Representation of parameter uncertainty in probabilistic inflow models of SDDP

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ICSP 2016

Búzios, Brazil, June 2016

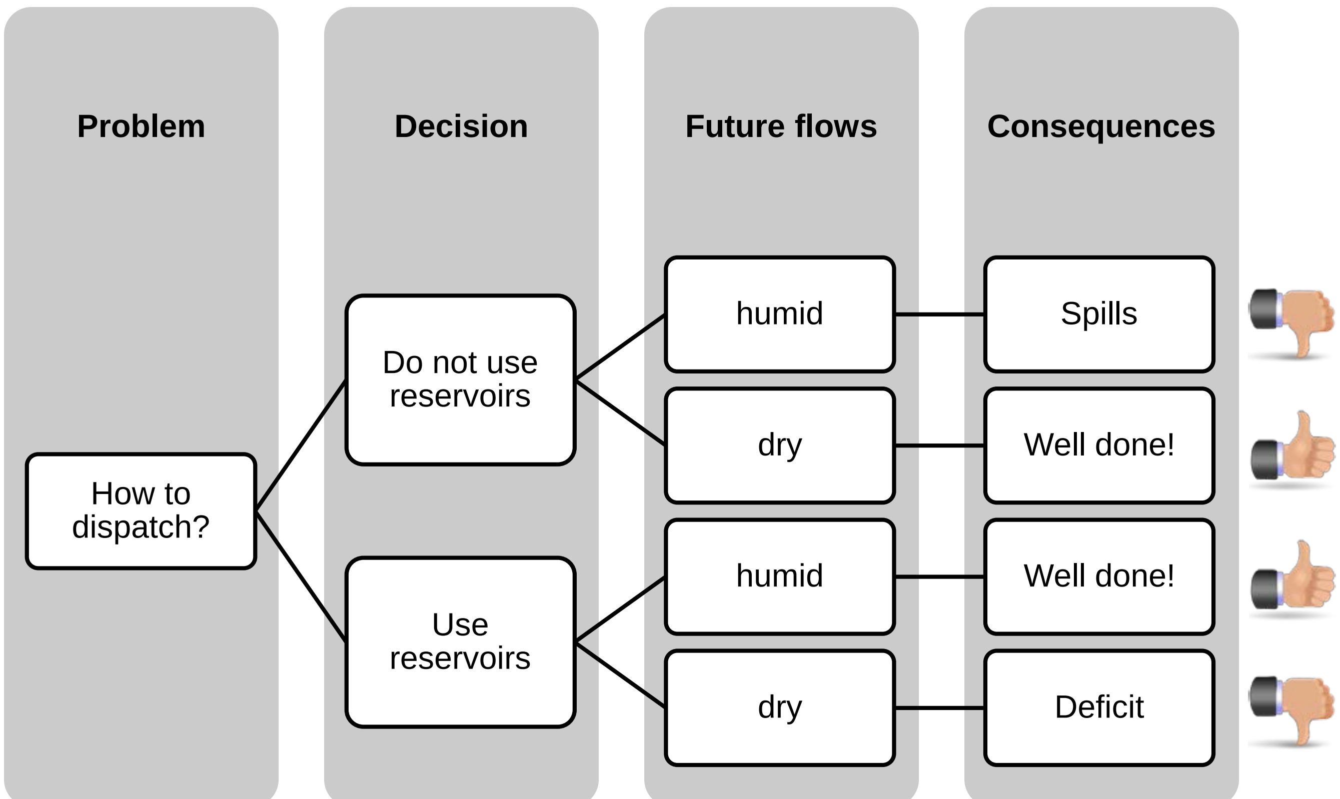
Outline

- Objective
- Generation of inflow scenarios with parameter uncertainty
- Selection of best inflow model
- SDDP policy calculation with different inflow models
- Conclusions

Hydrothermal dispatch p

The scheduling problem is solved by SDP or SDDP, where the original problem is decomposed in one-stage sub-problems

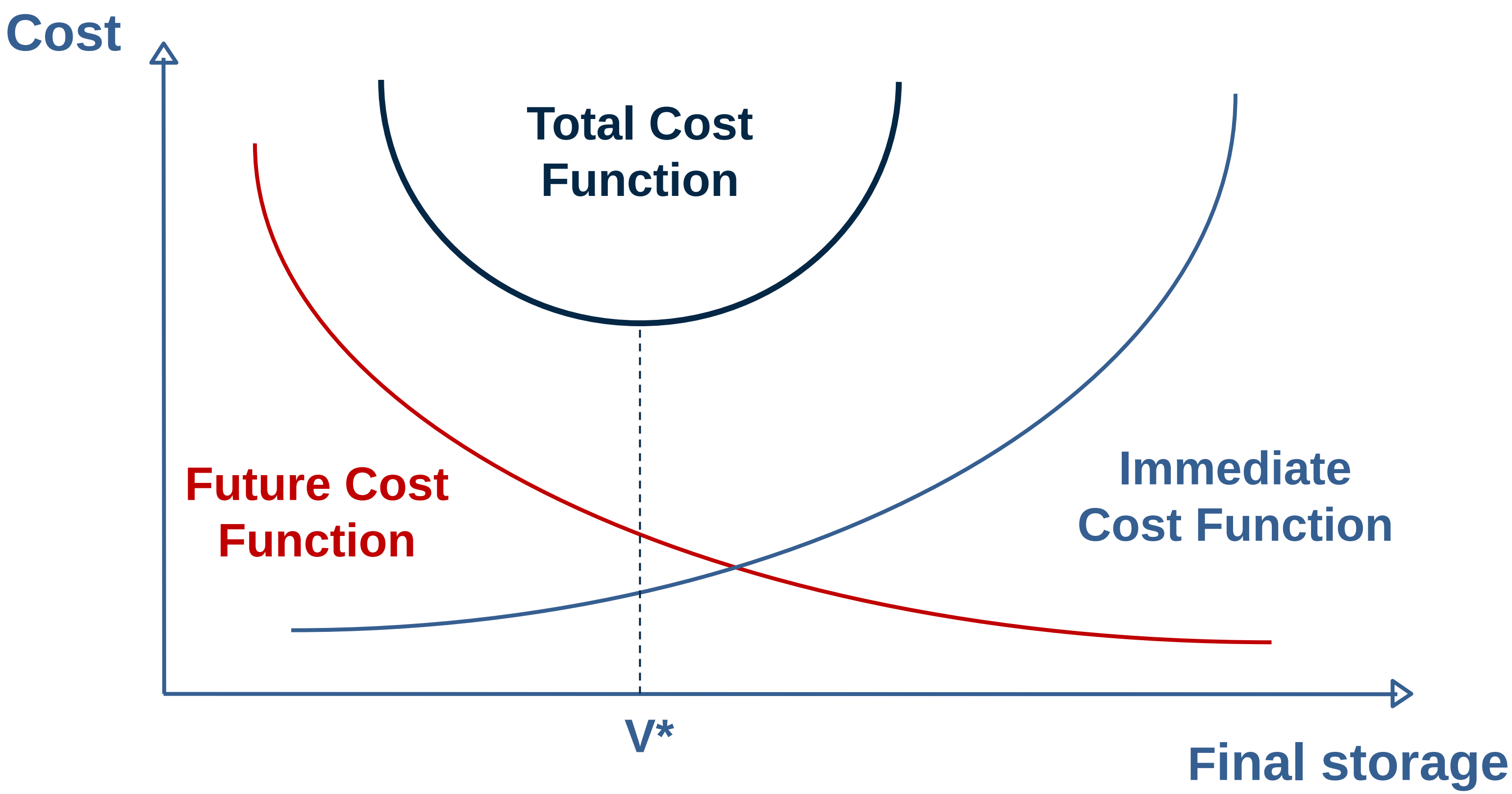
- Minimize present value of expected operation cost
  - Fuel costs + penalties for violation of operational constraints



Objective function: minimize total cost

$$\alpha_t(\hat{v}_t^s, \tilde{a}_t^s) = Min \sum_j c_j \sum_\tau g_{t,\tau,j} + \frac{1}{L} \sum_l \alpha_{t+1}^l$$

FCF with state variable $\tilde{a}_t^s$

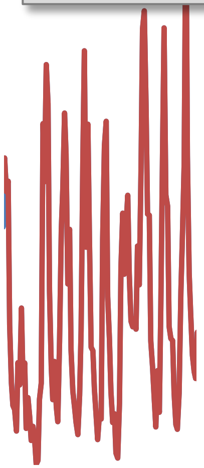


Parameter estimation

Hydro Furnas (1216 MW)

- Because it is impossible to have perfect forecasts of future inflows, uncertainty is represented through scenarios
  - Monte Carlo simulation based on **PAR(p) models**
  - Linearity of PAR(p) suitable for SDDP (convexity)

The PAR(p) model parameters are estimated from **historical data**



Historical data

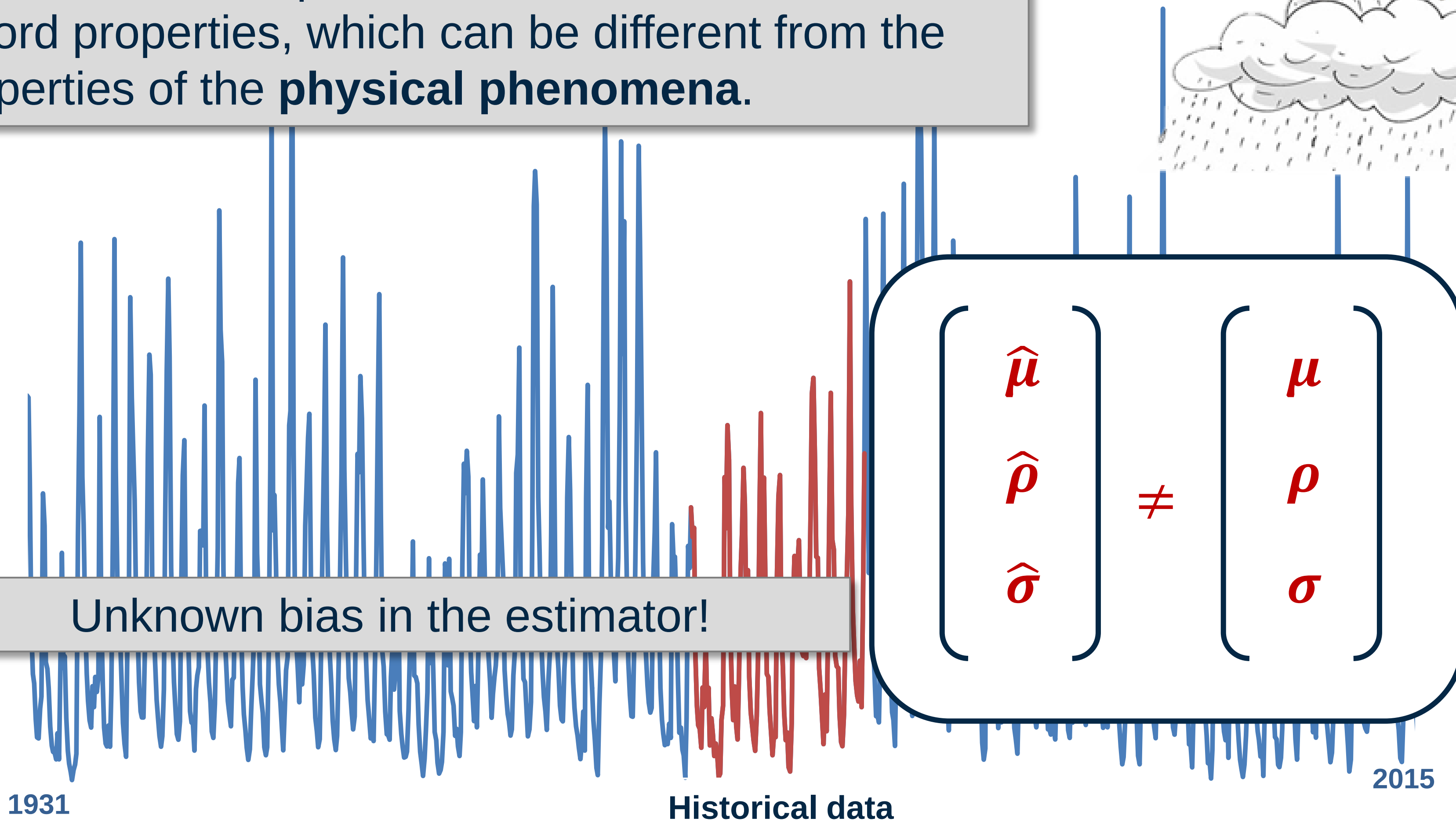
$\boldsymbol{mean}$: $\hat{\mu}$

$\boldsymbol{stdev}$: $\hat{\sigma}$

$\boldsymbol{corr}$: $\hat{\rho}$

Does the historical record truly represents the physical inflow process?

Synthetic streamflow generation based on these parameters will reproduce the historical inflow record properties, which can be different from the properties of the **physical phenomena**.



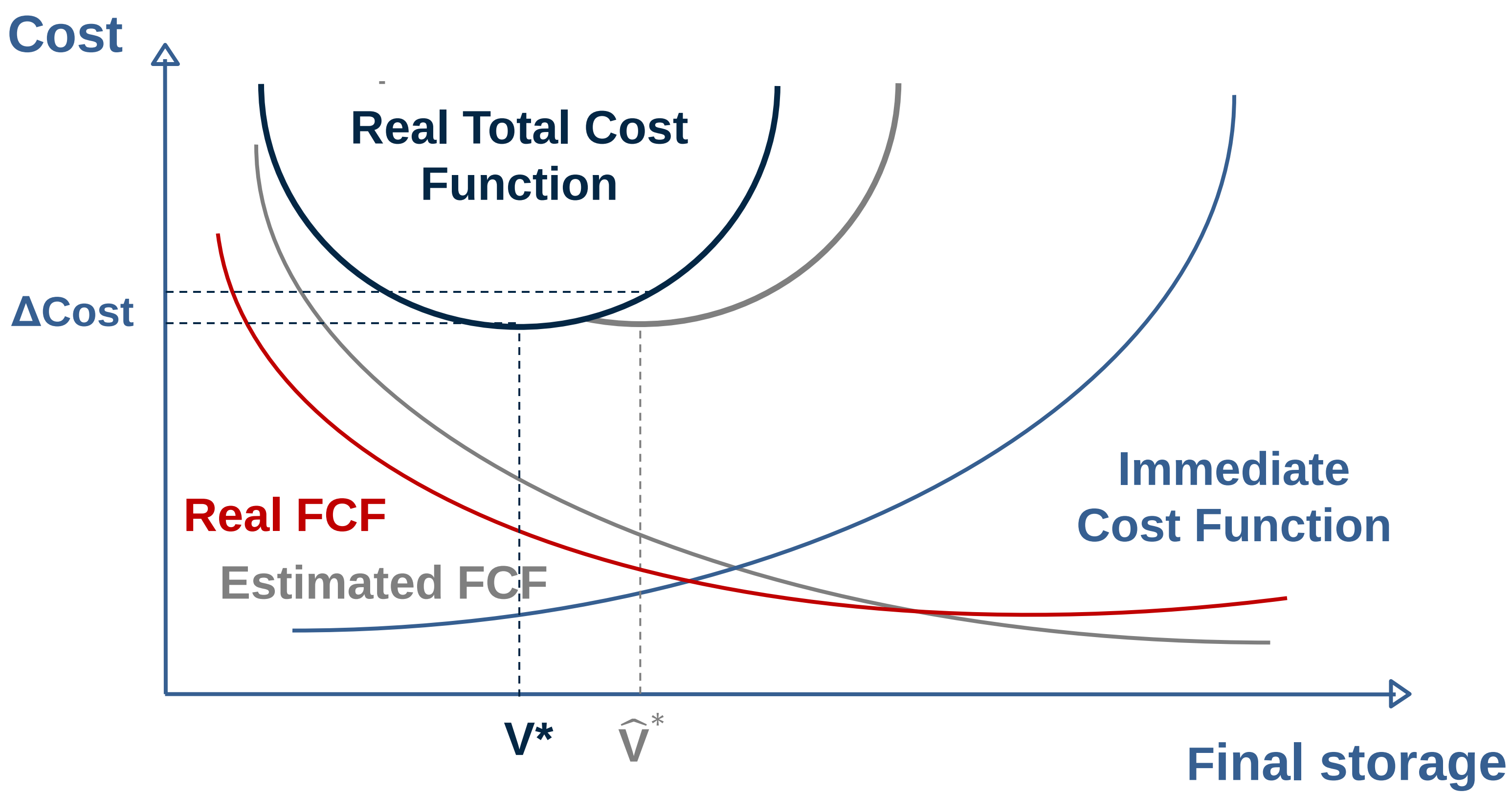
Unknown bias in the estimator!

$$\begin{bmatrix} \hat{\mu} \\ \hat{\rho} \\ \hat{\sigma} \end{bmatrix} \neq \begin{bmatrix} \mu \\ \rho \\ \sigma \end{bmatrix}$$

*São Pedro (St. Peter) is “responsible” for the rain in Brazil

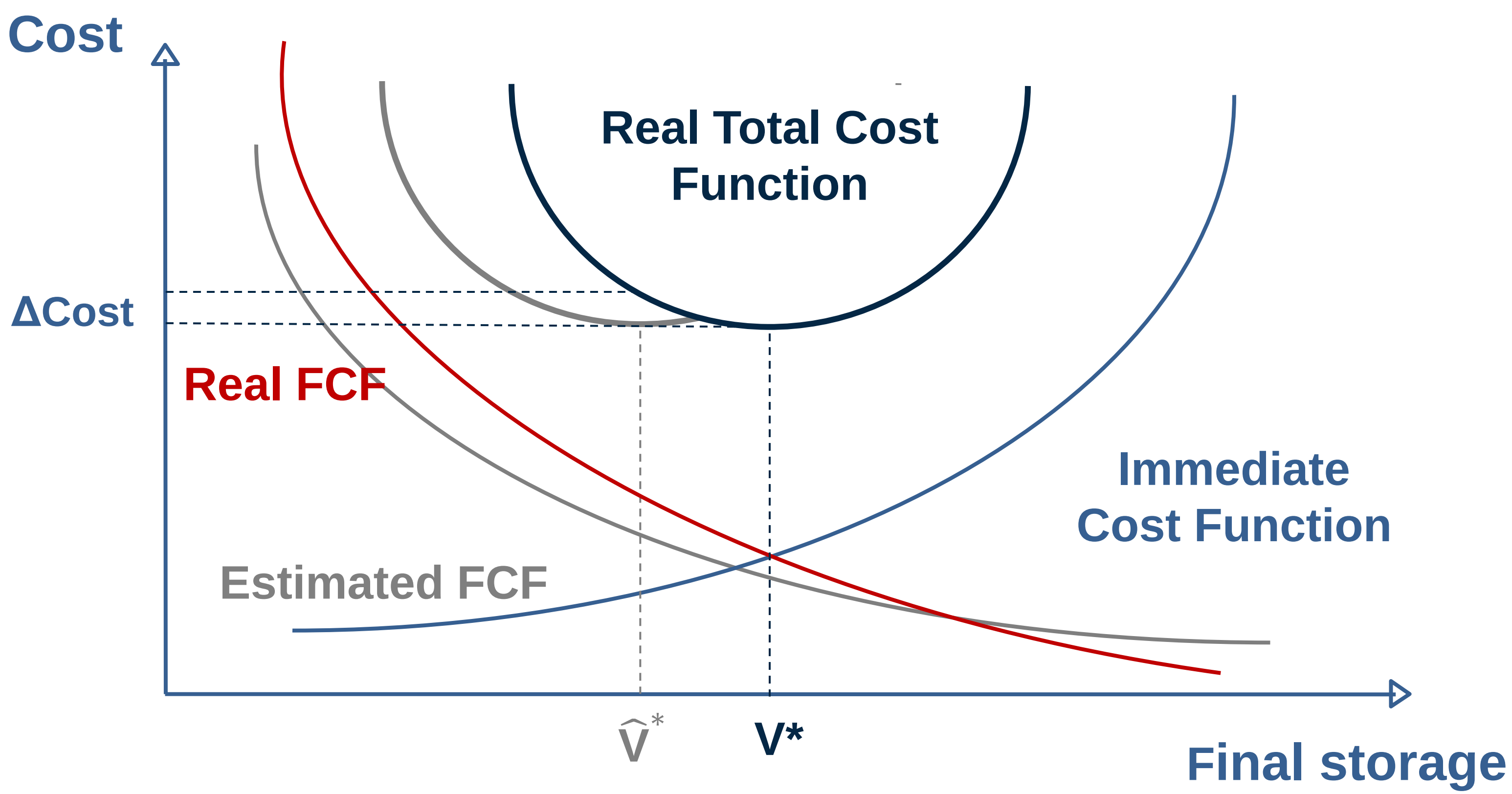
Impact on operation policy: negative bias

- Water may be unnecessarily stored and is likely to be spilled in the future.



Impact on operation policy

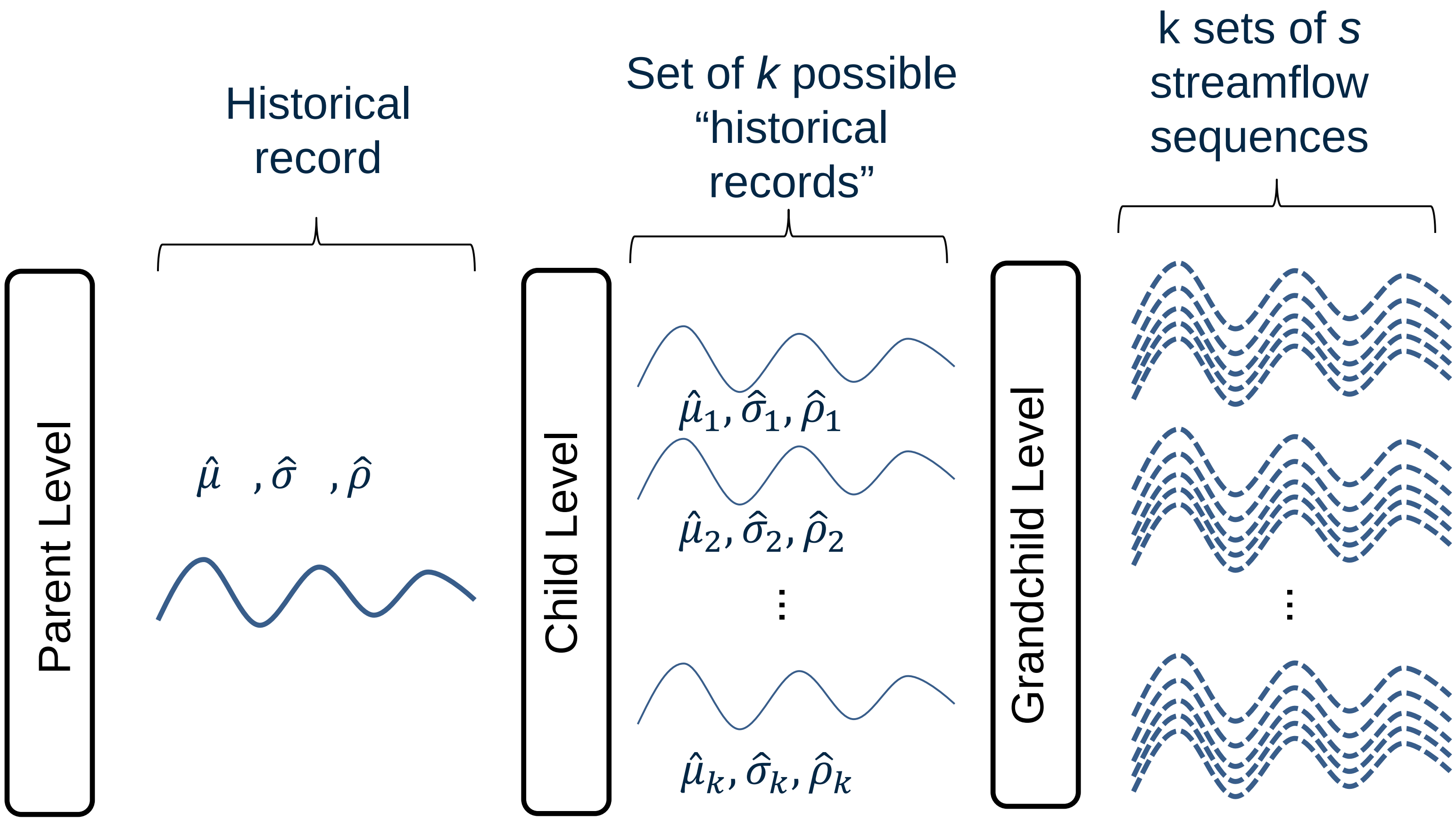
- Hydro reservoirs are depleted faster than needed, resulting in the dispatch of costly thermal plants in the future.



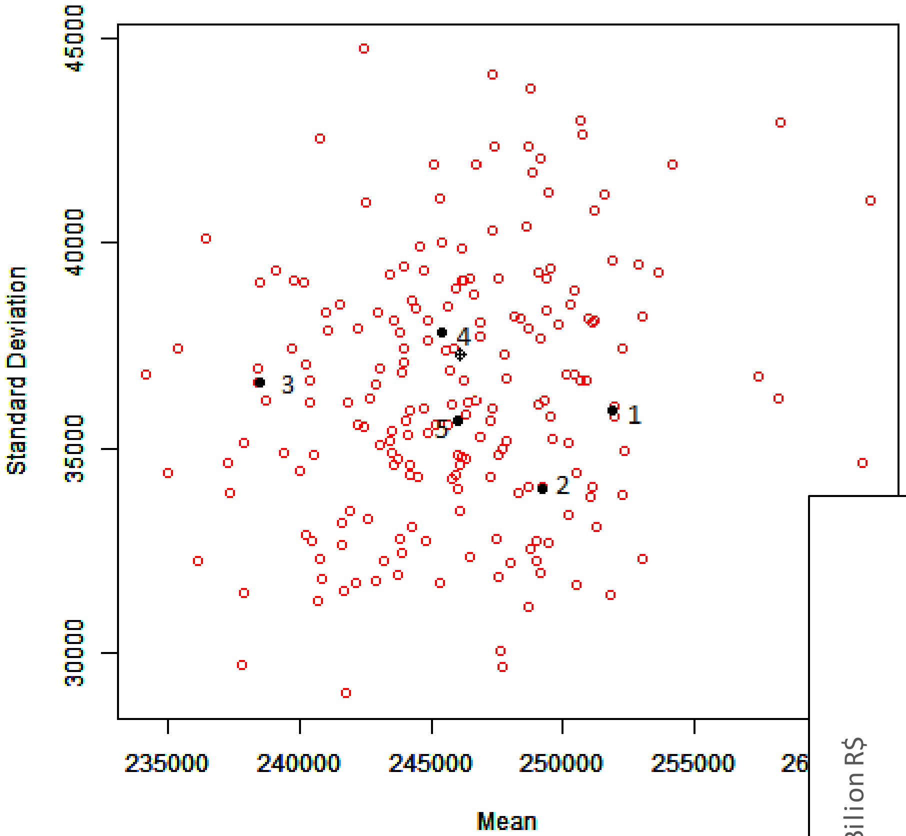
Objectives of this work

- Assess the impacts of incorporating the uncertainty of the PAR model parameters in the stochastic hydrothermal scheduling model.
- Develop a methodology to calculate a SDDP policy taking into account parameter uncertainty

Generation of inflow scenarios with parameter uncertainty

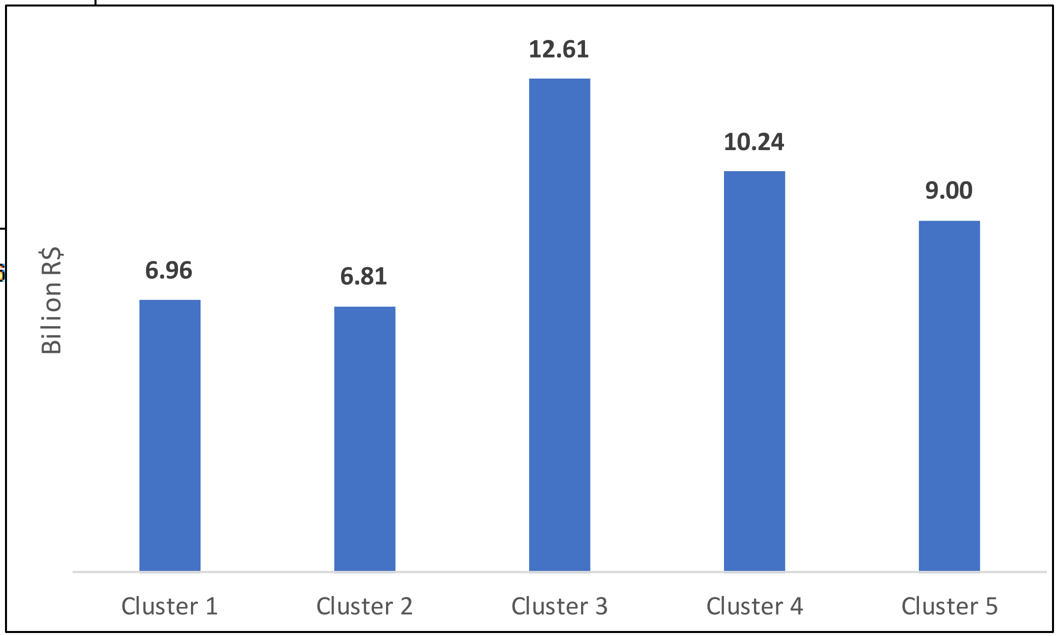


Impact of parameter uncertainty on operation costs



1. Calculate the operating policy with inflow model parameters from the historical record
2. Simulate the system operation with inflows produced by other sets of parameters

Case study for the Brazilian Power System (~140 GW), 10 year horizon, 200 hydrological scenarios



Parameter estimation as part of stochastic optimization

1. Selection of inflow best model

2. SDDP policy calculation with different inflow models

1. Selection of best inflow model: key idea

- Calculate “taylor made” operating policies for each set of inflow model parameters $m = 1, \dots, M$; simulate system operation with inflows produced by all the other parameters

- Decision criteria:

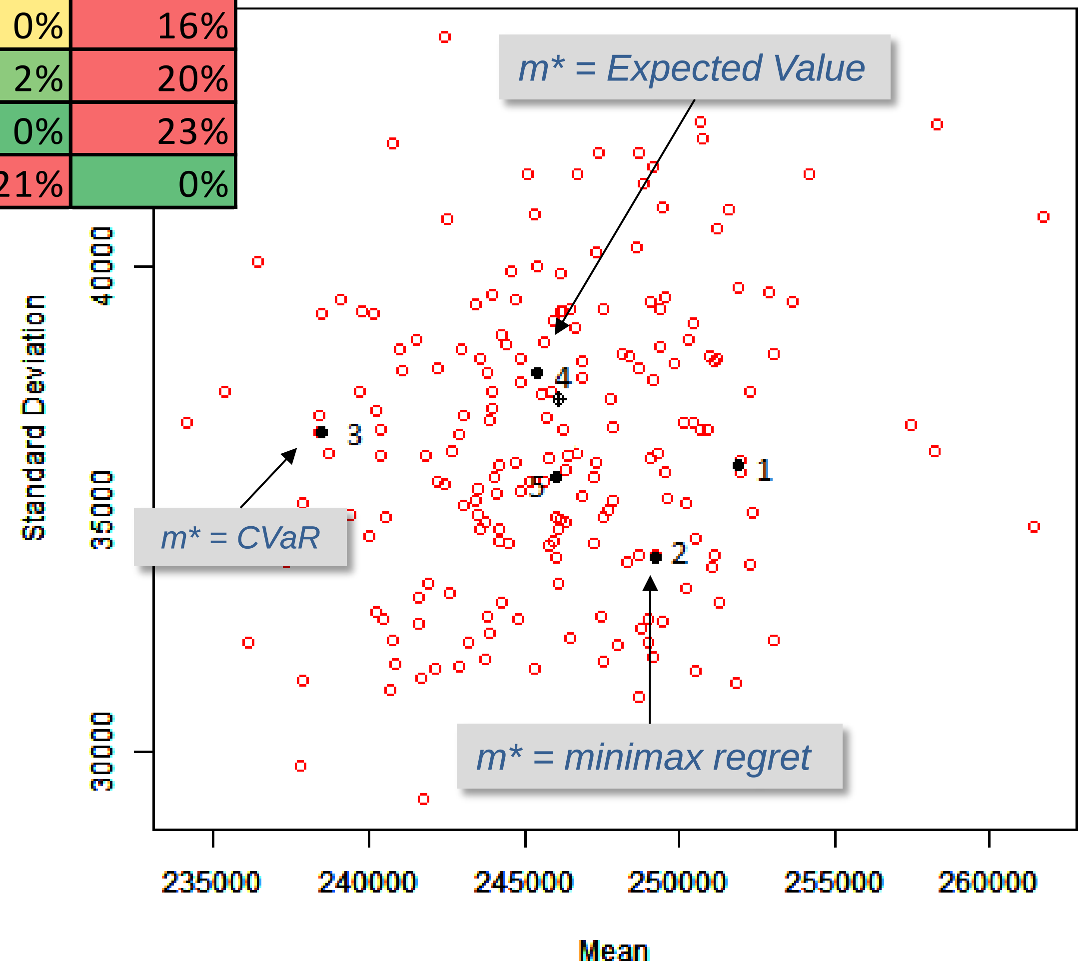
$$\begin{vmatrix} p_1 & \cdots & p_m & \cdots & p_{\mathcal{M}} \\ \tilde{z}_{11} & \dots & \tilde{z}_{1m} & \dots & \tilde{z}_{1\mathcal{M}} \\ \vdots & \ddots & & & \vdots \\ \tilde{z}_{m1} & & \tilde{z}_{mm} & & \tilde{z}_{m\mathcal{M}} \\ \vdots & & & \ddots & \vdots \\ \tilde{z}_{\mathcal{M}1} & \dots & \tilde{z}_{\mathcal{M}m} & \dots & \tilde{z}_{\mathcal{M}\mathcal{M}} \end{vmatrix}$$

  - Expected value: $m^* = \underset{m}{\operatorname{argmin}} \sum_n p_n \tilde{z}_{mn}$
  - Minimax regret: $m^* = \underset{m}{\operatorname{argmin}} \underset{n}{\operatorname{Max}} \{\tilde{z}_{mn} - \tilde{z}_{mm}\}$
  - Convex combination: $m^* = \underset{m}{\operatorname{argmin}} \left[\lambda \sum_n p_n \tilde{z}_{mn} + (1-\lambda) \underset{n}{\operatorname{Max}} \{\tilde{z}_{mn} - \tilde{z}_{mm}\}\right]$
  - CVaR: $m^* = \underset{m}{\operatorname{argmin}} \left[\lambda \sum_n p_n \tilde{z}_{mn} + (1-\lambda) \operatorname{CVaR}_q\{\tilde{z}_{mn}\}\right]$

1. Selection of best inflow model: case study

	Policies				
Simulations	P1	P2	P3	P4	P5
S1	0%	2%	6%	2%	18%
S2	0%	0%	5%	0%	16%
S3	6%	8%	0%	2%	20%
S4	4%	6%	2%	0%	23%
S5	18%	16%	18%	21%	0%

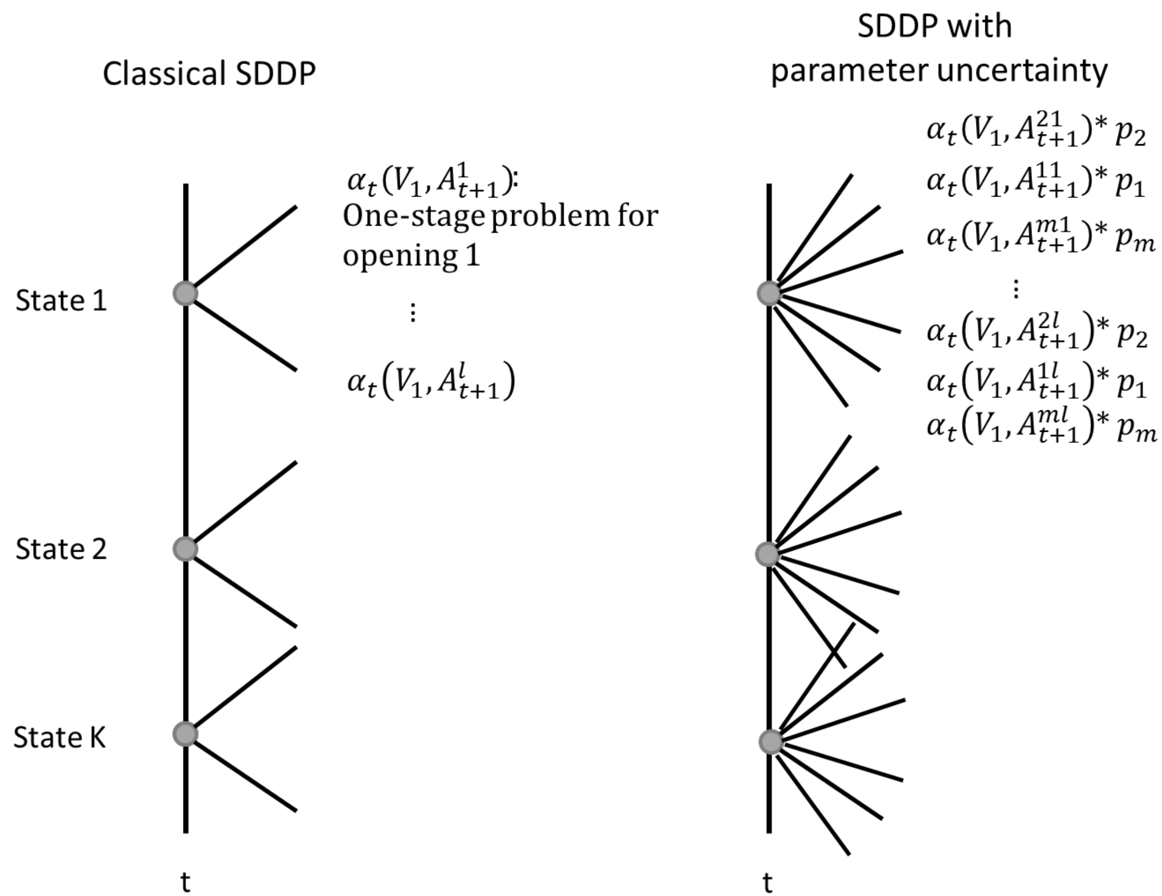
The "taylor made" policies are the best ones, as expected.



The "best" inflow model based on the policy calculation / simulation depends on the decision criteria.

2. SDDP policy with different inflow models: key idea

- Represent all the M alternative inflow models as part of the SDDP recursion



2. SDDP policy with different inflow models: formulation

- $\mathcal{M}$ PAR(1) models with probabilities $\{p_m, m = 1, \dots, \mathcal{M}\}$

$$\alpha_t(\hat{v}_t^s, \tilde{a}_t^s) = Min \sum_j c_j \sum_\tau g_{t,\tau,j} + \sum_m p_m \left[\frac{1}{L}\sum_l \alpha_{t+1}^{ml}\right]$$

FCF with state variable $\tilde{a}_t^s$

$$v_{t+1,i} = \hat{v}_{t,i}^s + \tilde{a}_{t,i}^s - (u_{t,i} + s_{t,i}) + \sum_{\eta \in M_i} (u_{t,\eta} + s_{t,\eta})$$

water balance

$$\sum_i e_{t,\tau,i} + \sum_j g_{t,\tau,j} = \hat{d}_{t,\tau} - \sum_n \hat{r}_{t,\tau,n}^s$$

demand balance

$$\frac{\left(a_{t+1,i}^{ml} - \hat{\mu}_{w(t+1),i}^m\right)}{\hat{\sigma}_{w(t+1),i}^m} = \hat{\rho}_{w(t),i}^m \times \frac{\left(\tilde{a}_{t,i}^s - \hat{\mu}_{w(t),i}^m\right)}{\hat{\sigma}_{w(t+1),i}^m} + \sqrt{1 - \left[\hat{\rho}_{w(t),i}^m\right]^2} \times \hat{\xi}_{t,i}^l \quad \forall i, l, m$$

stochastic model

$$\alpha_{t+1}^{ml} \geq \sum_i \hat{\varphi}_{ht+1,i}^p \times v_{t+1,i} + \sum_i \hat{\varphi}_{at+1,i}^p \times a_{t+1,i}^{ml} + \hat{\varphi}_{0t+1}^p \qquad \forall p, m, l$$

Benders cuts

2. SDDP policy with different inflow models: case study

- Test system with 1 hydro and 3 thermal plants
- 12 month study period; 4096 hydrological scenarios

		Final Simulation				
		Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5
Policy	Ω_1 Cluster 1	0.0%	0.2%	2.3%	1.2%	2.1%
	Ω_2 Cluster 2	0.6%	0.0%	2.2%	1.2%	2.1%
	Ω_3 Cluster 3	1.1%	0.8%	0.0%	0.5%	0.8%
	Ω_4 Cluster 4	34.5%	29.1%	50.8%	0.0%	81.4%
	Ω_5 Cluster 5	0.9%	1.1%	0.4%	0.4%	0.0%
	Ω_6 Parameter Uncertainty	0.6%	0.4%	0.0%	0.2%	0.5%

- The policy with parameter uncertainty minimizes both expected operation cost and maximum regret

And there are more improvements!

- The policy can be refined by using the probability of each model *conditioned* to the current inflow value $\tilde{a}^s_{t,i}$.

$$\alpha_t\left(\hat{v}^s_t, \tilde{a}^s_t, \mathcal{C}^{\mathscr{k}(s)}_t\right) = Min \sum_j c_j \sum_\tau g_{t,\tau,j} + \sum_\nu p^{\mathscr{k}\nu}_t \left[\sum_m p^{\mathscr{k}}_{tm} \left(\frac{1}{L}\sum_l \alpha^{ml\nu}_{t+1}\right)\right]$$

inflow cluster as state variable

- For each cluster $\mathcal{C}^{\mathscr{k}}_t$ there is an associated vector of model probabilities $\{p^{\mathscr{k}}_{tm}\}$
- The transition probability from cluster $\mathscr{k}$ in stage t to cluster ν in stage $t+1$ is $p^{\mathscr{k}\nu}_t$.

Conclusions

- Parameter uncertainty has a significant impact on system operating costs
- The representation of uncertainty in the operating policy minimized both expected operation cost and maximum regret
- The quality of the proposed policy can be improved by modeling the inflows as a Markov Chain, with transition probabilities between each cluster.

References

- B. Bezerra; A. Veiga Filho; L. A. Barroso; M. Pereira, "Stochastic Long-term Hydrothermal Scheduling with Parameter Uncertainty in Autoregressive Streamflow Models," in *IEEE Transactions on Power Systems,* 2016
- B. Bezerra, A Veiga, L. A. Barroso and M. Pereira, "Assessment of parameter uncertainty in autoregressive streamflow models for stochastic long-term hydrothermal scheduling," 2012 IEEE Power and Energy Society General Meeting, San Diego, CA, 2012, pp. 1-8.
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