Representation of uncertainties in fuel cost and load growth in SDDP-based hydrothermal scheduling

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Motivation

- Large scale multi-stage stochastic planning problems are in the "everyday" routine of energy planning (ISO's, agents, regulators): optimal generation scheduling, expansion planning etc
  - Multi-stage characteristic (= time coupling of decisions) comes from storage mechanisms (mainly hydro reservoirs)

- Those problems are successfully solved in real life via Benders decomposition-based methods, such as SDDP (Pereira and Pinto, 1991).

Motivation

- In centralized systems (e.g. Brazil), the ISOs solve a long-term stochastic least cost dispatch problem
  - Uncertainty on inflows and production of renewables

- In deregulated markets (e.g. Nordpool), the problem is to determine the dispatch schedule that maximizes revenues under uncertainty on inflows, production of renewables and spot prices
  - Price-taker modeling: Gjelsvik *et al.* (1999), Fosso *et al.* (1999) => Hybrid SDDP+SDP with Markov chain

Motivation

- Solving the least cost dispatch problem by SDDP:
  - Inflows: historical record is given => scenarios of future inflows are generated by Monte Carlo (stochasticity)
  - Availabilities of the plants and circuits are given (including modifications over time, planned maintenance schedules etc); outage sampling of equipment can be represented by Monte Carlo (stochasticity)
  - Production of renewables can be represented via scenarios, sampled by Monte Carlo (stochasticity)
  - Demand forecast is given (deterministic)
  - Fuel costs forecast is given (deterministic)

Motivation

- Usually planners perform what-if analysis w.r.t demand and fuel costs forecasts (base case, high, low) => stochasticity not considered => resulting scheduling is not prepared for the possible variability in those key elements.

- How to incorporate the uncertainty on both demand growth and fuel costs?

SDDP formulation - indices

Indices

- $t = 1, \ldots, T$ time stages (typically weeks or months)
- $\tau = 1, \ldots, \mathrm{T}$ intra-stage time blocks (e.g. peak, medium and low demand or 168 hours in a week)
- $s = 1, \ldots, S$ scenarios for each stage t produced by the stochastic models (typically inflows and renewable generation; also loads, equipment availability and fuel costs)
- $l = 1, \ldots, L$ set of scenarios for stage $t + 1$ conditioned to scenario s in stage t
- $i = 1, \ldots, I$ storage devices (typically hydro plants; also fuel storage, batteries, emission limits and some types of contracts)
- $m \in M_i$ set of hydro plants immediately upstream of plant i
- $j = 1, \ldots, J$ dispatchable devices (typically, thermal plants; also, some controllable renewables and price-responsive demand)
- $n = 1, \ldots, N$ non-dispatchable devices (typically, wind, solar and biomass)
- $\wp = 1, \ldots, \mathcal{P}$ number of hyperplanes (Benders cuts) in the future cost function

SDDP formulation – decision variables

Decision variables for the operation problem in stage t, scenario s

- $v_{t+1,i}$ stored volume of hydro i by the end of stage t
- $u_{t,i}$ turbined volume of hydro i stage t
- $s_{t,i}$ spilled volume of hydro i in stage t
- $e_{t,\tau,i}$ energy produced by hydro i in block τ, stage t
- $g_{t,\tau,j}$ energy produced by thermal plant j in block τ, stage t
- α_{t+1}^{l} present value of expected future cost from $t+1$ to T conditioned to scenario l in $t+1$

Note: for notational simplicity, we will not include the transmission network model in the formulations.

SDDP formulation – constants

Known values for the operation problem in stage t, scenario s

- $\hat{a}_{t,i}^{s}$ lateral inflow to hydro i in stage t, scenario s ($\hat{a}_{t}^{s}$ set of inflows for all hydro plants)
- $\hat{v}_{t,i}^{s}$ stored volume of hydro i in the beginning of stage t, scenario s ($\hat{v}_{t}^{s}$ set of stored volumes for all hydro plants)
- $\overline{v}_i$ maximum storage of hydro i
- $\overline{u}_i$ maximum turbined outflow of hydro i
- ρ_i production coefficient (kWh/m^3) of hydro i
- $\overline{g}_j$ maximum generation of thermal plant j
- c_j variable operating cost of thermal plant j
- $\hat{r}_{t,\tau,n}^{s}$ energy produced by renewable plant n in stage t, block τ, scenario s
- $\hat{d}_{t,\tau}$ demand of block τ, stage t

SDDP formulation – multipliers

Multipliers (dual variables)

- $\pi_{ht,i}$ multiplier of the storage balance equation of hydro i
- $\pi_{at,i}$ multiplier of the conditioned inflow equation of hydro i

SDDP formulation – Benders cut coefficients

$\mathscr{p}^{th}$ *Benders cut coefficients*

- $\hat{\varphi}^{\mathscr{p}}_{ht+1,i}$ coefficient of hydro plant i's storage, $v_{t+1,i}$
- $\hat{\varphi}^{p}_{at+1,i}$ coefficient of hydro plant i's inflow, $a^{l}_{t+1,i}$
- $\hat{\varphi}^{p}_{0t+1}$constant term

SDDP formulation – obj. function

Objective function (SDDP recursion)

$$\alpha_t(\hat{v}_t^s, \hat{a}_t^s) = Min \sum_j c_j \sum_\tau g_{t,\tau,j} + \frac{1}{L} \sum_l \alpha_{t+1}^l$$

SDDP formulation – storage balance

Storage balance for each stage

$$v_{t+1,i} = \hat{v}^s_{t,i} + \hat{a}^s_{t,i} - \left(u_{t,i} + s_{t,i}\right) + \sum_{m \in M_i} \left(u_{t,m} + s_{t,m}\right) \qquad \leftarrow \pi_{ht,i}$$

Note: for notational simplicity, we will not represent real-life features of the storage balance equations such as evaporation, filtration, water diversion for irrigation and city supply, transposition and others.

SDDP formulation – storage & turb. outflow limits

Storage and turbined outflow limits

$$v_{t+1,i} \leq \overline{v}_i$$

$$u_{t,i} \leq \overline{u}_i$$

SDDP formulation – energy balance & limits

Generation and demand balance for each block

$$e_{t,i} = \rho_i u_{t,i}$$

$$\sum_{\tau} e_{t,\tau,i} = e_{t,i}$$

$$e_{t,\tau,i} \leq \overline{e}_i$$

$$g_{t,\tau,j} \leq \overline{g}_j$$

$$\sum_{i} e_{t,\tau,i} + \sum_{j} g_{t,\tau,j} = \hat{d}_{t,\tau} - \sum_{n} \hat{r}^{s}_{t,\tau,n}$$

SDDP formulation – inflow model

Conditioned inflow scenarios for t+1

For simplicity of presentation, we show a multivariate $AR(1)$ model (in practice, SDDP uses a multivariate periodic autoregressive model (PAR) with lag up to 6):

$$\frac{\left(a_{t+1,i}^{l} - \hat{\mu}_{t+1,i}\right)}{\hat{\sigma}_{t+1,i}} = \hat{\phi}_{t,i} \times \frac{\left(\hat{a}_{t,i}^{s} - \hat{\mu}_{t,i}\right)}{\hat{\sigma}_{t,i}} + \sqrt{1 - \hat{\phi}_{t,i}^{2}} \times \hat{\xi}_{t,i}^{l} \quad \leftarrow \pi_{at,i}$$

Where the parameters $\left\{\hat{\mu}_{t,i}, \hat{\sigma}_{t,i}, \hat{\phi}_{t,i}\right\}$ are respectively the mean, standard deviation and serial correlation of the lateral inflow to hydro i in stage t. Spatial dependence is represented through a correlation matrix in the sampling of the innovation values $\hat{\xi}_{t,i}^{l}$ for all hydro plants.

Note: For clarity of presentation, the stochastic streamflow models are shown explicitly here. In the actual SDDP implementation, they are represented implicitly.

SDDP formulation – FCF

Future cost functions

As it is well known, the FCFs in SDDP are represented by a set of hyperplanes:

$$\alpha_{t+1}^{l} \geq \sum_{i} \hat{\varphi}_{ht+1,i}^{\wp} \times v_{t+1,i} + \sum_{i} \hat{\varphi}_{at+1,i}^{\wp} \times a_{t+1,i}^{l} + \hat{\varphi}_{0t+1}^{\wp}$$

$$\forall \wp = 1, \ldots, \mathcal{P}; l = 1, \ldots, L$$

SDDP – backward recursion

Backward recursion step

- After each one-stage dispatch problem is solved, we can generate a Benders cut to improve the future cost function approximation of the previous stage.
- Assuming that the FCF for the previous stage already has $\mathcal{P}$ hyperplanes, the Benders cut will correspond to the $(\mathcal{P}+1)^{th}$ FCF constraint:

$$\alpha_t^l \geq \sum_i \hat{\varphi}_{ht,i}^{\mathcal{P}+1} \times v_{t,i} + \sum_i \hat{\varphi}_{at,i}^{\mathcal{P}+1} \times a_{t,i}^l + \hat{\varphi}_{0t}^{\mathcal{P}+1}$$

SDDP – backward recursion

$$\alpha_t^l \geq \sum_i \hat{\varphi}_{ht,i}^{\mathcal{P}+1} \times v_{t,i} + \sum_i \hat{\varphi}_{at,i}^{\mathcal{P}+1} \times a_{t,i}^l + \hat{\varphi}_{0t}^{\mathcal{P}+1}$$

The Benders cut coefficients $\hat{\varphi}_{ht,i}^{\mathcal{P}+1}$, $\hat{\varphi}_{at,i}^{\mathcal{P}+1}$ and $\hat{\varphi}_{0t}^{\mathcal{P}+1}$ are calculated from a linear expansion of the optimal solution α_t^* of the one-stage dispatch problem:

$$\alpha_t(v_t, a_t) \approx \alpha_t^* + \sum_i \frac{\partial \alpha_t}{\partial v_{t,i}} \times \left(v_{t,i} - \hat{v}_{t,i}^s\right) + \sum_i \frac{\partial \alpha_t}{\partial a_{t,i}} \times \left(a_{t,i} - \hat{a}_{t,i}^s\right)$$

$\hat{\varphi}_{ht,i}^{\mathcal{P}+1} = \partial \alpha_t / \partial v_{t,i}$, which is the simplex multiplier $\pi_{ht,i}$.

$\hat{\varphi}_{at,i}^{\mathcal{P}+1} = \partial \alpha_t / \partial a_{t,i}$, calculated as: $\pi_{ht,i} + \left(\hat{\phi}_{t,i} / \hat{\sigma}_{t,i}\right) \times \pi_{at,i}$.

The constant term is obtained by adding all the constants of the linear expansion:

$$\hat{\varphi}_{0t}^{\mathcal{P}+1} = \alpha_t^* - \sum_i \hat{\varphi}_{ht,i}^{\mathcal{P}+1} \hat{v}_{t,i}^s - \sum_i \hat{\varphi}_{at,i}^{\mathcal{P}+1} \hat{a}_{t,i}^s$$

SDDP – forward step

Upper bound calculation

In stage t, scenario s of the forward simulation step, we calculate the immediate operation cost associated to the optimal solution (indicated by the superscript "*").

$$z_t^s = \sum_j c_j \sum_\tau g_{t,\tau,j}^*$$

As in the traditional SDDP formulation, the upper bound is calculated as:

$$\overline{z} = \frac{1}{S} \sum_t \sum_s z_t^s$$

SDDP – SDDP – Inflow scenario for stage $\boldsymbol{t}$+1

Conditional scenario (backward scenario)

We also calculate in the forward simulation step the inflow scenario vector for the next stage $t+1$: $\{\hat{a}^s_{t+1,i}, i = 1, \ldots, I\}$. This is done by sampling from the expression for the conditioned inflows:

$$\frac{\left(a^l_{t+1,i} - \hat{\mu}_{t+1,i}\right)}{\hat{\sigma}_{t+1,i}} = \hat{\phi}_{t,i} \times \frac{\left(\hat{a}^s_{t,i} - \hat{\mu}_{t,i}\right)}{\hat{\sigma}_{t,i}} + \sqrt{1 - \hat{\phi}^2_{t,i}} \times \xi^l_{t,i}$$

$$\forall l = 1, \ldots, L;\ \forall i = 1, \ldots, I$$

SDDP – Inflow scenario for stage $\boldsymbol{t}$+1

Forward scenario

Basically, $\hat{s}$ is randomly sampled from the set $\{1, \dots, L\}$, and the inflows $\{\hat{a}^{s}_{t+1,i}, i = 1, \dots, I\}$ are calculated for the corresponding innovation vector $\{\hat{\xi}^{\hat{s}}_{t,i}, i = 1, \dots, I\}$.

$$\frac{\left(a^{s}_{t+1,i} - \hat{\mu}_{t+1,i}\right)}{\hat{\sigma}_{t+1,i}} = \phi_{t,i} \times \frac{\left(\hat{a}^{s}_{t,i} - \hat{\mu}_{t,i}\right)}{\hat{\sigma}_{t,i}} + \sqrt{1 - \hat{\phi}^{2}_{t,i}} \times \hat{\xi}^{\hat{s}}_{t,i} \quad \forall i = 1, \dots, I$$

We have two sampling possibilities: either we pre calculate $a_{t,i}$ for all t before even starting the SDDP recursion or we repeat the above described procedure in each iteration (re-sampling) so that we have higher probability of generating new cuts

Markov chains

- The Markov chain scheme allows the representation of stochastic processes that are not well represented by autoregressive models, such as annual load growth rates, as well as uncertainties in coefficients of the objective function, such as the operating costs of thermal plants (which result from uncertainties in fuel costs) and spot prices.

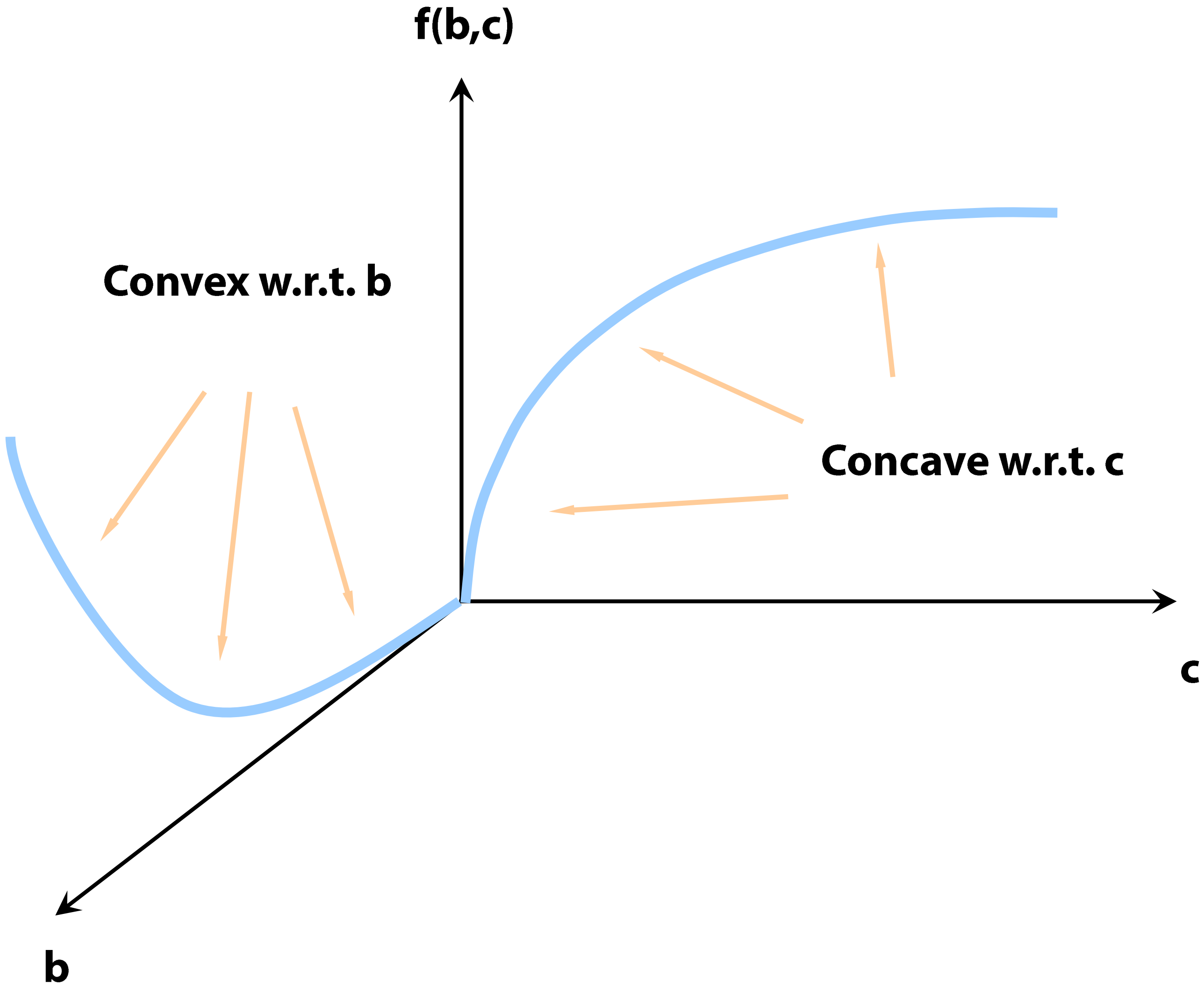
A few comments on the convexity of the FCF

The function:

$$f(b,c) = \underset{\substack{s.to \\ Ax \geq b}}{Min\,cx} = \underset{\substack{s.to \\ \pi A \leq c}}{Max\,\pi b}$$

- For a fixed c, $f(b,c)$ is convex and can be piecewise-linearly approximated on b.
- For a fixed b, $f(b,c)$ is concave and can be piecewise-linearly approximated on c.
- But as a joint function of b and c, $f(b,c)$ is neither convex nor concave (is saddle-shaped).

A few comments on the convexity of the FCF

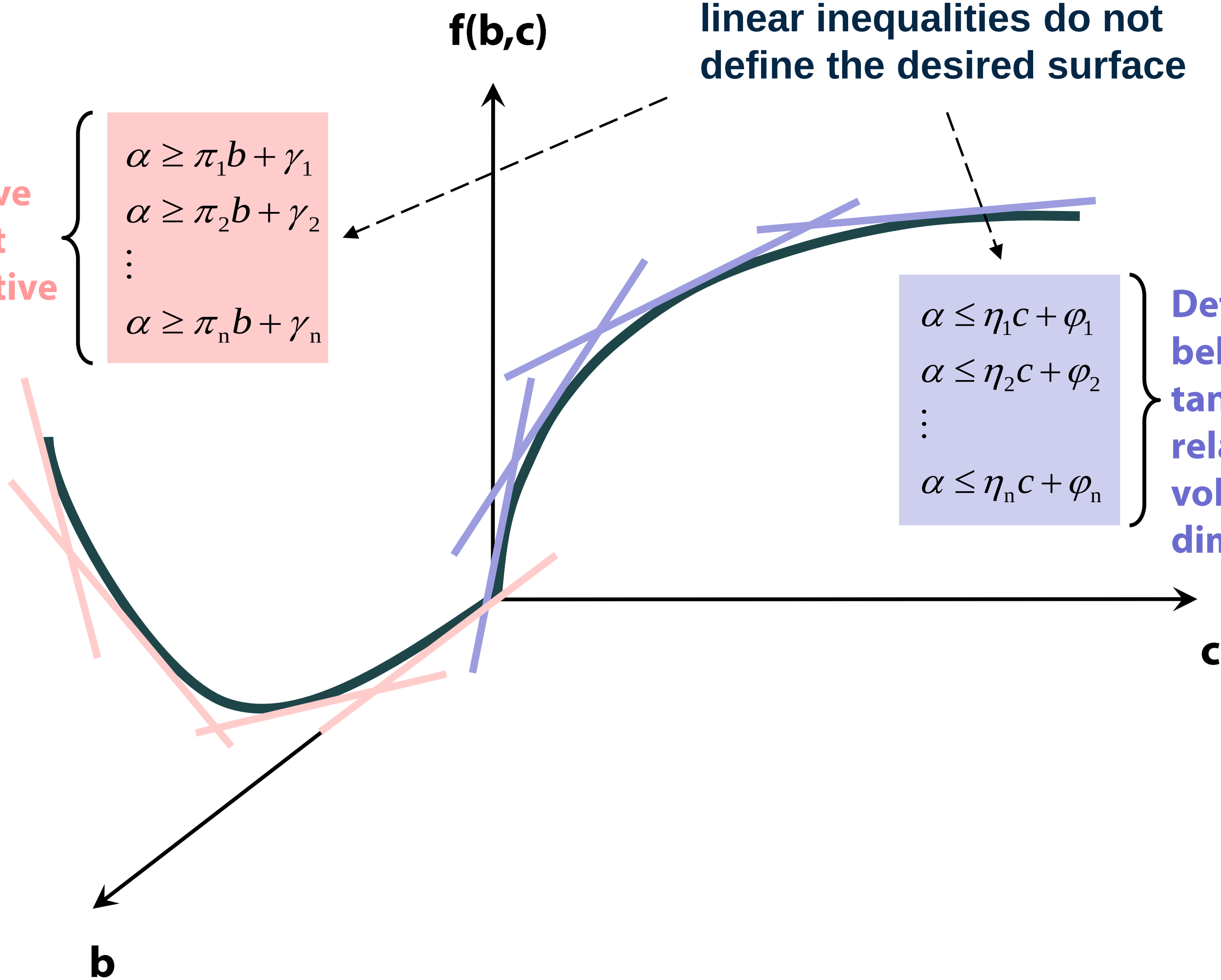


A few comments on the convexity of the FCF

Define the region above the tangent planes relative to the price dimension

$$
\left\{
\begin{aligned}
&\alpha \geq \pi_1 b + \gamma_1 \\
&\alpha \geq \pi_2 b + \gamma_2 \\
&\vdots \\
&\alpha \geq \pi_n b + \gamma_n
\end{aligned}
\right.
$$

Jointly, these sets of linear inequalities do not define the desired surface

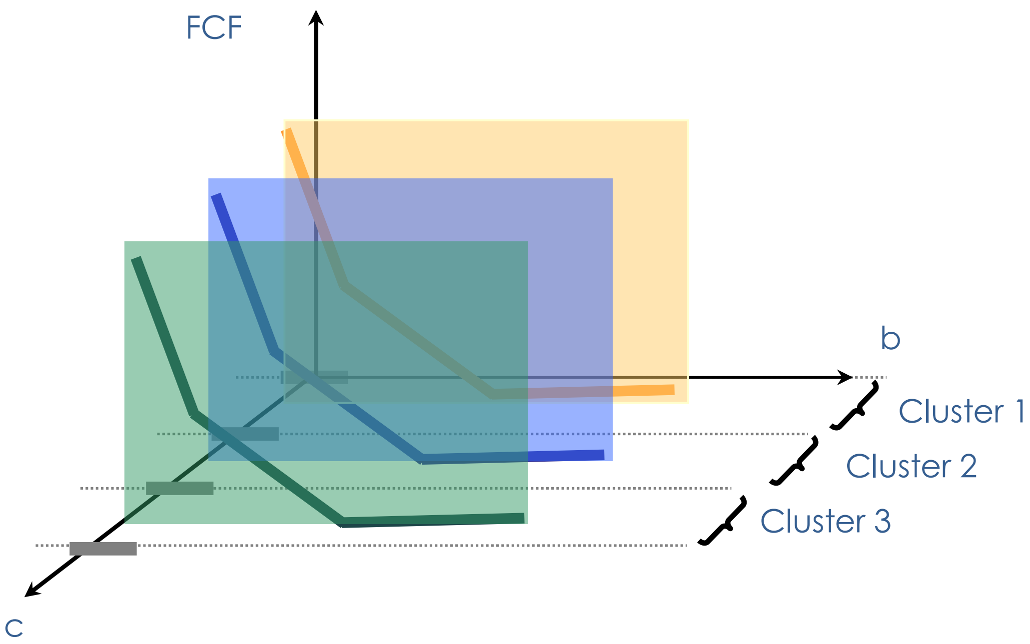


$$
\left.
\begin{aligned}
&\alpha \leq \eta_1 c + \varphi_1 \\
&\alpha \leq \eta_2 c + \varphi_2 \\
&\vdots \\
&\alpha \leq \eta_n c + \varphi_n
\end{aligned}
\right\}
$$

Define the region bellow the tangent planes relative to the volume dimension

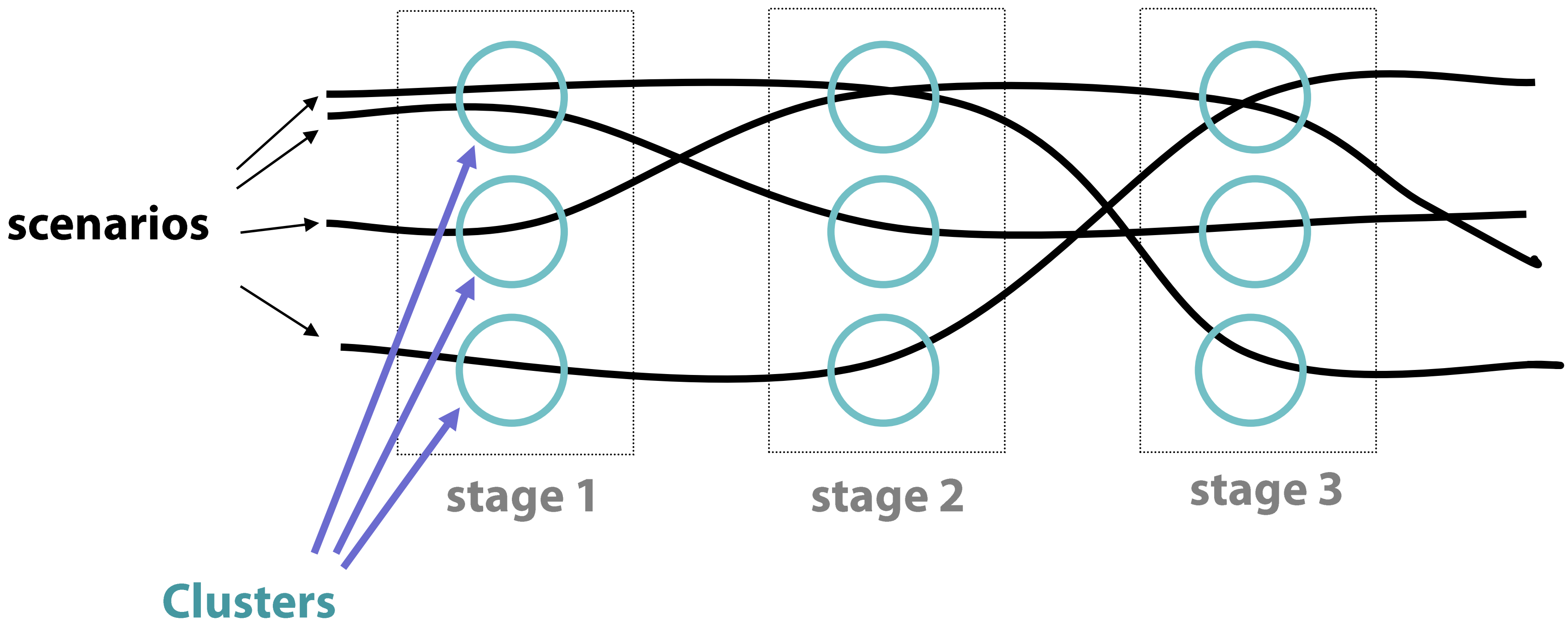
Back to Markov chains

- The Markov chain is represented by $k = 1, \ldots, K$ clusters (states) (for notational simplicity, we assume that K is the same in all stages).



Back to Markov chains

- The Markov chain transition probability from state k in stage t to state m in stage $t+1$ is represented by p_t^{km}.



Markov chains

- Each cluster k contains $n = 1, \ldots, \mathcal{N}_t^k$ sets of *values*, where each set may contain a vector of operating costs for the thermal plants; or a vector of spot prices for each load block; or an annual load growth rate.
- With the Markov scheme we have K separate piecewise linear future cost functions (FCFs) in each stage, represented by $\alpha_t^k(\hat{v}_t^s, \hat{a}_t^s)$, $k = 1, \ldots, K$. This means that, for each scenario s and stage t, in addition to the inflows $\hat{a}_t^s$, we need to sample: (i) one of the states of the Markov chain, represented by $\hat{k}_t^s$; and (ii)one of the $\mathcal{N}_t^k$ *values* (or vector of values) contained in that cluster.

Markov chains

- We also need to sample Markov states for each of the L conditioned inflow vectors for stage $t+1$, $\left\{a_{t+1}^{l}, l = 1, \ldots, L\right\}$. This is done by sampling L times from the transition probabilities from (the already sampled) state $\hat{k}_t^s$ to states in stage $t+1$: $\left\{p_t^{\hat{k}^s m}, m = 1, \ldots, K\right\}$.

Uncertainty on fuel costs

- In this case, each cluster k in stage t contains $\mathcal{N}_t^k$ vectors of operating costs for the J thermal plants: $\left[c_{tj}^{kn}, j = 1, \dots, J\right]$, n= 1, …, $\mathcal{N}_t^k$. As previously mentioned, one of those vectors, represented as c_{tj}^{ks}, will be sampled and used in the dispatch problem.

Uncertainty on fuel costs

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$$\alpha_t^{\hat{k}^s}(\hat{v}_t^s, \hat{a}_t^s) = Min \sum_j c_{tj}^{ks} \sum_\tau g_{t,\tau,j} + \frac{1}{L} \sum_l \alpha_{t+1}^{m_l}$$

Uncertainty on fuel costs

- In order to understand the FCF formulation in this case, we should recall that in the basic SDDP formulation there is only *one* future cost function whose value is calculated L times, each with a different a_{t+1}^{l}, that is, $\alpha_{t+1}\left(v_{t+1}, a_{t+1}^{l}\right), l = 1, \ldots, L$.
- In the Markov formulation, however, we have K different FCFs for stage $t+1$, each corresponding to one of the clusters in the Markov matrix. As mentioned, we sampled one of the clusters – and, thus, one of the FCFs - for each of the L conditioned inflow values. The sampled cluster for conditioned inflow l is represented as $m(l)$. For this reason, the FCF is given by:

$$\alpha_{t+1}^{m_l} \geq \sum_i \hat{\varphi}_{ht+1,i}^{m_l \wp} \times v_{t+1,i} + \sum_i \hat{\varphi}_{at+1,i}^{m_l \wp} \times a_{t+1,i}^{l} + \hat{\varphi}_{0t+1}^{m_l \wp} \qquad \forall \wp = 1, \ldots, \mathcal{P}_t^{m_l}; l = 1, \ldots, L$$

Uncertainty on fuel costs

- Note that the SDDP problem size with the Markov model is the same as the basic formulation; the difference is on the values of the Benders cut coefficients.
- As a consequence, it is reasonable to expect that a Markov chain representation will not have a significant impact on SDDP's execution time.
  - The situation is similar to the representation of higher-order autoregressive inflow models, which does not affect running times.

Uncertainty on fuel costs

Benders cut calculation for the previous stage (FCF improvement):

- The Benders cut calculation is similar to the basic formulation. We just have to remember that the cut will be added to only one of the K FCFs of stage t, corresponding to the sampled cluster $\hat{k}_t^s$.

Uncertainty on annual load growth rates

- In this case, the Markov chain transition for December to January (or last week to the first week) of each year contains possible values of load growth rates $\{\zeta_t^{kn}\}, n = 1, \ldots, \mathcal{N}_t^k$ for each state (for example, "low", "medium" or "high" growth).
  - For the other stages in the year, the clusters contain the value 1.0, i.e. no growth.
  - Also, the transition probabilities in the Markov chains from January to February, February to March etc. until November to December have "1.0" in the diagonal and zero elsewhere, indicating that there are no Markov transitions within the year; only the Markov chain from December to the following January have transition probabilities.

Uncertainty on annual load growth rates

- In the beginning of each year, the average annual load for that year is obtained by multiplying the previous year's average load by one of the load growth rates in the sampled cluster.
  - This means that the average annual load becomes a state variable.

- The load $d_{t,\tau}$ in each stage t and block τ is obtained by multiplying the average annual load by a *disaggregation factor* $\widehat{\Delta}_{t,\tau}$. (Note that, by construction, $\sum_{\tau} \widehat{\Delta}_{t,\tau} = 1.$)

Uncertainty on annual load growth rates

The dispatch problem is formulated as:

$$\alpha_t^{\hat{k}^s}\left(\hat{v}_t^s, \hat{a}_t^s, \bar{d}_t^s\right) = Min \sum_j c_j \sum_\tau g_{t,\tau,j} + \frac{1}{L}\sum_l \alpha_{t+1}^{m_l}$$

$$\ldots$$

$$\hat{d}_{t,\tau}^s = \widehat{\Delta}_{t,\tau} \times \bar{d}_t^s \qquad\qquad \leftarrow \pi_{\bar{d}t,\tau}$$

$$\sum_i e_{t,\tau,i} + \sum_j g_{t,\tau,j} = \hat{d}_{t,\tau}^s - \sum_n \hat{r}_{t,\tau,n}^s$$

$$\frac{\left(a_{t+1,i}^l - \hat{\mu}_{t+1,i}\right)}{\hat{\sigma}_{t+1,i}} = \hat{\phi}_{t,i}^1 \times \frac{\left(\hat{a}_{t,i}^s - \hat{\mu}_{t,i}\right)}{\hat{\sigma}_{t,i}} + \hat{\phi}_{t,i}^2 \times \hat{\xi}_{t,i}^l \qquad\qquad \leftarrow \pi_{at,i}$$

$$\bar{d}_{t+1}^s = \hat{\zeta}_t^{\hat{k}^s n} \times \bar{d}_t^s \qquad\qquad \leftarrow \pi_{\bar{d}t}$$

$$\alpha_{t+1}^{m_l} \geq \sum_i \hat{\varphi}_{ht+1,i}^{m_l \wp} \times v_{t+1,i} + \sum_i \hat{\varphi}_{at+1,i}^{m_l \wp} \times a_{t+1,i}^l + \hat{\varphi}_{\bar{d}t+1}^{m_l \wp} \times \bar{d}_{t+1}^s + \hat{\varphi}_{0t+1}^{m_l \wp} \qquad \forall \wp, l$$

Uncertainty on annual load growth rates

- As seen, $\hat{\zeta}_t^{\hat{k}^s n}$ is equal to 1.0 for all stages, except December (or the last week) of each year.
- Finally, the Benders cut coefficient $\hat{\varphi}_{\bar{d}t}^{m_l\mathcal{P}+1}$ for the previous stage is given by:

$$\hat{\varphi}_{\bar{d}t}^{m_l\mathcal{P}+1} = \hat{\zeta}_t^{\hat{k}^s n} \times \pi_{\bar{d}t} + \sum_{\tau} \widehat{\Delta}_{t,\tau} \times \pi_{\bar{d}t,\tau}$$

PSR

Thank you!

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