the values $\{\alpha_t(v_t^m,a_t^k),\ m = 1, ..., M;\ k = 1, ..., K\}$

next

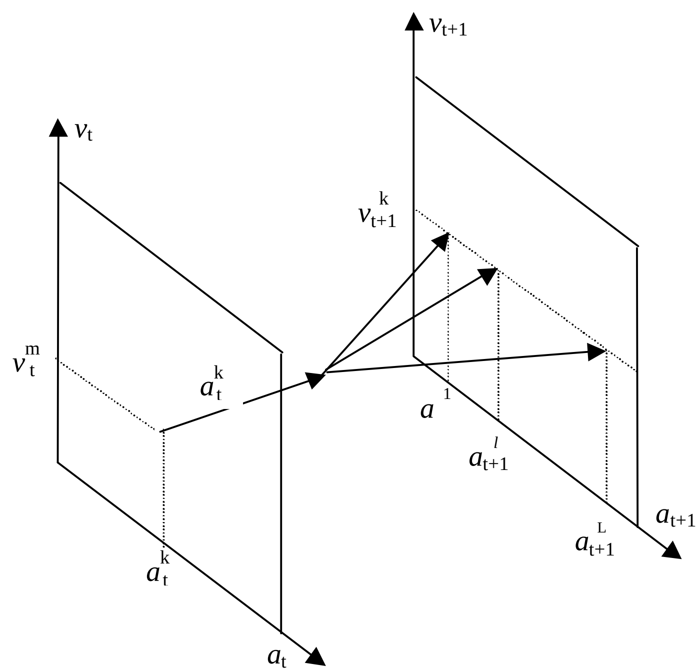


Figure 4.7 – SDP with a_t as a state variable

4.5 Alternative Representation of Serial Correlation

An alternative – and mathematically equivalent - way to represent serial correlations is to use as a state variable the inflow in the *previous* stage, a_{t-1}. The procedure is then:

initialize the end-of-horizon future cost function $\alpha_{T+1}(v_T, a_{T-1}) \leftarrow 0$
for t = T, T-1, ..., 1
    for each storage value $v_t = v_t^1, ..., v_t^m, ..., v_t^M$
        for each *previous* inflow scenario $a_{t-1} = a_{t-1}^1, ..., a_{t-1}^k, ..., a_{t-1}^K$
            for each inflow in t conditioned to previous inflow a_{t-1}^k: $a_t = a_t^1, ..., a_t^l, ..., a_t^L$
                solve the dispatch for initial storage v_t^m and conditioned inflow a_t^l:

$$\begin{aligned} \alpha_t^l(v_t^m, a_{t-1}^k) = \; & \text{Min } c_t(u_t) + \alpha_{t+1}(v_{t+1}^l, a_t^l) \\ & \text{subject to} \\ & \quad v_{t+1}^l = v_t^m - u_t - s_t + a_t^l \\ & \quad v_{t+1}^l \le \bar{v};\; u_t \le \bar{u} \end{aligned} \tag{4.4}$$

            next
            calculate the expected operation cost over all inflow scenarios:

$$\alpha_t(v_t^m, a_{t-1}^k) = \sum_{l=1}^{L} p_{kl} \times \alpha_t^l(v_t^m, a_{t-1}^k)$$

        next
    next
    create a complete future cost function $\alpha_t(v_t, a_{t-1})$ for the previous stage by interpolation over the values $\{\alpha_t(v_t^m, a_{t-1}^k), m = 1, ..., M; k=1, ..., K\}$.
next

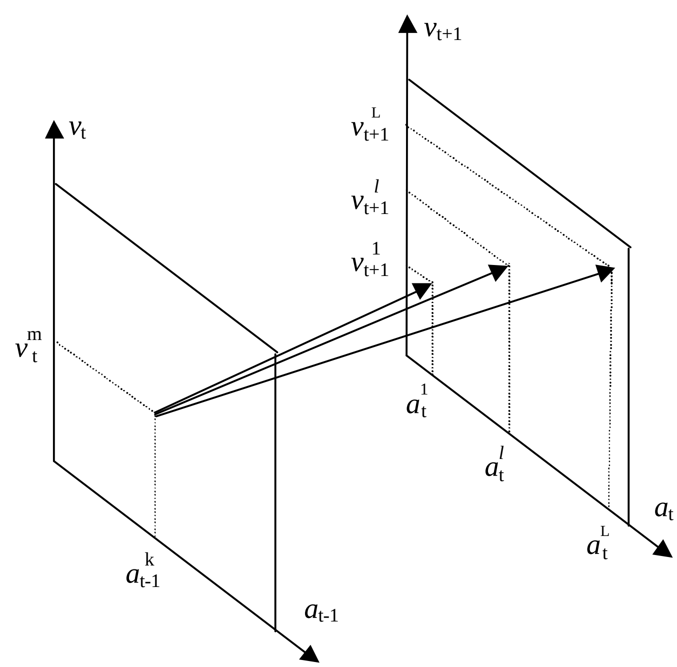


Figure 4.7 – SDP with a_{t-1} as a state variable

4.6 SDP Scheme Limitations

The SDP scheme is straightforward to implement and has been used for several years in most hydro-dominated countries (e.g. [2],[3]). However, as seen above, the SDP recursion requires the enumeration of all *combinations* of initial storage values and previous inflows. As a consequence, computational effort increases exponentially with the number of reservoirs, the well-known “curse of dimensionality” of dynamic programming. This is illustrated in the table below, which shows the number of combinations for different system sizes, assuming that that each of the state variables for reservoir storage and inflow is divided into 20 levels.

# of plants	# of states
1	$20^2 =$ 400
2	$20^4 =$ 160 thousand
3	$20^6 =$ 64 million
4	$20^8 \approx$ 25 billion
5	$20^{10} \approx$ 10 trillion

For this reason, it has become necessary to develop computationally feasible state-space schemes. The traditional approach, still adopted in many countries, has been to reduce system dimensionality by the aggregating system reservoirs into one reservoir that represents the energy production capability of the cascade [3]. This scheme is in some cases coupled with the use of partial dynamic programming schemes (typically, calculation of separate future cost functions for each basin) [4]-[7].

More recently, an approach based on the analytical representation of the future cost function, known as stochastic *dual* dynamic programming (SDDP) [8]-[10] has been applied in several countries in South and Central America, plus USA, New Zealand, Spain and Norway[1]. The SDDP scheme does not require discretization of the state space and, as a consequence, alleviates the computational requirements of the stochastic DP recursion.

[1] A related scheme, called constructive dynamic programming, has been applied to the Australian system [11].

5. THE DUAL DYNAMIC PROGRAMMING SCHEME

5.1 The SDDP algorithm

5.1.1 Piecewise Approximation of the Future Cost Function

The Dual DP scheme is based on the observation that the FCF can be represented as a piecewise linear function, i.e. there is no need to create an interpolated table. Furthermore, it is shown that the *slope* of the FCF around a given point can be analytically obtained from the one-stage dispatch problem (4.1). Figure 5.1 illustrates the Dual DP calculation of expected operation cost and FCF slope for the last stage, initial state = 100% (step (c) of the traditional DP procedure)

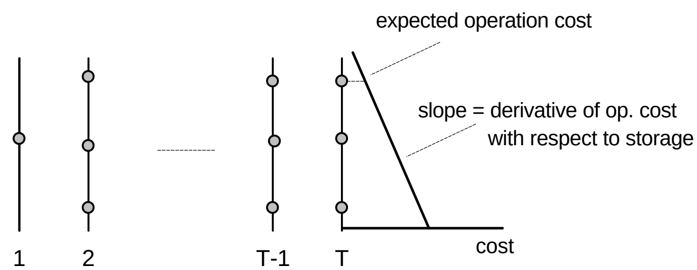


Figure 5.1 - Dual DP - Calculation of First FCF Segment

The last-stage dispatch problem is shown below (note that the future cost function in this stage, $\alpha_{T+1}(v_{T+1})$, is set to zero):

$$
\begin{array}{llr}
z_T = \text{Min } c_T(u_T) & \text{multipliers} & \\
\quad \text{subject to} & & (5.1) \\
\quad\quad v_{T+1} = v_T - u_T - s_T + a_T & \pi_h & \\
\quad\quad v_{T+1} \le \overline{v} & \pi_v & \\
\quad\quad u_T \le \overline{u} & \pi_u &
\end{array}
$$

It is well known from LP theory that there is a set of simplex multipliers associated to the constraints of problem (5.1) at the optimal solution. These multipliers represent the derivative of the optimal solution value (operation cost in this case) with respect to a perturbation in the constraint right-hand side. In particular, the multiplier associated to the water balance equation, π_h, represents the derivative of z_T with respect to a variation in initial storage v_T:

$$\pi_h = \partial z_T / \partial v_T \qquad (5.2)$$

We see in Figure 5.1 that expression (5.2) corresponds to the slope of the future cost function for stage T-1. The linear segment can also be interpreted as a (linear) series expansion of the FCF around the initial storage v_T.

Figure 5.2 shows the calculation of operation cost and FCF slopes for each state in stage T. We see that the FCF $\alpha_T(v_T)$ for stage T-1 corresponds to the *piecewise* cost surface produced by taking the linear segment with the highest cost value in each state (convex hull).

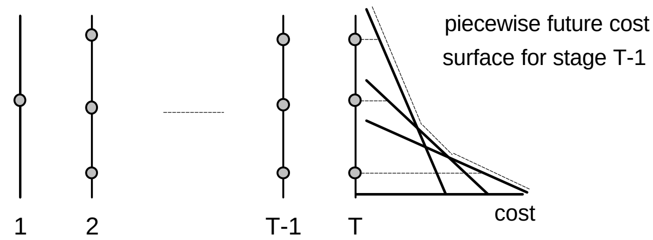


Figure 5.2 - Calculation of a Piecewise FCF for Stage T-1

The hydrothermal dispatch for the previous stage T-1 is represented as a LP problem:

$$
\begin{aligned}
\alpha_{T-1}(v_{T-1}) = \quad & \text{Min } c_{T-1}(u_{T-1}) + \alpha_T \\
& \text{subject to} \\
& \quad v_T = v_{T-1} - u_{T-1} - s_{T-1} + a_{T-1} \\
& \quad v_T \le \overline{v} \\
& \quad u_{T-1} \le \overline{u} \\
& \quad \alpha_T \ge \varphi_T^n v_T + \delta_T^n \qquad \text{for } n = 1, ..., N
\end{aligned}
\qquad (5.3)
$$

The future cost function is represented by the scalar variable α_T and N linear constraints $\{\alpha_T \ge \varphi_T^n v_T + \delta_T^n\}$, where N is the number of linear segments. As shown in Figure 5.3, the inequalities $\{\alpha_T \ge ...\}$ represent the piecewise characteristic of this function (for any v_T, the segment with the highest value $\varphi_T^n v_T + \delta_T^n$ will always be binding).

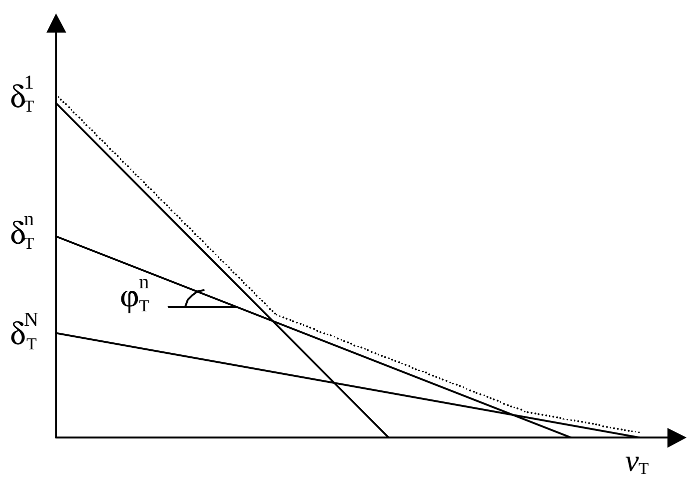


Figure 5.3 – piecewise linear future cost function

5.1.2 Backward Recursion Scheme

The recursive calculation of the piecewise linear future cost functions is very similar to the standard stochastic DP scheme:

set number of linear segments N = number of initial storage values M
initialize future cost function for the last stage as zero: $\{\varphi^n_{T+1}$ and $\delta^n_{T+1}\} = 0$ for $n = 1, ..., N$

for $t = T, T\text{-}1, ..., 1$

  for each storage value $v_t = \{v^m_t, m = 1, ..., M\}$

    for each inflow scenario $a_t = a^1_t, ..., a^k_t, ..., a^K_t$

      solve the one-stage scheduling problem for initial storage v^m_t and inflow a^k_t:

$$\begin{array}{lll} \alpha^k_t(v^m_t) = \text{Min}\, c_t(u_t) + \alpha_{t+1} & \text{simplex} & \\ \quad\text{subject to} & \text{multiplier} & (5.4) \\ \qquad v_{t+1} = v^m_t - u_t - s_t + a^k_t & \pi^k_{ht} & \\ \qquad v_{t+1} \le \overline{v} & & \\ \qquad u_t \le \overline{u} & & \\ \qquad \alpha_{t+1} \ge \varphi^n_{t+1} v_{t+1} + \delta^n_{t+1} \quad n = 1, ..., N & & \end{array}$$

    next
    calculate the coefficient and constant term for the m^{th} linear segment of the future cost function in the previous stage:

$$\varphi^m_t = \sum_{k=1}^{K} p_k \times \pi^k_{ht} \qquad \text{and} \quad \delta^m_t = \sum_{k=1}^{K} p_k \times \alpha^k_t(v^m_t) - \varphi^m_t \times v^m_t$$

  next
next

5.1.3 Lower bound calculation

At first sight, there are no substantial differences between the Dual DP procedure in 5.2 and the traditional DP scheme in 4.3.2. Note, however, that the traditional scheme had to create a new future cost function table in each stage by *interpolation* of the discrete values $\{\alpha_t(v^m_t)\}$. As a consequence, the required number of points in the table for a system of I hydro plants is at least equal to the 2^I combinations of extreme points (full/empty).

In the Dual DP scheme, the piecewise linear segments can be used to *extrapolate* the future cost function values, i.e. it not necessary to use all combinations of points to obtain a complete (although approximate) future cost function. Moreover, if a smaller number of initial storage values is used, a smaller number of linear segments will be generated. As seen in previous Figure 5.3, the resulting future cost function, which is based on the maximum value over all segments, will then be a *lower bound* to the “true” function.

As a consequence, the future cost function for the first stage is a lower bound $\underline{z}$ to the optimal solution of the hydrothermal scheduling problem:

$$\underline{z} = \alpha_1(v_1) \tag{5.5}$$

5.1.4 Upper bound calculation

If we also had a way of calculating an upper bound $\overline{z}$ to the optimal solution value, this would allow an strategy of incrementally increasing the accuracy of the problem solution, i.e. start with a small set of initial volumes, calculate upper and lower bounds, and increase the number of points if necessary.

5.1.4.1 Simulation Scheme

This upper bound can be obtained by a *Monte-Carlo simulation* of system operation, using the set of future cost functions produced by the recursion scheme in 5.2. This is due to the fact that the only future cost function that can result in the optimal expected operation cost is the optimal function itself; all others, by definition, have to result in higher operation costs.

The simulation scheme is implemented in the following steps:

define a set of inflow scenarios $a_t = \{a_t^1, ..., a_t^m, ..., a_t^M\}$ for all stages t = 1, ..., T

for each inflow scenario $a_t = a_t^1, ..., a_t^m, ..., a_t^M$

initialize storage value for stage 1 as $v_t^m = v_1$

for t = 1, ... , T

solve the one-stage scheduling problem for initial storage v_t^m and inflow a_t^m:

$$\begin{aligned} &\text{Min } c_t(u_t^m) + \alpha_{t+1} \\ &\text{subject to} \\ &\qquad v_{t+1}^m = v_t^m - u_t^m - s_t + a_t^m \\ &\qquad v_{t+1}^m \le \overline{v} \\ &\qquad u_t^m \le \overline{u} \\ &\qquad \alpha_{t+1} \ge \varphi_{t+1}^n v_{t+1}^m + \delta_{t+1}^n \quad n = 1, ..., N \end{aligned} \tag{5.6}$$

next

calculate total operation cost z^m for simulation scenario m as the sum of all immediate costs along the study period:

$$z^m = \sum_{t=1}^{T} c_t(u_t^m)$$

next

5.1.4.2 Confidence Interval

The expected operation cost is *estimated* as the mean total cost over all simulation scenarios:

$$\hat{z} = \frac{1}{M} \sum_{t=1}^{T} z^m \tag{5.7}$$

Because we used a Monte-Carlo simulation, there is an uncertainty around the "true" (population) expected value for $\overline{z}$. The 95% confidence interval is given by:

$$\overline{z} \in [\hat{z} - 1.96\hat{\sigma}; \hat{z} + 1.96\hat{\sigma}] \tag{5.8}$$

where $\hat{\sigma}$ is the standard deviation of the estimator, given by:

$$\hat{\sigma} = [\frac{1}{M-1} \sum_{t=1}^{T} (z^m - \overline{z})^2]^{½} \tag{5.9}$$

5.1.5 Optimality Check

Optimality is achieved when the lower bound $\underline{z}$ calculated in (5.5) is inside the confidence interval (5.8). Note that, because of sampling variation, the lower bound can sometimes exceed the upper bound *mean estimate* $\hat{z}$.

5.1.6 New Iteration

If the lower bound is outside the confidence interval, the backward recursion step 5.2 is executed again with an additional set of storage values. The natural candidates for the new values are the volumes $\{v_t^m, m = 1, ..., M\}$produced in the simulation step 5.1.4.1.

Note that the linear segments calculated in the previous iteration are retained, because the piecewise future cost function is given by the maximum over all segments. In other words, it is possible to gradually improve the future cost function representation.

6. ANALYTICAL REPRESENTATION OF UNCERTAINTIES IN DUAL DP

6.1 Time-Independent Uncertainties

In the same way as traditional DP, the Dual DP scheme can represent uncertainties that have no time dependence (e.g. equipment outage and short-term load forecast uncertainty) in a straightforward way (see section ---).

6.2 Time-Dependent Uncertainties

In addition, the Dual DP scheme can represent *analytically* several types of uncertainty resulting from *time-dependent* stochastic processes without increasing computational effort. This feature will be initially illustrated for the case of inflows and then extended to other variables.

6.2.1 Representation of Inflow Serial Correlation

In order to represent serial correlation in SDDP, we model inflows as a continuous Markov *process* (linear autoregressive model) rather than a Markov chain:

$$\frac{(a_t - \mu_t)}{\sigma_t} = \phi_1 \frac{(a_{t-1} - \mu_{t-1})}{\sigma_{t-1}} + \phi_2 \xi_t \qquad (6.1)$$

where:

μ_t	inflow mean
σ_t	standard deviation
ϕ_1 and ϕ_1	model parameters
ξ_t	independent random variable

We use the formulation of section 4.5, where the state variables are the reservoir storage at the beginning of stage *t*, v_t, and the inflow in the *previous* stage, a_{t-1}. This is illustrated in the equations below, which show the dispatch problem for stage *t*, with initial storage v_t^m and previous inflow a_{t-1}^m:

$$\begin{array}{lll}
\alpha_t^l(v_t^m, a_{t-1}^m) = & \text{Min } c_t(u_t) + \alpha_{t+1} & \text{simplex} \\
& \text{subject to} & \text{multiplier} \\
& \quad v_{t+1} = v_t^m - u_t - s_t + a_t^l & \pi_{ht}^l \\
& \quad v_{t+1} \le \overline{v} & \\
& \quad u_t \le \overline{u} &
\end{array} \qquad (6.2)$$

The inflow for stage t, a_t^l, is obtained by applying the Markov process (6.1):

$$a_t^l = \sigma_t \times [\phi_1 \frac{a_{t-1}^m - \mu_{t-1}}{\sigma_{t-1}} + \phi_2 \, \xi_t^l] + \mu_t \qquad (6.3)$$

where ξ_t^l is sampled from the corresponding probability distribution.

The linear approximation of the future cost function for the previous stage will now have two coefficients, based on the derivatives of $\alpha_t^l(v_t^m, a_{t-1}^m)$ with respect to v_t^m and a_{t-1}^m at the optimal solution.

The first coefficient is calculated in the same way as the independent case:

$$\partial\alpha_t^l/\partial v_t = \pi_{ht}^l \qquad (6.4)$$

The second is calculated through the chain rule:

$$\partial\alpha_t^l/\partial a_{t-1} = \partial\alpha_t^l/\partial a_t \times \partial a_t/\partial a_{t-1} \qquad (6.5)$$

The term $\partial\alpha_t^l/\partial a_t$ also corresponds to π_{ht}^l, because both v_t^m and a_t^l are in the same water balance equation. In turn, the term $\partial a_t/\partial a_{t-1}$ is calculated from the inflow model parameters in (6.3):

$$\partial a_t/\partial a_{t-1} = \sigma_t \times \phi_1/\sigma_{t-1} \qquad (6.6)$$

The recursion scheme is implemented as follows:

1) generation of M inflow sequences along the study period

initialize $\{a_0^m\}$, $m = 1, ..., M$
for $t = 1, ..., T$
    for $m = 1, ..., M$
        sample a random variable ξ_t^m
        calculate the inflow for stage t conditioned to previous inflow a_{t-1}^m:

$$a_t^m = \sigma_t \times [\phi_1 \frac{a_{t-1}^m - \mu_{t-1}}{\sigma_{t-1}} + \phi_2 \, \xi_t^m] + \mu_t$$

    next
next

2) backward recursion

for t = T, T-1, ..., 1

  for m = 1, ..., m

    retrieve the *pair* [storage, past inflow] $\{v_t^m, a_{t-1}^m\}$

    for l = 1, ..., L

      sample a random variable ξ_t^l

      calculate the inflow for stage t conditioned to previous inflow a_{t-1}^m:

$$a_t^l = \sigma_t \times [\phi_1 \frac{a_{t-1}^m - \mu_{t-1}}{\sigma_{t-1}} + \phi_2 \xi_t^l] + \mu_t$$

      solve the one-stage scheduling problem for v_t^m and a_t^k:

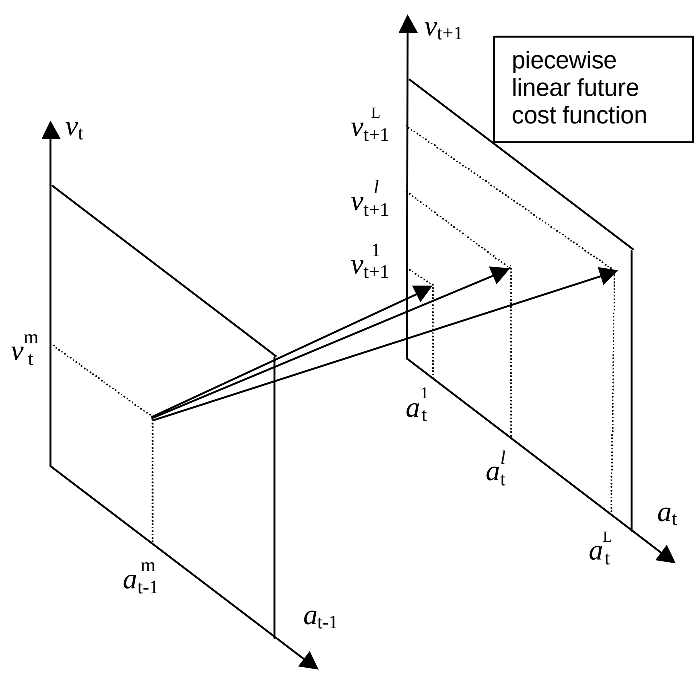
$$\begin{aligned} \alpha_t^l(v_t^m, a_{t-1}^m) = \ & \text{Min } c_t(u_t) + \alpha_{t+1} \\ & \text{subject to} \\ & \quad v_{t+1}^l = v_t^m - u_t - s_t + a_t^l \\ & \quad v_{t+1}^l \le \overline{v} \\ & \quad u_t \le \overline{u} \\ & \quad \alpha_{t+1} \ge \varphi_{t+1}^n v_{t+1} + \gamma_{t+1}^n a_t^l + \delta_{t+1}^n \qquad n = 1, ..., \text{N} \end{aligned} \tag{6.7}$$

      calculate the coefficients of the future cost function approximation for the previous stage as shown in (6.4) through (6.6)

    next

  next

next



Note that a^{l}_{t} is a known value when problem (6.7) is solved. As a consequence, $\gamma^{n}_{t+1}a^{l}_{t}$ is added to the constant term δ^{n}_{t+1} when the one-stage dispatch problem is solved. In other words, the computational effort of the Dual DP scheme does not change with the increase in the state space, as the derivatives are calculated analytically.

6.2.2 Representation of Additional Uncertainties

The same procedure of the previous section can be applied to represent uncertainties on all parameters in the problem right-hand side. For example, load in the stage can be modeled as another auto-regressive process:

$$d_t = \sigma_{dt}\times[\phi_{d1}\frac{d_{t-1} - \mu_{dt-1}}{\sigma_{dt-1}} + \phi_2\,\xi_t] + \mu_{dt} \tag{6.8}$$

Replacing the implicit definition of immediate cost function $c_t(u_t)$ – see (3.5) - in the dispatch problem (6.7), we have:

$$\begin{array}{llll}
\alpha^{l}_{t}(v^{m}_{t},a^{m}_{t-1},d^{m}_{t-1}) = & \text{Min} \sum_{j=1}^{J} c_j\,g_t + \alpha_{t+1} & \text{simplex} & \\
& \text{subject to} & \text{multipl.} & (6.9) \\
& \quad v_{t+1} = v^{m}_{t} - u_t - s_t + a^{l}_{t} & \pi^{l}_{ht} & \\
& \quad v_{t+1} \le \bar{v} & & \\
& \quad u_t \le \bar{u} & & \\
& \quad \sum_{j=1}^{J} g_{tj} = d^{l}_{t} - \rho\, u_{ts} & \pi^{l}_{dt} & \\
& \quad \alpha_{t+1} \ge \varphi^{n}_{t+1}v_{t+1} + \gamma^{n}_{t+1}a^{l}_{t} + \theta^{n}_{t+1}d^{l}_{t} + \delta^{n}_{t+1} & & n = 1, ..., \text{N}
\end{array}$$

where the load value d^{l}_{t} in stage t is obtained from the application of the auto-regressive model to the previous value d^{m}_{t-1}, similarly to the inflow modeling. The multiplier π^{k}_{dt} associated to the load supply equation represents the system "spot price" ($/MWh).

The derivatives of $\alpha^{k}_{t}(v^{m}_{t},a^{m}_{t-1},d^{m}_{t-1})$ with respect to storage and previous inflow are as discussed previously. The derivative with respect to the previous load is given by:

$$\begin{aligned}
\partial\alpha^{l}_{t}/\partial d_{t-1} &= \partial\alpha^{l}_{t}/\partial d_t \times \partial d_t/\partial d_{t-1} \\
&= \pi^{k}_{dt}\times\sigma_{dt}\times\phi_{d1}/\sigma_{dt-1}
\end{aligned} \tag{6.8}$$

6.3 Limitations in the Analytical Representation of Uncertainties

The analytical representation of time-dependent uncertainties cannot be directly extended to the unit operation cost of thermal units, $\{c_j\}$. Because those costs are in the objective function, the optimal solution value α_t^l is a *concave* function of $\{c_j\}$ (see Figure 6.2). In contrast, as shown in Figure 6.3, α_t^l is a *convex* function of the values discussed so far (storage, inflows and load), all of which are in the problem right-hand side.

As a consequence, the optimal solution value has a *saddle shape* when those parameters are jointly taken into account, and cannot be approximated by piecewise linear cost functions. In this case, it becomes necessary to represent cost uncertainty explicitly, as will be described next.

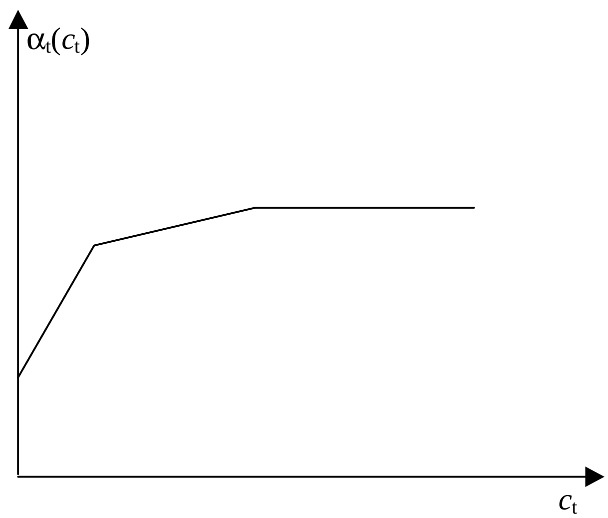


Figure 6.2 – Variation of operation cost with fuel costs

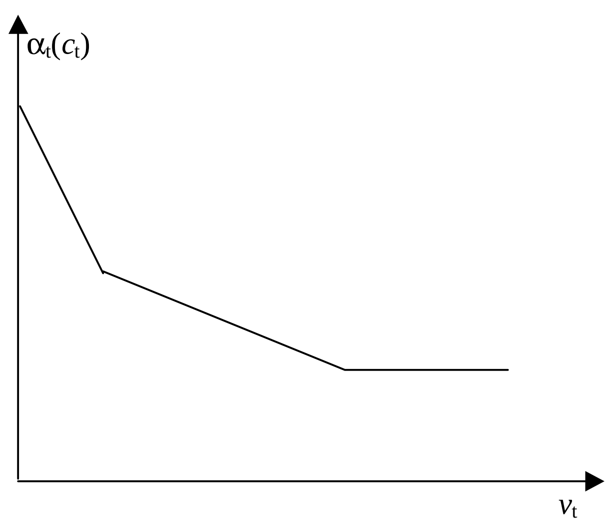


Figure 6.3 – Variation of operation cost with storage

7. JOINT SCENARIO/ANALYTICAL REPRESENTATION OF UNCERTAINTY

Suppose there are K operating cost scenarios in each stage t, represented by the immediate cost functions $\{c_t^k(u_t)\}$ for k = 1, ..., K and t = 1, ..., T. The transition probabilities among scenarios are represented by the following Markov chain:

t+1→ t↓	c_{t+1}^1	...	c_{t+1}^l	...	c_{t+1}^L
c_t^1	p_{11}		p_{1l}		p_{1L}
	...		...		...
c_t^k	p_{k1}		p_{kl}		p_{kL}
	...		...		...
c_t^K	p_{K1}		p_{Kl}		p_{KL}

This uncertainty is represented in the Dual DP scheme as follows:

1) generation of M operating cost sequences along the study period

initialize the cost scenario for stage 0 as c_0^k for m = 1, ..., M
for t = 1, ..., T
    for m = 1, ..., M
        retrieve the cost scenario k associated to sequence m in stage t-1
        generate a cost scenario l for stage t conditioned to the previous scenario k:
            divide the [0,1] range into L intervals; the size of each interval is proportional to p_{kl}, the transition probability from k in stage t to l in stage t+1
            randomly sample from uniform random variable U(0,1) and identify the interval l corresponding to the sampled value
    next
next

2) generation of M inflow sequences along the study period

initialize $\{a_0^m\}$, m = 1, ..., M as the initial inflow scenario (observed in stage 0)
for t = 1, ..., T
    for m = 1, ..., M
        sample a random variable ξ_t
        calculate the inflow for stage t conditioned to previous inflow a_{t-1}^m:

$$a_t^m = \sigma_t \times [\phi_1 \frac{a_{t-1}^m - \mu_{t-1}}{\sigma_{t-1}} + \phi_2 \xi_t] + \mu_t$$

    next
next

3) backward recursion scheme

for $t = \mathrm{T}, \mathrm{T}-1, ..., 1$

for $m = 1, ..., \mathrm{M}$

retrieve the *pair* [storage, past inflow] associated with sequence m in stage t-1: $\{v^m_t, a^m_{t-1}\}$

identify the operating cost scenario c^k_t associated with sequence m in stage t

retrieve the transition probabilities $\{p_{kl}, l = 1, ..., \mathrm{L}\}$ from cost scenario k in stage t to scenario l in t+1

for $n = 1, ..., \mathrm{N}$

sample a random variable ξ^n_t

calculate the inflow for stage t conditioned to previous inflow a^m_{t-1}:

$$a^n_t = \sigma_t \times [\phi_1 \frac{a^m_{t-1} - \mu_{t-1}}{\sigma_{t-1}} + \phi_2 \xi^n_t] + \mu_t$$

solve the one-stage scheduling problem for v^m_t, a^n_t and c^m_t:

$$\alpha^k_t(v^m_t, a^m_{t-1} \mid c^k_t) = \quad \text{Min } c^k_t(u_t) + \sum_{l=1}^{L} p_{kl}\, \alpha^l_{t+1}(v^m_{t+1}, a^n_t \mid c^l_{t+1})$$

$$\text{subject to} \tag{7.1}$$

$$v_{t+1} = v^m_t - u_t - s_t + a^n_t$$
$$v_{t+1} \leq \overline{v}$$
$$u_t \leq \overline{u}$$

The conditioned future cost functions $\alpha^l_{t+1}(v^m_{t+1}, a^n_t \mid c^l_{t+1})$ are represented by their piecewise linear approximations:

			simplex multiplier
Min	α^l_{t+1}		
subject to			
	$\alpha^l_{t+1} \geq \varphi^n_{t+1} v_{t+1} \quad \gamma^n_{t+1} a^n_t + \delta^n_{t+1}$	for $n \in \mathrm{C}^l$	$\mu^n_{\alpha t}$

where $n \in \mathrm{C}^l$ indicates linear segments labeled with cost class #l in stage t+1

calculate the coefficients of the future cost function approximation for the previous stage as shown in (6.4) through (6.6); label this segment with the current cost scenario k

next

next

next

Figure 7.1 illustrates the recursion scheme. The storage and inflow state variables are represented analytically as piecewise linear functions. In turn, the cost scenarios are represented *as if* they were discrete state variables. Note, however, that this separation between future cost functions conditioned to scenarios 1 and 2 is carried out though the labeling of the linear segments, and there is no additional computational effort involved.

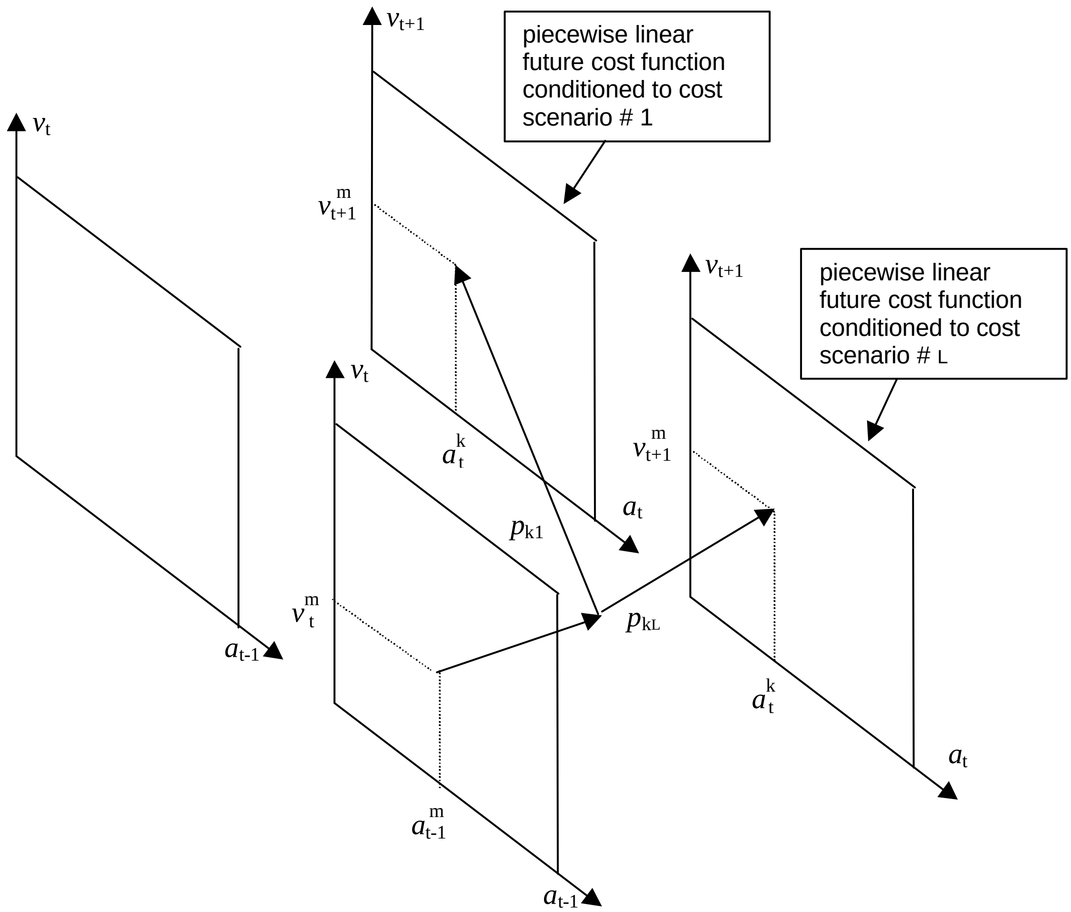


Figure 7.1 - joint scenario/analytical representation of uncertainty

8. MAXIMIZATION OF SPOT REVENUES

The procedure described in the previous section can be used to determine the operating policy that will maximize the expected spot revenues of a set of plants.

Let $\{i \in \mathrm{E}\}$ and $(j \in \mathrm{E}\}$ represent the set of hydro and thermal plants belonging to enterprise E (typically, a utility or an independent power producer).

Suppose there are K *spot price* scenarios in each stage *t*, represented as $\{\pi^k_{dt}\}$ for $k = 1, ..., \mathrm{K}$ and $t = 1, ..., \mathrm{T}$. As described in the previous section, the transition probabilities among those scenarios are represented by a Markov chain, as shown below.

$t+1\rightarrow$ $t\downarrow$	π^1_{dt+1}	...	π^l_{dt+1}	...	π^L_{dt+1}
π^1_{dt}	p_{11}		p_{1l}		p_{1L}
	...		...		...
π^k_{dt}	p_{k1}		p_{kl}		p_{kL}
	...		...		...
π^K_{dt}	p_{K1}		p_{Kl}		p_{KL}

The scheduling problem is implemented in the following steps:

1) generation of M spot price sequences along the study period

initialize the spot price scenario for stage 0 as π^k_{d0} for $m = 1, ..., \mathrm{M}$
for $t = 1, ..., \mathrm{T}$
    for $m = 1, ..., \mathrm{M}$
        retrieve the spot price scenario *k* associated to sequence *m* in stage *t*-1
        generate a price scenario *l* for stage *t* conditioned to the previous scenario *k* using the sampling procedure described in the previous section:
    next
next

2) generation of M inflow sequences along the study period

identical to the procedure in the previous section.

3) backward recursion scheme

for t = T, T-1, ..., 1

for m = 1, ..., M

retrieve the *pair* [storage, past inflow] associated with sequence m in stage t-1: $\{v_t^m, a_{t-1}^m\}$

identify the spot price scenario π_{dt}^k associated with sequence m in stage t

retrieve the transition probabilities $\{p_{kl}, l = 1, ..., L\}$ from price scenario k in stage t to scenario l in t+1

for n = 1, ..., N

calculate the inflow a_t^n for stage t conditioned to previous inflow a_{t-1}^m using the same procedure as in the previous section.

solve the one-stage scheduling problem for v_t^m, a_t^n and π_{dt}^k:

$$\alpha_t^k(v_t^m, a_{t-1}^m \mid \pi_{dt}^k) = \text{Max} \quad \sum_{i \in E} \pi_{dt}^k \times u_t(i) + \sum_{j \in E} (\pi_{dt}^k - c_j) \times g_{tj} + \sum_{l=1}^{L} p_{kl}\, \beta_{t+1}^l \tag{7.1}$$

subject to

$$v_{t+1} = v_t^m - u_t - s_t + a_t^n$$
$$v_{t+1} \le \overline{v}$$
$$u_t \le \overline{u}$$
$$\beta_{t+1}^l \le \varphi_{t+1}^n v_{t+1} + \gamma_{t+1}^n a_t^n + \delta_{t+1}^n \qquad \text{for } n \in C^l$$

where $n \in P^l$ indicates linear segments labeled with price class #l in stage t+1

calculate the coefficients of the future cost function approximation for the previous stage as shown in (6.4) through (6.6); label this segment with the current price scenario k

next

next

next