# Short-Term Scheduling of Transmission-Constrained Hydrothermal Systems A MILP-Based Approach

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***Abstract*— The optimal hourly scheduling of generation and transmission resources over the next day or week is a key function of both liberalized and centrally planned power sectors. The main difficulty in solving this short-term scheduling (STS) problem lies in the joint modeling of nonlinearities (for example, head variation in hydro plants and quadratic circuit losses); integer decisions/nonconvexities (e.g unit commitment); and time- and space-coupling constraints (such as the water balance in reservoirs and transmission network equations). Although several techniques, in particular Lagrangian Relaxation (LR), have been successfully applied to the solution of STS problems, some limitations appear when a large number of constraints has to be relaxed; also, the LR multiplier updating scheme often has to be "tuned" for each particular power system, thus reducing its flexibility. The approach presented in this paper is based on the transformation of STS nonlinearities and nonconvexities into piecewise mixed linear integer (MILP) constraints. This approach was found to be flexible, allowing the modeling of complex features of both hydrothermal generation and the transmission network. Also, by taking advantage of recent advances in commercial solver capabilities, the MILP scheme was found to be computationally efficient, as illustrated in case studies with seven countries in Latin America and Europe.**

**Index terms— Hydroelectric-thermal power generation, Power generation dispatch, Power generation scheduling**

## I. Introduction

THE objective of short-term scheduling is to determine the most economic hourly production schedule of the system resources (hydro and thermal generation, plus demand-side options) over the next day or week, subject to a set of operational constraints, such as: (i) water balance in cascaded reservoirs, including time delays between upstream releases and downstream arrivals, evaporation etc.; (ii) the operational characteristics of thermal plants (unit commitment, minimum up- and down-time etc.); and (iii) the transmission network equations, limits on circuit flows and quadratic losses. From the economics side, the objective in "traditional" systems may be to minimize the overall thermal production cost over the week, plus the expected future costs of hydro generation, given by end-of-period functions calculated by midterm scheduling models; or, in the case of liberalized systems, there may be an hourly price associated to each hydro and thermal resource. In both types of market, the short-term scheduling (STS) is a key function, and its optimal solution is a problem of great technical and commercial interest. The main difficulty in solving the STS problem lies in the joint modeling of several features, such as nonlinearities, integer decisions and coupling constraints for different time stages.

In the last decade, several computational tools have been developed to solve the STS problem. Most of the technical work is based on Lagrangian Relaxation (LR) [1]-[6]. The LR scheme is typically used to relax constraints that couple several plants, such as the load supply equation and reserve constraints, thus decomposing a problem with J plants and T time steps into J (simpler) subproblems of T time steps, which are then solved by a specialized technique such as dynamic programming. At each iteration, the Lagrange multipliers are updated by a subgradient or other method. The LR has been successfully applied to a wide range of problems, in particular for representing unit commitment constraints in predominantly thermal systems. The LR scheme limitations appear when there is a large number of time- and space-coupling constraints to be relaxed, such as ramp limits, water balance in reservoirs and power flow constraints. In this case, the multiplier updating scheme becomes more complex and may result in slower convergence.

Metaheuristic methods, such as genetic algorithms and simulated annealing [7]-[11] and [17] have also been extensively used. The main reasons for this type of approach include simplicity of implementation and flexibility (all kinds of constraints, and logical rules may be included; functions may be non smooth, etc.). Although optimality is not guaranteed with these methods, their proponents argue that the solutions found are robust.

The basic motivation for the work presented here is to develop a methodology that is flexible and robust enough to be used in several mid-sized systems, with different mixes of generation and other characteristics. The solution approach is to transform the STS nonlinearities into piecewise mixed linear integer (MILP) constraints. As will be seen in the paper, the MILP scheme allows the modeling of complex features for hydro, thermal and transmission. Also, by taking advantage of recent advances in commercial solver capabilities, the MILP scheme is found to be computationally efficient, as illustrated in case studies with seven countries in Latin America and Europe.

The paper is organized as follows. In section II, we present the STS problem modeling. Section III describes the piecewise MILP approximations. Section IV presents and discusses the case studies. Finally, the major conclusions are discussed in Section V.

## II. STS Problem Formulation

### A. Objective function

The STS objective is to determine the least-cost hourly generation schedule for a weekly horizon. As shown in (1), the cost function has three main components: (i) thermal variable operating and startup costs; (ii) energy rationing costs; and (iii) future cost function (FCF), which couples the end-of-week reservoir storage levels with future operating costs, calculated by mid-term scheduling models:

$$Z=\text{Min}\sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{k=1}^{K} c_{j,t,k}\, g_{j,t,k} + \sum_{t=1}^{T}\sum_{j=1}^{J} s_{j,t} y_{j,t} + \sum_{t=1}^{T}\sum_{n=1}^{N} \psi r_{n,t} + \alpha_T \tag{1}$$

where:

$t$ indexes hourly stages (T = 168 stages)

$j$ indexes thermal plants (J plants)

$k$ indexes thermal plant efficiency segment (K segments)

$n$ indexes transmission network buses (N buses)

$c_{j,t}$ variable cost (or price bid) of $j$ in hour $t$ and efficiency segment $k$ (\$/MWh)

$g_{j,t}$ energy produced by $j$ in hour $t$ and efficiency segment $k$ (MWh)

$s_{j,t}$ startup cost for $j$ in hour $t$ (\$)

$y_{j,t}$ binary variable that represents start-up decision (=1, if plant $j$ started in $t$; =0, otherwise)

$\psi$ rationing cost (\$/MWh)

$r_{n,t}$ energy rationing of bus $n$ in hour $t$ (MWh)

$\alpha_T$ Future cost function – FCF, which relates future expected costs with end-of horizon storage

### B. Hydro Plants

Hydro modeling includes: (i) water balance; (ii) limits on storage and outflow; (iii) energy generation; (iv) pumped storage; and (v) end-of-week storage conditions.

#### 1) Water balance in each plant

$$v_{i,t+1} = v_{i,t} + a_{i,t} - q_{i,t} - qr_{i,t} - w_{i,t} - qe_{i,t}(v_{i,t}) + \sum_{m\in M_i}[q_{m,t-\tau_{m,i}} - w_{m,t-\tau_{m,i}}] \tag{2}$$

where:

$v_{i,}$ stored volume of hydro plant $i$ in hour $t$

$q_{i,t}$ & $w_{i,t}$ turbined and spilled volumes, respectively

$\tau_{m,i}$ travel time between the hydro plants immediately upstream of plant $i$ ($m\in M_i$) and plant $i$

$a_{i,t}$ lateral (incremental) inflow plant $i$

$qr_{i,t}$ irrigation demand

$qe_{i,t}(v_{i,t})$ evaporation volume (varies linearly with storage)

#### 2) Limits on storage and outflow

$$\underline{v}_{i,t} \le v_{i,t} \le \overline{v}_{i,t} \tag{3}$$

$$\underline{q}_{i,t} \le q_{i,t} \le \overline{q}_{i,t} \tag{4}$$

#### 3) Hydro Generation

Fig. 1 shows a typical hydro plant energy production as a function of storage level and total (turbined + spilled) outflow. The different shades denote {volume, outflow} pairs with similar energy output.

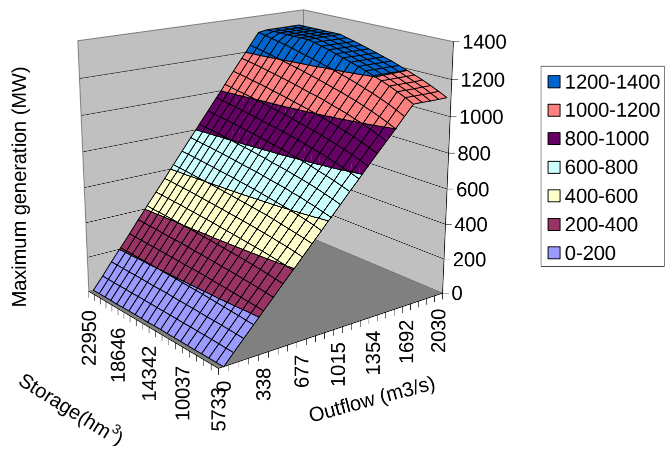


Fig.1 – Hydro production as a function of storage and outflow

The energy production $e_{i,t}$ is modeled as the product of turbine & generator efficiency $\eta$ × turbined outflow $q_{i,t}$ × net head $\delta h$.

The efficiency, in turn, is a function $\eta(q_{i,t})$ of the turbined outflow; the net head is represented as the difference between reservoir head – a function $f_1(v_{i,t})$ of storage, tailwater level (function $f_2(q_{i,t},w_{i,t})$ of turbined and spilled outflows) and head loss (function $f_3(q_{i,t})$ of outflow). Finally, the production is limited by the plant's generator capacity $\overline{P}_{i,t}$:

$$e_{i,t} = \text{Min}\{\overline{P}_{i,t}, [k\eta(q_{i,t})q_{i,t}[f_1(v_{i,t})-f_2(q_{i,t},w_{i,t})-f_3(q_{i,t})]]\} \tag{5}$$

where $k$ is a constant.

#### 4) Pumped storage

The modeling of pumped storage is similar to that of hydro generation. The difference between the lower pump level and the upper reservoir level results in a negative production factor, which multiplied by the "pumped flow" $q_{i,t}$ results in a negative power generation (power consumed).

#### 5) End-of-week storage conditions

The future cost function - FCF - signals the tradeoff between using the water along the week and storing it for future use.

The FCF is a multivariate, nonseparable function $F(\boldsymbol{v}_{T+1})$, where $\boldsymbol{v}_{T+1}$ is the vector of reservoir storage volumes at the end of the week. It is usually computed by mid-term scheduling models using either a traditional stochastic dynamic programming (SDP) recursion or, more recently, a stochastic *dual* dynamic programming (SDDP) algorithm [12-14].

### C. Thermal plants

Thermal modeling comprises minimum up and down time, and ramp constraints, discussed next.

#### 1) Unit commitment

If thermal plant $j$ is committed, its hourly production has lower and upper bounds; otherwise, these limits are zero:

$$\underline{g}_{j,t}\, x_{j,t} \le g_{j,t} \le \overline{g}_{j,t}\, x_{j,t} \qquad (6)$$

where $x_{j,t}$ is a binary variable ($x_{j,t}$=1 if $j$ is dispatched in hour $t$; =0 otherwise)

#### 2) Minimum up- and down- time

Once a plant is committed, it must remain in operation for a minimum number of hours:

$$x_{j,t-1} - x_{j,t} + x_{j,k} \ge 0, \quad k=t+1..min\{\mathrm{T}, t+\tau_{u,j}-1\} \qquad (7a)$$

Conversely, a decomitted plant must remain offline for a minimum number of hours:

$$x_{j,t-1} - x_{j,t} + x_{j,k} \le 1, \quad k=t+1..min\{\mathrm{T}, t+\tau_{d,j}-1\} \qquad (7b)$$

where $\tau_{d,j}$ and $\tau_{uj}$ are respectively the minimum downtime and uptime of plant $j$ (hours).

#### 3) Ramp constraints

Any increase/decrease in power production in successive hours must lie within a permissible range.

$$-\delta_d \le g_{j,t} - g_{j,t-1} \le \delta_u \qquad (8)$$

where $\delta_d$ and $\delta_u$ are the down and up ramp limits (MW/h).

#### 4) Non-convex thermal cost functions

Because of the non-linear relationship between power and costs, the plant total generation is modeled as the sum of the generation of different (linear) segments. Therefore, the installed capacity of the plant is discretized in K different segments of equal size. Certain types of thermal plants have higher efficiencies in converting fuel to power for higher power outputs. In other words, $c_{j,t,k+1} < c_{j,t,k+1}$. Because the model minimizes total costs, it would dispatch the segments in a reverse order for plants with this characteristic, which is physically meaningless. In order to prevent such distortion, additional 0-1 variables are included in the problem formulation which will enable the dispatch of segment $k$+1 if and only ff the variable associated to segment $k$ is in its upper bound. A 3-segment efficiency curve we require the following additional constraints:

$$g_{t,j,1} \le \overline{g}_{j,t}\, x_{j,t} / K \qquad (9a)$$

$$g_{t,j,2} \le \overline{g}_{j,t}\, \lambda_{j,t,2} / K \qquad (9b)$$

$$g_{t,j,3} \le \overline{g}_{j,t}\, \lambda_{j,t,3} / K \qquad (9c)$$

$$\lambda_{j,t,2} \le K\, g_{t,j,1} / \overline{g}_{j,t} \qquad (9d)$$

$$\lambda_{j,t,3} \le K\, g_{t,j,2} / \overline{g}_{j,t} \qquad (9e)$$

$$\lambda_{j,t,2}, \lambda_{j,t,3} \in \{0,1\} \qquad (9f)$$

### D. Transmission network model

We use a linearized active power flow model with quadratic power losses. The network equations are:

#### 1) Load supply balance

The first Kirchoff's law represents the load supply balance in each bus.

$$\sum_{k\in\Omega_n} f_{k,t} + p_{n,t} = d_{n,t} \quad \text{for } n = 1,., \mathrm{N};\ t=1,..,\mathrm{T} \qquad (10)$$

where:

| | |
|---|---|
| $n$ | indexes the system buses (N number of buses) |
| $p_{n,t}$ | generation at bus $n$, hour $t$ = $\sum_{\{i;j\}\in n} (e_{i,t}+g_{j,t})$ |
| $d_{n,t}$ | load at bus $n$, hour $t$ |
| $k$ | indexes the circuits (K number of circuits) |
| $f_{k,t}$ | power flow in circuit $k$, hour $t$ |
| $\Omega_n$ | set of circuits directly connected to bus $n$ |

#### 2) "loop flow" equation

This equation corresponds to Kirchhoff's second law:

$$f_{k,t} = \gamma_k[\theta_t(n_k) - \theta_t(m_k)] \text{ for } k = 1, ..., \mathrm{K} \qquad (11)$$

where:

| | |
|---|---|
| $f_{k,t}$ | power flow in hour circuit $k$, hour $t$. |
| $\gamma_k$ | circuit susceptance (inverse of reactance) |
| $\theta_t(n_k)$ | node voltage angle at the FROM bus $n_k$ |
| $\theta_t(m_k)$ | node voltage angle at the TO bus $m_k$ |

#### 3) power flow limits

$$-\overline{f}_{k,t} \le f_{k,t} \le \overline{f}_{k,t} \qquad (12)$$

#### 4) circuit losses

The power loss in each circuit $k$ is given by:

$$L_{k,t} = r_k f_{k,t}^2 \qquad (13)$$

where $r_k$ is the circuit resistance. Circuit losses are modeled as additional loads at the terminal (from-to) buses of each circuits. The power balance equation (10) becomes:

$$\sum_{k\in\Omega_n} f_{k,t} - 0.5L_{k,t} + p_{n,t} = d_{n,t} \text{ for } n = 1,., \mathrm{N};\ t=1,..,\mathrm{T} \qquad (14)$$

## III. MILP Solution Approach

As mentioned in the Introduction, the STS problem (1-14) is difficult to solve because of nonlinearities, integer variables and time- and space-coupling constraints.

In this section, we present the proposed transformations of the original STS into a MILP problem.

### A. Circuit losses

We use a piecewise linear approximation of (13) as illustrated in Fig.2.

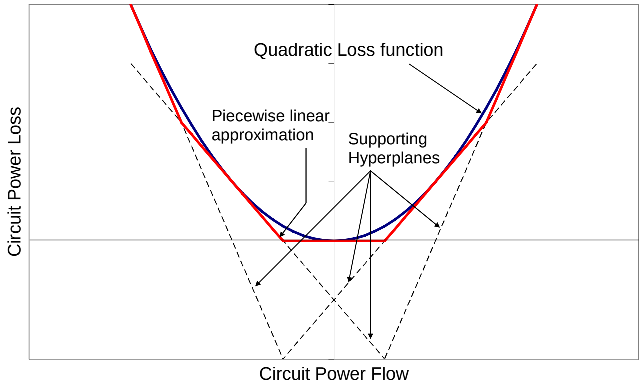


Fig. 2 – Piecewise linear approximation of circuit losses

The piecewise linear approximation is represented as the following set of constraints:

$$L_{kt} \geq a_{km} f_{kt} + b_{km}, \text{ for } k=1, .., \mathrm{K};\ m=1,.., \mathrm{M}_k;\ t=1, \ldots, \mathrm{T} \quad (15)$$

where $m$ indexes the segments ($\mathrm{M}_k$ is number of segments).

The coefficient $a_{km}$ is calculated as:

$$a_{km} = \frac{\partial \delta_k}{\partial f_k}\Big|_{f_k = f_{km}} = 2 r_k f_{km} \quad (16)$$

where $f_{km}$ is the circuit flow corresponding to the $m^{th}$ segment. In turn, the constant term $b_{kn}$ is calculated from $a_{km} f_{km} + \mathrm{b}_{km} = r_k f_{km}^2$, which gives $\mathrm{b}_{km} = - r_k f_{km}^2$.

Ref [19] describes a “compact” formulation of the piecewise loss approximation, currently being tested, which reduces the number of variables and can be applied to larger systems.

## B. Hydro production

We use a *convex hull* approximation [16] of the hydropower function, as proposed in [15], to transform the nonlinear (and mildly nonconvex) hydropower function into a piecewise linear function of storage ($v_{i,t}$) and total (turbined + spilled) outflow:

$$e_{i,t} \leq \phi_{i,h} + \gamma_{i,h}\, v_{i,t+1} + \psi_{i,h}\,(q_{i,t} + w_{i,t}) \text{ for } h_i=1..H_i \quad (17)$$

where $h_i$ indexes the piecewise segments required to build the convex hull ($H_i$ is the number of hyperplanes); $\phi_{i,h}$, $\gamma_{i,h}$ and $\psi_{i,h}$ are the hyperplane coefficients.

## C. Future cost function

As mentioned previously, the FCF is usually computed by a traditional stochastic DP recursion or by a stochastic dual (SDDP) scheme. The SDDP algorithm is particularly suitable because it automatically produces a piecewise multivariate FCF which can be easily “plugged” to the STS model.

$$\alpha \geq R_p + \sum_{i=1}^{I} \pi_{p,i}\, v_{i,T} \qquad \text{for} \quad p = 1..P \quad (18)$$

Where $R_p$ and $\pi_{p,i}$ are constants calculated by the SDDP recursion made in the mid-long term planning. Further details on the FCF modeling can be found in [14].

# IV. Study Cases

## A. Description of selected systems

We illustrate the MILP approach with realistic data derived from seven countries in Europe and Latin America: Bulgaria (BG), Slovenia (SL), Romania (RO), Brazil (BR), Nicaragua (NI), Bolivia (BO) and Ecuador (EC). As will be seen in the descriptions that follow, those systems have a variety of sizes, generation mixes and modeling requirements.

Unfortunately the data cannot be disclosed because of existing confidentiality agreements. The results shown for Bulgaria, Slovenia and Romania were provided by SEETEC "Southeastern Europe Electrical System Technical Support Project" http://www.seetec-balkans.org/, funded by the Canadian Development Agency (ACDI)

### 1) Bulgaria

There are 25 thermal plants (nuclear power, lignite, coal and natural gas) with startup costs and minimum uptime/ downtime constraints, plus 17 hydro plants (reservoirs, run-of-river and pump storage), arranged in a complex topology, with travel times. Imports/Exports to neighboring countries were modeled as dummy thermal plants and interruptible loads.

One of the objectives of the Bulgarian study is to investigate the use of hourly *water values* (dual variables associated to the water balance equations, expressed in terms of $/MWh) as the regulated bids of the hydro producers in the proposed “balancing market".

### 2) Slovenia

The Slovenian system has 11 thermal plants and 20 cascaded hydro plants, in three main river systems. Exports to Italy, imports from Austria and exports/imports with Croatia are also considered. As in the previous case, the main objective is to derive hydro bids from the water values.

### 3) Romania

The Romanian system has 59 thermal units (lignite, fuel oil and nuclear) and 150 hydro plants located in several river cascades. Most of hydro plants are small and were aggregated into larger equivalent plants. The number of hydro plants considered was then reduced to 41. Except for the nuclear power, all thermal units are modeled with commitment decision variables, but not minimum uptime/downtime or ramps. As in the previous cases, exports and imports to/from neighboring countries, Bulgaria and Serbia, are modeled.

### 4) Brazil

The Brazilian system is hydro-dominated (92% of the installed capacity) with 94 cascaded hydro plants in several river basins. Reservoir head variation is represented. The generation system also has 41 thermal plants, none with commitment requirements. A “zonal” representation of the high voltage transmission system is used, with five regions and six major interconnections.

#### 5) Nicaragua

The Nicaraguan system is thermal-dominated, with 18 power plants, some of which are modeled with commitment constraints. There are also 2 hydro plants and a pump. The HV transmission network has 65 buses and 75 circuits.

#### 6) Bolivia

Bolivia has 24 gas-fired thermal plants; most of them open cycle with non-convex production costs. There are also 21 hydro plants, either run of the river or with small reservoirs. The HV grid is composed of 33 buses and 34 circuits, Quadratic losses play a large role in determining the least cost dispatch.

#### 7) Ecuador

The Ecuadorian system has 114 thermal units (mostly diesel and fuel oil), most of which had commitment constraints. There are also 4 hydro plants with seasonal storage. The HV network considered has 56 buses and 96 circuits. Power exchanges with Colombia were also modeled.

#### 8) Summary of study parameters

Table I summarizes the system parameters, and Table II lists the features modeled in each case.

TABLE I – SYSTEM CHARACTERISTICS

| Country | BG | SL | RO | BR | NI | BO | EC |
|---|---|---|---|---|---|---|---|
| Thermal | 25 | 11 | 62 | 52 | 18 | 24 | 114 |
| Hydro | 17 | 20 | 41 | 94 | 3 | 21 | 4 |
| Buses | - | - | - | 5 | 65 | 33 | 56 |
| Circuits | - | - | - | 6 | 75 | 34 | 96 |

TABLE II – MODELING REQUIREMENTS FOR EACH SYSTEM

| Country | BG | SL | RO | BR | NI | BO | EC |
|---|---|---|---|---|---|---|---|
| Horizon (h) | 168 | 168 | 120 | 168 | 168 | 168 | 168 |
| Unit Commit. | Yes | Yes | Yes | No | Yes | No | Yes |
| Network | No | No | No | Yes[1] | Yes | Yes | Yes |
| Network Losses | No | No | No | No | No | Yes | No |
| Water travel time | Yes | No | No | Yes | No | Yes | Yes |
| Pump-storage | Yes | No | No | No | Yes | No | No |
| Convex hull | No | No | No | Yes | No | No | No |

### B. STS problem solution

Table III presents the number of constraints and decision variables (continuous and integer) in each STS problem. In all studies, the MILP model was able to find and prove the global optimum solution. The solution times (desktop PC with PIII, 1.6 GHz processor and 512Mb RAM, running Xpress 14 MILP Solver [18]) are also shown in the Table.

TABLE III – PROBLEM SIZE AND SOLUTION TIME

| Country | BG | SL | RO | BR | NI | BO | EC |
|---|---|---|---|---|---|---|---|
| Constraints | 39K | 23K | 40K | 219K | 59K | 129K | 419K |
| Continuous variables | 28K | 25K | 42K | 69K | 55K | 134K | 375K |
| Integer variables | 5280 | 3530 | 6960 | - | 960 | 4830 | 4520 |
| Solution Time | 1.5h | 1.4m | 18s | 9.5m | 14m | 2.5m | 14m |

### C. Model results

A graphical interface/database is used to extract a range of results from the model solution, such as thermal variable and startup costs, circuit flows and losses, reservoir storage level, spilled and turbined outflows, marginal costs, water values, and others.

As an illustration, Fig. 3 shows the hydro and thermal hourly power production for Romania.

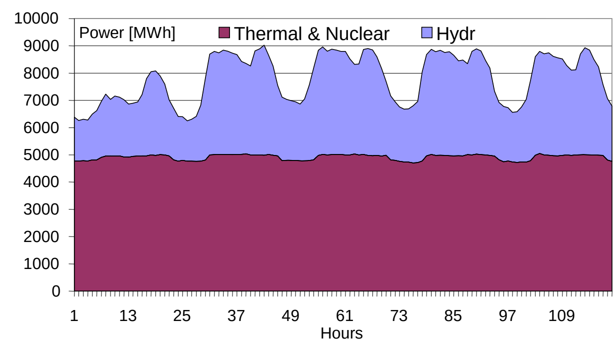


Fig. 3 – Total hourly hydro and thermal power production

### D. Computational aspects

As seen in Table II, the solution time does not necessarily increase with system dimensions. For example, the Romanian system solution took only 18 seconds, despite having almost 7,000 integer variables.

This behavior is typical of MILP systems. Also typical is the difference between finding the optimal solution and proving the optimality. For example, although 1.5 hours were spent to prove optimality for the Bulgarian system, the actual optimal solution had been found after 16 minutes.

## V. CONCLUSIONS

This paper presented a methodology for solving the transmission-constrained hydrothermal STS problem. The basic motivation is to develop a methodology that was flexible and robust enough to be used in several mid-sized systems, with different mixes of generation and other characteristics. The solution approach is to transform the problem nonlinearities and convexities into piecewise mixed linear integer (MILP) constraints. As shown in the paper, the MILP scheme allows the modeling of complex features for hydro, thermal and transmission. Also, by taking advantage of recent advances in commercial solver capabilities, the MILP scheme was found to be computationally efficient, as illustrated in case studies with seven countries in Latin America and Europe.

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