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Non-Convexities Representation on Hydrothermal Operation Planning using SDDP

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***Abstract*—This work describes an extension of the Stochastic Dual Dynamic Programming (SDDP) algorithm to represent non-convexities on the hydrothermal operation planning problem formulated as a mixed integer multistage stochastic model. One proposed methodology makes use of a non-conventional approach of the Lagrangian relaxation technique for convexification of the recourse function, and a special procedure is applied in order to find valid stronger Benders cuts to build these approximated convex future cost functions. Discussion is made over the topic of whether we should really worry about guarantying outer approximations of the original functions, which could lead to distorted strategies when dealing with highly non convex problems and, consequently, to non-economic or inadequate system operation. By focusing on the non-convexity introduced by the hydro production variation with storage, a more aggressive cut-generation procedure is also proposed using a non-linear variable transformation technique. Study cases of real hydrothermal systems are used to make comparisons and analysis over the dilemma of choosing the most suitable methodology for this problem.**

***Index Terms*—Convexification, Lagrangian relaxation, Multi-stage stochastic optimization.**

I. Introduction

THE objective of a hydrothermal operation planning problem consists in determining the optimal operating policy for the use of a system's generation resources in order to minimize the total expected cost for reliable electricity demand supply during a given time horizon. Hydrothermal systems are mainly characterized by the uncertainty of hydrological inflows and temporal coupling of the operative decisions, a result of the existence of limited water storage capacity in the reservoirs. This means that the problem consists in deciding whether a planner should use the water in the current stage (reducing immediate generation cost and possible rationing in the future if there is a drought) or store the water to be used in the future stages (leading to higher immediate cost and possible spillage in the future if there is a wet period). We are dealing, therefore, with a multistage stochastic problem traditionally solved by stochastic dynamic programming (SDP) algorithms. Generally speaking, these algorithms are based on a recursive procedure for construction of the so-called future cost functions (FCF), which establishes, for each system state, the expected value of the future cost associated to the best immediate operative decision.

Although SDP algorithms are very robust in terms of allowing the representation of non-convex models, they require a problem solution for each combination of the state variables of the system. It means that these algorithms suffer from the curse of dimensionality with state space exponential growth and, therefore, even mid-sized hydrothermal systems become computationally intractable.

The Stochastic Dual Dynamic Programming (SDDP) algorithm, developed by Pereira in 1991 [1], which is currently used in over 60 countries by different types of agents in the energy sector, is the state of the art of the methods that can handle multistage stochastic hydrothermal operation problems, with individual representation of reservoirs, and this is why this technique is applied in this particular work. The SDDP methodology uses Benders decomposition to separate the problem into single stage problems and iteratively builds approximations of the FCFs as piecewise linear functions, without requiring discretization of the state space.

Each iteration of the SDDP algorithm is composed by two phases: first a forward simulation, where a finite number of system states are selected for each stage; and second a backward recursion, where first order approximations of the FCFs are built for each of the selected states, using dual information of each single stage problem. These hyperplanes, or Benders cuts, are tangent to the original function which means that the algorithm builds outer approximations of the FCFs. The main limitation of this algorithm, however, is requiring convex FCFs in order to guarantee optimality, otherwise the Benders decomposition method cannot be immediately applied to generate distorted cuts for the problem.

In a hydrothermal system there may be several components which make the operation planning modeling a non-convex problem. Two examples are can be pointed out: hydro generator power output is given by the product of a production factor, as a function of the reservoir net head variable, and the water discharge variable, resulting in a non-convex function; moreover, the unit commitment of power plant's start-up and shut-down scheduling are discrete decisions and, therefore, a non-convex model.

Depending on the scope of the study, however, the components can be modeled differently in order to meet an appropriate trade-off between computational effort, technical consistency and quality results. When dealing with the hydrothermal operation planning problem, solved by SDDP algorithm, the use of oversimplified models, despite accelerating the problems solution, may sometimes derive in inadequate operation strategy, which means bad management of reservoir levels. In particular, net head variations may often significantly affect the system operation. This way, the application of the SDDP algoritm on realistic hydrothermal systems require the use of specialized convexification techniques for generating the FCFs approximations.

In this paper we use a MILP formulation for the hydrothermal problem, where thermal unit commitment are binary decisions and hydro power production is modeled as a bilinear function represented by a piecewise linear approximation, accurately improved by incorporation of binary variables. The objective of this work is to discuss, compare and propose convexification approaches for the hydrothermal operation problem, classified into two categories:

1) FCF convexification - Non-convex components of the system are kept represented in the problem's formulation and Lagrangian relaxation technique is applied to the recourse equations in order to obtain convex FCF approximations [3]. Since the recourse equations contain the state variables of the problem, the resulting Lagrangian subproblem becomes convex with respect to these variables and it can be shown how valid Benders cuts can be extracted for any Lagrange multiplier vector.

2) Components convexification - The original formulation of the problem is replaced by convex approximations of the functions which model the non-convex components, so that traditional SDDP algorithm can be applied to the problem. Examples of Components convexification techniques include McCormick envelope models [4] or convex piecewise linear functions, which can be adjusted in order to minimize the approximation mean error.

Each convexification approach has its up and downsides: the first one ensures that, besides constructing an outer approximation of the FCF, it could result in the convex envelope of the function. Nevertheless, if we are dealing with a highly non-convex problem, external approximation may never get close enough to the original function in certain regions and convergence may never be achieved. The second approach can drive to a more realistic solution, by reducing approximation errors, but since it's not necessarily an outer approximation, then optimality is not guaranteed. The selection of the most suitable method to be applied on a specific problem is not a trivial task and depends on the system's characteristics and its practical appliance.

An interesting property of the FCF convexification approach is related to a non-conventional application of the Lagrangian relaxation technique, since it's not being used to actually solve the original problem but to generate cuts for convex approximations of the FCFs. In fact, the Lagrange multipliers don't necessarily have to be optimal in order to build a valid cut, however, they lead to stronger approximations. In the literature, we can find several techniques for the optimization of the Lagrangian multipliers, including, proximal bundle methods [5], sequential refinement [6], ascending directions [7] and outer-approximations [8]. In general, these methods require a large number of iterations to converge so, when incorporated in a SDDP solution scheme, it's even crucial the use of efficient multipliers search procedure.

In [9] the Lagrangian relaxation approach was proposed for a disjunctive McCormick envelope formulation but disregarding the Lagrange multipliers optimization and using a simplified SDDP approach by considering a small scenario tree to model hydrological inflows uncertainties. One significant contribution of the work presented in this paper consists in incorporating a new efficient algorithm for searching the Lagrange multipliers in a FCF convexification approach. Based on the fact that each single stage MILP problem is, in practice, relatively easy to solve, then the algorithm uses the problem's optimal solution for two purposes: to build a locally convexified problem whose dual solution generates a good initial value for the multipliers; and to generate a good lower bound for the Lagrangian subproblem solution to be used in the update step evaluation during the iterative multipliers search procedure.

While exploring the comparison between both convexification criteria, this work is specially focused on solving the non-convexity of the hydro production function. In this context, a new component convexification methodology is also proposed, based on the work of Suanno [10] where the problem's non-convexity is eliminated by applying a non-linear variable transformation of water volume to stored energy. Unlike the original work, this new formulation allows production function variability representation of hydros in cascade, which consisted in the main limitation of the original model. Such contribution was made possible through the incorporation of an approximated parallel modelling of the hydro system, which consists of a spacial decoupling technique of the cascade.

This paper has the following structure: In section II the MILP formulation of the hydrothermal problem formulation is presented. In section III we discuss the two FCF convexification criteria and describe the proposed improvement in the Lagragian relaxation approach. In section IV the new component convexification methodology is presented. The results obtained for all approaches are presented for real hydrothermal systems in section V. Finally, section VI contains the conclusions of this work.

II. Hydrothermal Problem Formulation

The hydrothermal operation problem (PO) is modeled in this paper by the following set of variables and constraints:

Sets:

$\mathbb{T}$	Set of time stages
$\mathbb{I}$	Set of hydro plants
$\mathbb{J}$	Set of thermal plants
$\mathbb{M}_i$	Set of upstream plants of hydro plant i

Parameters:

$c_{t,j}$	Operative cost of thermal plant	\$/MWh
d_t	System energy demand	MWh
$a_{t,i}$	Inflow volume of hydro plant	hm^3
η_i	Efficiency factor of hydro plant	p.u.
$\overline{gh}_i$	Maximum generation of hydro plant	MWh
$\underline{u}_i$	Minimum turbining of hydro plant	hm^3
$\overline{u}_i$	Maximum turbining of hydro plant	hm^3
$\underline{v}_i$	Minimum volume of hydro plant	hm^3
$\overline{v}_i$	Maximum volume of hydro plant	hm^3
$\underline{gt}_j$	Minimum generation of thermal plant	MWh
$\overline{gt}_j$	Maximum generation of thermal plant	MWh

Variables:

$gt_{t,j}$	Generation of thermal plant	MWh
$xt_{t,j}$	Unit commitment of thermal plant	$\{0,1\}$
$gh_{t,i}$	Generation of hydro plant	MWh
$v_{t,i}$	Stored volume of hydro plant	hm^3
$u_{t,i}$	Turbined volume of hydro plant	hm^3
$s_{t,i}$	Spilled volume of hydro plant	hm^3
$h_{t,i}$	Net head of hydro plant	m
w_t	Operative expected cost of stage t	\$
α_t	Approximate future cost of stage t	\$

Energy balance: this equation ensures the system's demand supply for each stage:

$$\sum_{i\in\mathbb{I}} gh_{t,i} + \sum_{j\in\mathbb{J}} gt_{t,j} = d_t. \tag{1}$$

Hydro balance: this equation follows the principle of mass conservation given by the stored volume at the end of a stage must be equal to the stored volume at the beginning of the stage plus the inflow volumes and discounted by the outflow volumes of the hydro plant during the stage:

$$v_{t,i} + u_{t,i} + s_{t,i} - \sum_{m\in\mathbb{M}_i} (u_{t,m} + s_{t,m}) = v_{t-1,i} + a_{t,i}. \tag{2}$$

Hydro production: this non-linear equation represents the hydro energy production given by a turbined water volume $u_{t,i}$ and reservoir net head $h_{t,i}$:

$$gh_{t,i} = k \cdot \eta_i \cdot u_{t,i} \cdot h_{t,i}. \tag{3}$$

where constant k is the product of gravity acceleration in m/s^2 by the water relative density in kg/m^3 and a conversion factor to MWh.

The net head of the reservoir is given by the difference between head and tailwater levels, discounted by hydraulic losses. When neglecting losses and tailwater variations with the plant's outflow volume, the net head $h_{t,i}$ is approximated by a function of the stored volume $v_{t,i}$. The relationship between the two variables is given by the head x volume curve, which reflects the topographical characteristics of the reservoir flooded area. Thus, the hydro production equation can be expressed as a bilinear equation given by the multiplication of a production function and the turbined volume:

$$gh_{t,i} = \rho_i\left(v_{t,i}\right) \cdot u_{t,i}. \tag{4}$$

In this paper the bilinear constraints are represented by a MILP model $gh^L_{t,i}(v_{t,i}, u_{t,i})$ given by a piecewise linear approximation using binary variables:

$$\begin{aligned}
& gh_{t,i} = \sum_{n=1}^{N}\sum_{m=1}^{M} gh_{i,n,m} \cdot \gamma_{i,n,m} \\
& v_{t,i} = \sum_{n=1}^{N}\sum_{m=1}^{M} v_{i,n} \cdot \gamma_{i,n,m} \\
& u_{t,i} = \sum_{n=1}^{N}\sum_{m=1}^{M} u_{i,m} \cdot \gamma_{i,n,m} \\
& \sum_{n=1}^{N}\sum_{m=1}^{M} \gamma_{i,n,m} = 1; \sum_{n=1}^{N} x_{i,n} = 1; \sum_{m=1}^{M} y_{i,m} = 1 \\
& \gamma_{i,n,m} \le x_{i,n} + x_{i,n+1}, n = 1..N, m = 1..M \\
& \gamma_{i,n,m} \le y_{i,m} + y_{i,m+1}, n = 1..N, m = 1..M \\
& \gamma_{i,n,m} \ge 0, n = 1..N, m = 1..M \\
& x_{i,n} \in \{0,1\}, n = 1..(N-1) \\
& y_{i,m} \in \{0,1\}, m = 1..(M-1) \\
& gh_{t,i} \le \overline{gh}_i \\
& \underline{v}_i \le v_{t,i} \le \overline{v}_i \\
& \underline{u}_i \le u_{t,i} \le \overline{u}_i
\end{aligned} \tag{5}$$

where N and M are the number of discretized points of variables v and u, respectively, and variables $\gamma_{n,m}$ represent convex combination of the points.

Thermal unit commitment: this model aims at finding optimal scheduling of plant's start-up/shut-down decisions:

$$\begin{aligned}
& gt_{t,j} - \overline{gt}_{t,j} \cdot xt_{t,j} \le 0 \\
& gt_{t,j} - \underline{gt}_{t,j} \cdot xt_{t,j} \ge 0 \\
& xt_{t,j} \in \{0,1\}
\end{aligned} \tag{6}$$

Objective function: minimization of total cost given by the immediate cost and expected future cost associated to operative decisions:

$$w_t = \min \sum_{j\in\mathbb{J}} c_{t,j} \cdot gt_{t,j} + \alpha_{t+1} \tag{7}$$

Future cost function: In this paper we suppose, without loss of generality, that v_{t-1} are the only state variables of the problem.

$$\alpha_{t+1} \ge w^k_{t+1} + \sum_{i\in\mathbb{I}} \pi^k_{t+1,i}\left(v_{t,i} - v^k_{t,i}\right), \quad k = 1..K \tag{8}$$

where K is the number of hyperplanes obtained so far during the SDDP iterative procedure. For a purely linear problem, the slopes π of these hyperplanes are given by the dual variables associated to the coupling constraints (2) in the optimal solution. However, the problem we are considering is non-convex due to constraints (5) and (6), so no dual information can be extracted from the solution of this problem. Since SDDP convergence depends on the FCF convexity property, then it's necessary to use techniques to either convexify the FCF after solving a non-convex problem (FCF convexification), or convexify the problem before solving it (Component convexification).

III. Convexification Approaches

Convexification approaches are used in the construction of FCF approximations in order to allow the use of SDDP algorithm in the solution of hydrothermal operation problems. The convergence of the algorithm depends directly on the quality of these approximations, that is, the mismatch in relation to the original functions, so the motivation in searching for tighten convex approximations.

In the FCF Convexification technique based on Lagrangian relaxation, the violation of the coupling constraints, given by the hydro balance equations, is penalized in the objective function by the vector of Lagrange multipliers. The multipliers together with the solution of this relaxed MILP problem are used to build a new hyperplane for the FCF convex approximation. The focus of this paper, however, is on improving the search for Lagrange multipliers which generate better lower bounds for the FCF approximations. This means that, although we are not specially interested in obtaining the optimal multipliers, it's important to consider the use of an efficient strategy to ensure a strong approximation.

Let the original non-convex problem (PO) be formulated as:

$$w_t^{PO}\left(v_{t-1}\right) = \min \sum_{j\in\mathbb{J}} c_{t,j}\cdot gt_{t,j} + \alpha_{t+1} \tag{9}$$

$$\begin{aligned}
\text{s/t}\quad & v_{t,i} + u_{t,i} + s_{t,i} - \sum_{m\in\mathbb{M}_i}\left(u_{t,m} + s_{t,m}\right) = v_{t-1,i} + a_{t,i}\\
& \alpha_{t+1} \geq w_{t+1}^k + \sum_{i\in\mathbb{I}} \pi_{t+1,i}^k\left(v_{t,i} - v_{t,i}^k\right), \quad k = 1..K\\
& \sum_{i\in\mathbb{I}} gh_{t,i} + \sum_{j\in\mathbb{J}} gt_{t,j} = d_t\\
& gh_{t,i} = gh_{t,i}^L\left(v_{t,i}, u_{t,i}\right)\\
& gt_{t,j} - \overline{gt}_{t,j}\cdot xt_{t,j} \leq 0\\
& gt_{t,j} - \underline{gt}_{t,j}\cdot xt_{t,j} \geq 0\\
& xt_{t,j} \in \{0,1\}
\end{aligned}$$

Then the Lagrangian subproblem (LS) can be expressed as follows:

$$\begin{aligned}
w_t^{LS}\left(\lambda_t, v_{t-1}\right) = \min & \sum_{j\in\mathbb{J}} c_{t,j}\cdot gt_{t,j} + \alpha_{t+1} + \\
& + \sum_{i\in\mathbb{I}} \lambda_{t,i}\left(v_{t-1,i} + a_{t,i} - v_{t,i} - u_{t,i} - s_{t,i} + \right.\\
& \left. + \sum_{m\in\mathbb{M}_i}\left(u_{t,m} + s_{t,m}\right)\right)
\end{aligned} \tag{10}$$

$$\begin{aligned}
\text{s/t}\quad & \alpha_{t+1} \geq w_{t+1}^k + \sum_{i\in\mathbb{I}} \pi_{t+1,i}^k\left(v_{t,i} - v_{t,i}^k\right), \quad k = 1..K\\
& \sum_{i\in\mathbb{I}} gh_{t,i} + \sum_{j\in\mathbb{J}} gt_{t,j} = d_t\\
& gh_{t,i} = gh_{t,i}^L\left(v_{t,i}, u_{t,i}\right)\\
& gt_{t,j} - \overline{gt}_{t,j}\cdot xt_{t,j} \leq 0\\
& gt_{t,j} - \underline{gt}_{t,j}\cdot xt_{t,j} \geq 0\\
& xt_{t,j} \in \{0,1\}
\end{aligned}$$

Let $\hat{w}_t^{LS}$ be the optimal solution of problem (LS) for state vector $\hat{v}_{t-1}$, note that:

$$w_t^{LS}\left(\lambda_t, v_{t-1}\right) = \hat{w}_t^{LS} + \sum_{i\in\mathbb{I}} \lambda_{t,i}\left(v_{t-1,i} - \hat{v}_{t-1,i}\right) \tag{11}$$

Since problem (LS) is a relaxation of (PO), then then $w_t^{LS}\left(\lambda_t, v_{t-1}\right)$ is an outer approximation of the original FCF for every value of λ_t. However, the closer the Lagrange multipliers are to the optimal value, the tighter is the approximation, where optimal Lagrange multipliers are obtained from the solution of the following maximization problem (LR):

$$w_t^{LR}\left(v_{t-1}\right) = \max_{\lambda_t} w_t^{LS}\left(\lambda_t, v_{t-1}\right) \tag{12}$$

As $w_t^{LR}\left(v_{t-1}\right)$ is convex on v_{t-1} then is corresponds to the lower convex envelope of the original FCF [2]. Nevertheless, the Lagrangian maximization problem can lead to a high computational effort to be solved by traditional iterative procedures. In this work we developed the following algorithm to Lagrange multipliers optimization:

1) Solve original problem (PO) and obtain optimal values for objective function w^{PO} and binary variables xt^{PO} and those associated to function $gh^{PO} = gh^L\left(v^{PO}, u^{PO}\right)$.

2) Solve locally convexified problem (LC), which corresponds to problem (PO) with fixed binary variables, and obtain optimal value of dual variables π^{LC}.

3) Use dual variables from problem (LC) as initial value for Lagrange multipliers: $\lambda = \pi^{LC}$.

4) Solve Lagrangian subproblem (LS) and obtain optimal value for objective function w^{LS} and variables v^{LS}, u^{LS} and s^{LS}.

5) Check stop criteria: if $w^{LS} << w^{PO}$ then vector λ must be updated in order to get closer to original problem optimal solution and then go back to step 4). However, if no significant progress is achieved towards this goal then no further effort should be done to decrease the duality gap of the Lagrangian relaxation.

The Lagrange multipliers updating is done by taking steps using the subgradient method [11] which, in many applications, has slow convergence and therefore fails to be of real practical interest. Nevertheless, unlike those applications where actual lower bounds of the Lagrangian subproblem are typically unknown, in this case, the optimal solution of the original problem (PO) is the best possible lower bound for evaluation of step sizes:

$$\lambda_{t,i} = \lambda_{t,i} - \mu\cdot\phi_i \tag{13}$$

where step size μ and subgradient ϕ_i are calculated as:

$$\mu = \frac{w^{LS} - w^{PO}}{\sum_{i\in\mathbb{I}}\left(\phi_i\right)^2} \tag{14}$$

$$\phi_i = v_{t,i}^{LS} - v_{t-1,i} - a_{t,i} + u_{t,i}^{LS} + s_{t,i}^{LS} - \sum_{m\in\mathbb{M}_i}\left(u_{t,m}^{LS} + s_{t,m}^{LS}\right)$$

In summary, this methodology has an interesting approach by taking advantage from availability of the original non-convex problem optimal solution to efficiently drive the Lagrange multipliers searching towards its optimal value, first by providing a way to locally convexify the problem, and second by guarantying a good lower bound for the subgradient method. Additionally, it also offers a measure of the problem's non-convexity level what bring us to questioning if lower bound approximations of the FCF are effectively suitable for all sort of problems.

The Component Convexification approach is a more widely applied technique and consists in replacing the constraints which introduce non-convexity into the problem, by a convex formulation of these components. In problem (PO) the sources of non-convexity are: the hydro production function and thermal unit commitment decision. The linearization of this model is done by constructing piecewise-linear convex functions, obtained from convex hull technique combined with hyperplanes adjustment for minimizing the approximation mean error.

In the case of thermal unit commitment model, the following procedure is done: Let gH_t be the sum of all hydro generation in the system such that $gH_t = \sum_{i \in \mathbb{I}} gh_{t,i}$, then an immediate thermal cost function can be obtained by discretization of variable gH_t and solution of the following MILP problem for each gH_t^n, $n = 1..N$:

$$
\begin{aligned}
ct_t\left(gH_t^n\right) = & \min \sum_{j \in \mathbb{J}} c_{t,j} \cdot gt_{t,j} \qquad (15)\\
\text{s/t} \quad & \sum_{j \in \mathbb{J}} gt_{t,j} = d_t - gH_t^n \\
& gt_{t,j} - \overline{gt}_{t,j} \cdot xt_{t,j} \leq 0 \\
& gt_{t,j} - \underline{gt}_{t,j} \cdot xt_{t,j} \geq 0 \\
& xt_{t,j} \in \{0, 1\}
\end{aligned}
$$

where function $ct_t\left(gH_t\right)$ can be adjusted as a convex piecewise linear function $ch_t^C\left(gH_t\right)$, as illustrated in figure 1. In a similar way, the hyperplanes of the hydro production function $gh_{t,i}^L\left(v_{t,i}, u_{t,i}\right)$ given by constraint (5) can also be adjusted as a convex piecewise linear function $gh_{t,i}^C\left(v_{t,i}, u_{t,i}\right)$.

The interesting aspect of this methodology lies on the fact that all convexification effort is done only one time, before the application of the SDDP algorithm. The new formulation of the convexified problem (PC), used to construct the FCF approximation, is obtained from replacing the non-convex constraints of problem (PO) by the piecewise-linear functions:

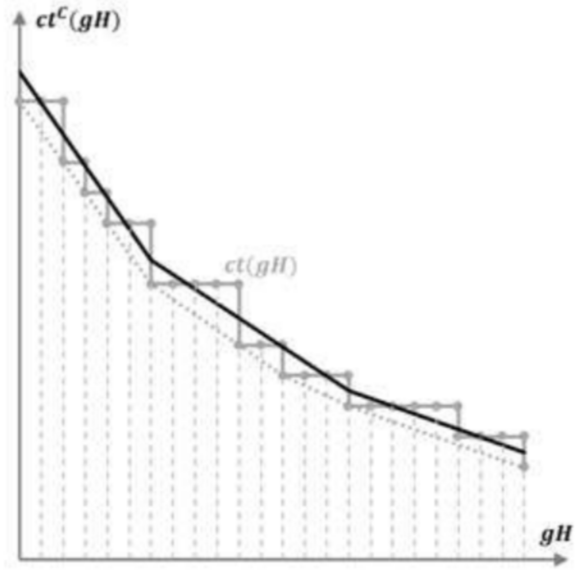

Fig. 1. Linearization of non-convex component

$$
\begin{aligned}
w_t^{PC}\left(v_{t-1}\right) = & \min ct_t^C\left(gH_t\right) + \alpha_{t+1} \qquad (16)\\
\text{s/t} \quad & gH_t = \sum_{i \in \mathbb{I}} gh_{t,i} \\
& \alpha_{t+1} \geq w_{t+1}^k + \sum_{i \in \mathbb{I}} \pi_{t+1,i}^k \left(v_{t,i} - v_{t,i}^k\right), \quad k = 1, K \\
& gH_t = d_t \\
& v_{t,i} + u_{t,i} + s_{t,i} - \sum_{m \in \mathbb{M}_i} \left(u_{t,m} + s_{t,m}\right) = v_{t-1,i} + a_{t,i} \\
& gh_{t,i} = gh_{t,i}^C\left(v_{t,i}, u_{t,i}\right)
\end{aligned}
$$

Note that both FCF and Component Convexification approaches presented above are generically enough to allow more complex formulation for this problem in order to consider further details of the system. Nevertheless, special attention should be given to the hydro production function due to the effect on the reservoir level management. For this reason, a new Component Convexification methodology, specifically developed for this function, is described in the next section.

IV. New Approach on Component Convexification

The work of Suanno discuss all physical interpretation of the non-linear variable transformation of water volume into stored energy in order to overcome convexity limitations of a hydrothermal operation problem. Nevertheless, the proposed methodology neglects the variability on downstream plants production function in a hydro system cascade because, since there is no biunivocal relationship between the volume and energy variables for those plants, the same water volume of a head plant is associated to a different energy amount according to each downstream plants production factor.

This paper proposes an extension of this approach by generating approximate conversion relationships between the plants stored energy in a hydro cascade. The relationship

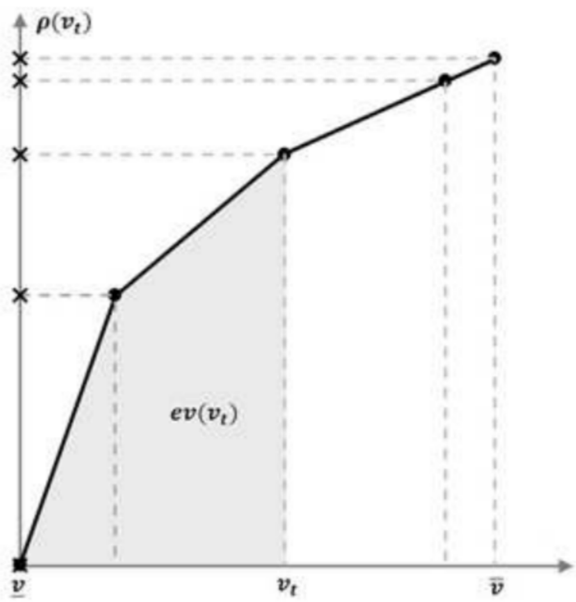

Fig. 2. Production function in terms of stored energy

between the reservoir stored volume and the associated energy amount is given by the following expression:

$$ev_t = \int_{\underline{v}}^{v_t} \rho(x) \cdot dx \qquad (17)$$

From this equation, one can express the production function of a plant in terms of the stored energy $\rho(v_t) \rightarrow \rho'(ev_t)$, according to figure 2.

Lets suppose the hydro balance equation of a head plant 1:

$$v_{t,1} + u_{t,1} + s_{t,1} = v_{t-1,1} + a_{t,1} \qquad (18)$$

By applying the variable transformation, we have:

$$\begin{aligned} &ev_{t,1} + gh_{t,1} + es_{t,1} = ev_{t-1,1} + ea_{t,1} \\ &ea_{t,1} = \rho_1^e(ev_{t,1}) \cdot a_{t,1} \end{aligned} \qquad (19)$$

where ea_t is the reservoir inflow energy in MWh, ev_t is the stored energy in MWh and es_t is the spilled energy in MWh.

We observe that, for this model, the non-convexity associated to the hydro production no longer exists and the reformulated problem is convex. Now, lets suppose the hydro balance equation of plant 2 which is downstream to plant 1:

$$v_{t,2} + u_{t,2} + s_{t,2} - u_{t,1} - s_{t,1} = v_{t-1,2} + a_{t,2} \qquad (20)$$

The difficulty associated to the variable transformation of this equation is in the inflow volume that comes from the upstream plant $u_{t,1} + s_{t,1}$. The problem is that the equivalent outflow energy of plant 1 and inflow energy of plant 2 associated to this same water volume, are function of their own production coefficients. That means we are unable to directly remove the problem's non-convexity and then an approximate relationship is proposed.

First step is to build an equivalent parallel representation of the hydro system cascade by replacing each downstream

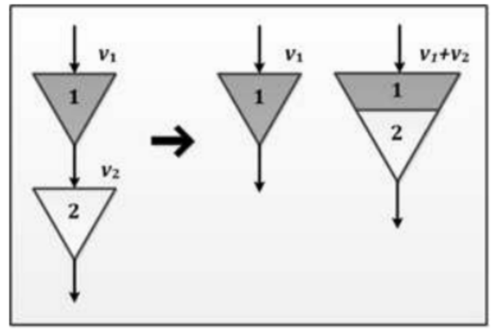

Fig. 3. Parallel representation of hydro system cascade

reservoir by the sum of the upstreams ones. This way, the equations of a downstream plant become:

$$\begin{aligned} &v_{t,12} + u_{t,2} + s_{t,2} = v_{t-1,12} + a_{t,1} + a_{t,2} \\ &gh_{t,2} = \rho_2(v_{t,12} - v_{t,1}) \cdot u_{t,2} \\ &v_{t,12} - v_{t,1} \leq \overline{v}_2 \end{aligned} \qquad (21)$$

where $v_{t,12} = v_{t,1} + v_{t,2}$, as shown in figure 3.

Now, by applying the variable transformation, this formulation results:

$$\begin{aligned} &ev_{t,12} + gh_{t,2} + es_{t,2} = ev_{t-1,12} + ea_{t,12} \\ &ea_{t,12} = \rho_2^e(ev_{t,12} - \tilde{ev}_{t,1}) \cdot (a_{t,1} + a_{t,2}) \\ &ev_{t,12} - \tilde{ev}_{t,1} \leq \overline{ev}_2 \end{aligned} \qquad (22)$$

unlike the previous formulation where the aggregated storage is the sum of both storages, in this case $ev_{t,12} = \tilde{ev}_{t,1} + ev_{t,2}$ where $\tilde{ev}_{t,1} \neq ev_{t,1}$ because the volume in the first reservoir represents a different amount of energy in each plant.

The heuristic procedure described next was developed to generate convex approximations of the stored energy relationship between plants in a cascade.

1) First step consists in performing an operative simulation of the system in order to capture the reservoirs water draining dynamics. Considering full storage level situation for all reservoirs as an initial condition of the system, a non-linear model was used to generate a water draining profile of the reservoirs by optimizing the total system output energy. The draining profile represents an estimation of the best management of the reservoirs storage level in a hydro system cascade. Basically, it's used to indicate what's the optimal storage level associated to each reservoir in a cascade for a specific system state, and consequently obtain the relationship between the head plant level and the production function of downstream plants, according to figure 4.

2) By constructing function $\tilde{\rho}_2(v_1)$ from step 1, and using the known function $ev_1(v_1)$, we are able to evaluate the approximate functions $\tilde{ev}_1(ev_1)$, according to figure 5.

3) By using the draining profile to represent a good estimation of reservoir levels at each system state, we assume that $\tilde{ev}_1(ev_{12}, ev_1) = \tilde{ev}_1(ev_1)$, and the problem is formulated by the following convex approximate model:

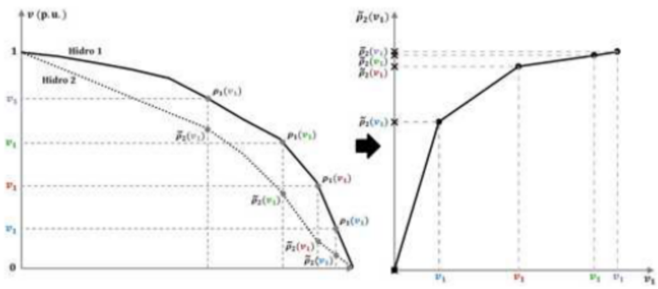

Fig. 4. Reservoir draining profile

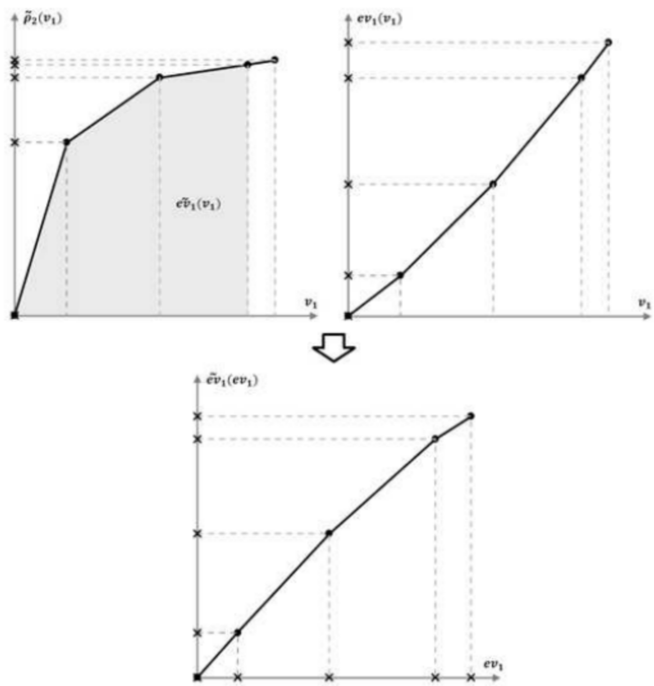

Fig. 5. Approximate production function

$$ev_{t,12} = ev_{t-1,12} + ea_{t,12} - gh_{t,2} - es_{t,2} \qquad (23)$$
$$ea_{t,12} = \rho_2^e \left(ev_{t,12} - \tilde{ev}_{t,1}\left(ev_{t,1}\right)\right) \cdot \left(a_{t,1} + a_{t,2}\right)$$
$$ev_{t,12} - \tilde{ev}_{t,1}\left(ev_{t,1}\right) \leq \overline{ev}_2$$

It's important to highlight that the extension of this approximation procedure is straight forward for the cases with several downstream plants in the same hydro cascade.

V. Case Study

In order to evaluate the convexification criteria applied to hydrothermal operation problema consideréing the non-convex hydro production functions, the following procedure was adopted: first we obtained the operative policy, or the FCFs approximations, for each one of the discussed methodologies on this work and, then, operative simulations were held for those functions, considering the original non-convex model for a realistic representation of the system. Comparisons were made for the following methods, according to the FCF Convexification criteria:

TABLE I
HONDURAS - OPERATIVE POLICY CONVERGENCE RESULTS

Method	Num. Iter.	LBnd. (M$)	UBnd. (M$)	Min.UBnd (M$)	Max.UBnd (M$)
LR1	3	16.84	20.40	19.35	21.46
LR2	3	16.93	20.39	19.35	21.44
LR3	3	16.96	20.39	19.36	21.43
CNT	2	17.65	17.71	17.05	18.38
PWL	2	20.32	20.55	19.53	21.57
ENR	2	20.33	20.69	19.83	21.54

TABLE II
BRAZIL - OPERATIVE POLICY CONVERGENCE RESULTS

Method	Num. Iter.	LBnd. (M$)	UBnd. (M$)	Min.UBnd (M$)	Max.UBnd (M$)
LR1	6	843.0	995.5	918.5	1072.5
LR2	6	853.1	982.9	907.2	1058.6
LR3	6	879.9	977.0	904.5	1049.6
CNT	3	1213.6	1214.7	1142.5	1286.9
PWL	3	983.6	993.1	914.9	1071.3
ENR	3	1059.0	1083.1	1003.4	1162.8

[LR1]: Lagrangian relaxation using linear relaxation solution to obtain Lagrange multipliers;

[LR2]: Lagrangian relaxation using local convexification solution to obtain Lagrange multipliers;

[LR3]: Lagrangian relaxation using multipliers optimization after local convexification solution.

And, for the Components Convexification, the following methods were considered:

[CNT]: Average production factor, neglecting hydro production variability with storage level;

[PWL]: Approximation of hydro production by convex piecewise-linear function adjusted for error minimization;

[ENR]: Non-linear transformation of the water volume variables into stored energy variables.

Two different cases were used in the results analysis. The first case contains the El Cajon dam from Honduras, whose combination of high net head and high storage capacity of the reservoir makes the case particularly interesting due to the large variation of over 140% of the plant's production factor, depending on the reservoir storage level.

The second case represents the southeast (and part of the mid-west) regions of the Brazilian electric system, responsible for approximately 70% of the whole system's energy consumption. It's energy production is predominantly hydro based with high regulating capacity reservoirs. The system is represented by 38 thermal plants and 98 hydro plants, whereas 18 of them have variable production factor.

For each case we used a 12 monthly stage horizon, 30 scenarios for simulation and 10 inflow discretization values for each scenario in recursion phase. Tables I and II contain information associated to operative policy convergence. Columns *LBnd.* and *UBnd.* correspond respectively to the lower and upper bounds of the operative expected cost. Columns *Min.UBnd* and *Max.UBnd* determine the upper bound confidence interval.

Graphics shown in figures 6 and 7 illustrate the computational time required for obtaining the FCFs approximations and the operative expected cost resulted from the final simulation for each case.

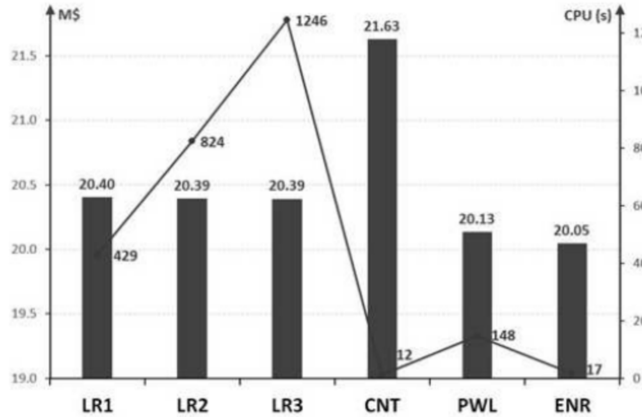

Fig. 6. Honduras - CPU time and Operative cost

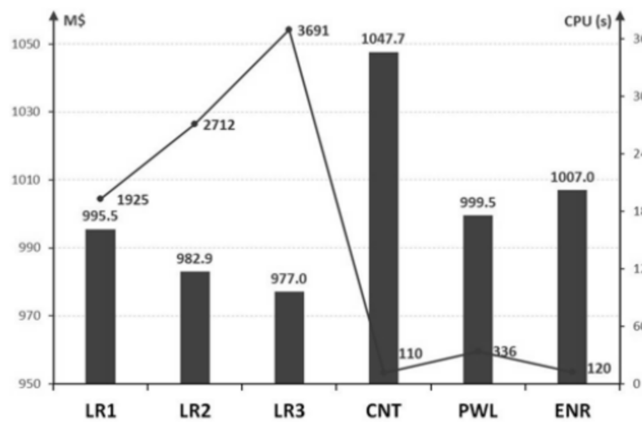

Fig. 7. Brazil - CPU time and Operative cost

The results obtained for models [LR1], [LR2] and [LR3] show that the incorporation of the solution of a locally convexified problem and the multipliers optimization procedure have granted more refined FCFs approximations by increasing the lower bound of the operative cost.

For the Components Convexification models, we observed that the decision of ignoring production factors variability, in case [CNT], has resulted in the worst operative strategy as noticed by the high associated expected cost. Regarding case [PWL] and, specially case [ENR], it was verified that, for El Cajon system, the convexification criteria that doesn't guarantee outer approximation has performed better with regard to computational effort and total operative cost. Nevertheless, for the Brazilian system, there has been observed total operative cost reduction with application of Lagrangian relaxation in the convexification criteria, despite its high CPU time.

VI. Conclusions

The results showed that the performance of an operative policy is based on the quality of the FCFs approximation and, for this reason, the benefits achieved by the incorporation of non-convexities on the hydrothermal operation model are the warranty for investments in efficient convexification techniques that allow the use of SDDP algorithm in solving these type of problems.

It was showed that the method based on the Lagrangean relaxation was clearly improved by the incorporation of the procedures proposed in this work, although the performance for the El Cajon case indicated that the use of FCF outer approximations may not ensure the best operative strategy for all systems. The Components Convexification methods, however, may not provide outer approximations for the FCFs, but can generate particularly interesting results, specially when applied to highly non-convex problems.

We conclude, therefore, that the selection of the most suitable convexification methodology to be applied on a particular hydrothermal operation planning problem is a very important task, but not a trivial one, since it depends primarily on the characteristics of the system and on the study practical application.

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