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Optimal capacity expansion planning applied to risk-averse nested optimization problems

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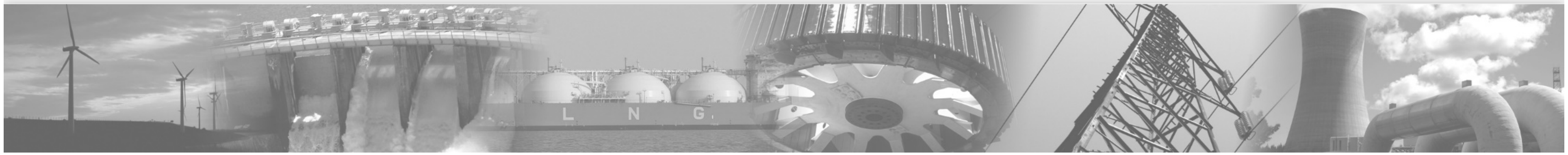


Integrated Expansion-Operation problem

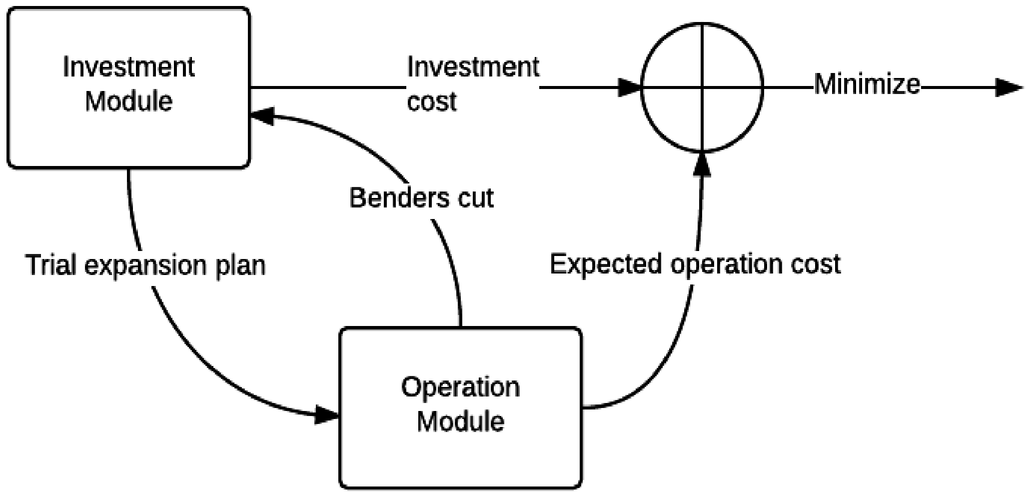
- The main interest of this presentation is proposing a revised solution strategy for the investment problem coupled with the dispatch problem, for systems which involve highly time-coupled stochastic operation decisions
- The problem is well suited to a master problem + slave problem structure, in which an optimization of the dispatch problem (maintaining investment decisions constant) returns Benders cuts to the investment problem
- We are interested in using an SDDP approach for the operations problem

Integrated Expansion-Operation problem

$$Min \sum_{t\in\mathbb{T}} \sum_{i\in\mathbb{I}} I_{t,i} x_{t,i} + w$$

$$x_{t,i} \geq x_{t-1,i} \quad \forall t \in \mathbb{T}\backslash\{0\}; \forall i \in \mathbb{I}$$

$$w \geq w_0 + \sum_{t\in\mathbb{T}} \sum_{i\in\mathbb{I}} \mu_{t,i}^k x_{t,i} \quad \forall k \in \mathbb{K}$$



$t \in \mathbb{T}$ periods

$i \in \mathbb{I}$ expansion candidates

$k \in \mathbb{K}$ Benders cuts from slave problem

$\mu_{t,i}^k$ linear Benders coefficients from slave problem

$I_{t,i}$ fixed cost of the expansion decision

$x_{t,i}$ expansion decision

Problem loop

I. Initialize investment decisions $\hat{x}_t$

II. Repeat until convergence of the expansion problem:

  1. Initialize state variables $\hat{z}_t^s$
  2. Repeat until convergence of operational problem:
     i. Backward iteration: determine lagrange multipliers φ_{t} for each stage t and each trajectory s and add cuts to stage $t-1$
     ii. Forward iteration: determine $\hat{z}_{t+1}^s$ for each stage t and each trajectory s and update the state variable at $(t+1, s)$
  3. Using the converged case, determine lagrange multipliers μ_t for each stage t and each trajectory s and add cuts to the expansion problem

Approximation to the operative cost

- Because the slave operation problem is solved via SDDP, there are two "natural" ways to represent the operational costs as a function of the investment decisions:

Bounds:	$UB = \frac{1}{\lvert\mathbb{S}\rvert}\sum_{t\in\mathbb{T}}\sum_{s\in\mathbb{S}} C_t^s\,(\hat{z}_t^s, \hat{x}_t)$	$LB = \alpha_0(\hat{z}_0)$
Operative cost estimate:	$w \geq w_0 + \frac{\partial UB}{\partial x_i}\cdot x_i$	$w \geq w_0 + \frac{\partial LB}{\partial x_i}\cdot x_i$
Marginal cost estimate:	$\frac{\partial UB}{\partial x_i} = \frac{1}{\lvert\mathbb{S}\rvert}\sum_{t\in\mathbb{T}}\sum_{s\in\mathbb{S}} \underbrace{\frac{\partial C_t^s}{\partial x_i}}_{\substack{dual\\ variable}}$	$\frac{\partial LB}{\partial x_i} = \underbrace{\frac{\partial \alpha_0}{\partial x_i}}_{\substack{dual\\ variable}}$

Approximation to the operative cost

- The “classical” way of handling the investment problem in this master-slave structure is to use the upper bound representation, seeing that the dual variables can be easily derived from the slave problem’s constraints for each t, s

$$g^T_{t,s,\tau} \leq \hat{x}^T_t \cdot \bar{g}_t \qquad \leftarrow \mu^T_{t,s,\tau} \quad \text{[thermal]}$$

$$g^R_{t,s,\tau} = \hat{x}^R_t \cdot \bar{g}^R_t \cdot \hat{r}_{t,s,\tau} \qquad \leftarrow \mu^R_{t,s,\tau} \quad \text{[renewable]}$$

$$u^H_{t,s,\tau} \leq \hat{x}^H_t \cdot \bar{u}^H_t \qquad \leftarrow \mu^H_{t,s,\tau} \quad \text{[hydro]}$$

$$v_{t,s,\tau} = \hat{x}^H_t \cdot \bar{v}^H_t \qquad \leftarrow \mu^H_{t,s,\tau} \quad \text{[hydro]}$$

- Generally, we may write these multipliers as follows:

$$x_t = \hat{x}_t \qquad \leftarrow \mu_t$$

Lower bound-based cuts

- However, the cuts obtained in this manner are only guaranteed to be lower bounds to the operational cost if the slave problem has already reached convergence
- Therefore, we propose a new formulation for the cuts based on the simulation's **lower bound**
- To represent this dual variable dependency, we wish to write the first-period lower bound α_0 as a function of each of the decision variables $\{x_\tau\}_{\tau \in \mathbb{T}}$:

$$\alpha_0 \geq \left(\Phi^p + \varphi_0^p \cdot \hat{z}_0\right) + \sum_{t \in \mathbb{T}} \xi_{t,0}^k \cdot x_t \quad \forall k \in \mathbb{K}$$

Lower bound-based cuts

- The core principle to obtain these marginal values would involve introducing all subsequent stages' capacities as state variables represented in the future cost function:

$$\pi_t^{s,p,l} \quad \rightarrow \quad \alpha_{t+1}^{l} \geq \Phi^p + \varphi_0^p \cdot z_{t+1}^{s,l} + \sum_{\theta \in \mathbb{T}_{t+1}} \xi_{\theta,t+1}^{p} \cdot x_\theta \quad \forall p \in \mathbb{P}$$

$$\varphi_t^s \quad \rightarrow \quad z_t^s = \hat{z}_t^s$$

$$\xi_{\theta,t}^{s} \quad \rightarrow \quad x_\theta = \hat{x}_\theta \quad \forall \theta \in \mathbb{T}_{t+1} = \{t+1, \dots T\}$$

Lower bound-based cuts

- Due to the problem structure, it is possible to avoid explicitly representing a large number of additional constraints – it suffices to represent the constraint for the current period:

$$\pi_t^{s,p,l} \rightarrow \quad \alpha_{t+1}^l \geq \Phi^p + \varphi_0^p \cdot z_{t+1}^{s,l} + \sum_{\theta \in \mathbb{T}_{t+1}} \xi_{\theta,t+1}^p \cdot x_\theta \quad \forall p \in \mathbb{P}$$

$$\varphi_t^s \rightarrow \quad z_t^s = \hat{z}_t^s$$

$$\xi_{t,t}^s \rightarrow \quad x_t = \hat{x}_t$$

- The coefficients for periods $\theta > t$ are calculated recursively in the backward simulation (moving from T to 0):

$$\xi_{\theta,t}^s = \sum_{l \in \mathbb{L}} \sum_{\mathrm{p} \in \mathbb{P}} \xi_{\theta,t+1}^p \times \pi_t^{s,p,l} \quad \forall \tau \in \mathbb{T}_{t+1} = \{t+1, \ldots T\}$$

Benefits of lower bound-based cuts

- Even though the calculation of the lower bound coefficients requires additional computational effort, this alternative implementation has superior theoretical properties and several practical applications
- Many problems exhibit slow convergence of the operation policy in SDDP – a heuristic that interrupts the slave problem after few iterations to make a new estimate of the investment decision could significantly speed up optimization, especially in the first few iterations of the investment problem

Case study

- Validation of the lower bound cut generation strategy using a sample electricity system from Costa Rica
  - Substantial hydro capacity including large reservoirs, implying that the operations problem would be substantially time-coupled
  - 33 existing hydro plants + 17 existing thermal plants
  - Small problem – 5 years, 2 thermal expansion candidates, linear expansion decisions limited to the first period
- Minimal differences from the upper bound cut generation and within the convergence gap

Problem	Cut generation strategy	Investment cost (M$)	Operative cost UB (M$)	Total cost UB (M$)	Operative cost LB (M$)	Total cost LB (M$)
Risk-neutral	Upper Bound	492.94	510.24	1003.18	491.36	984.30
Risk-neutral	Lower Bound	499.22	500.42	999.64	491.2	990.42

Application: nested CVaR objective function

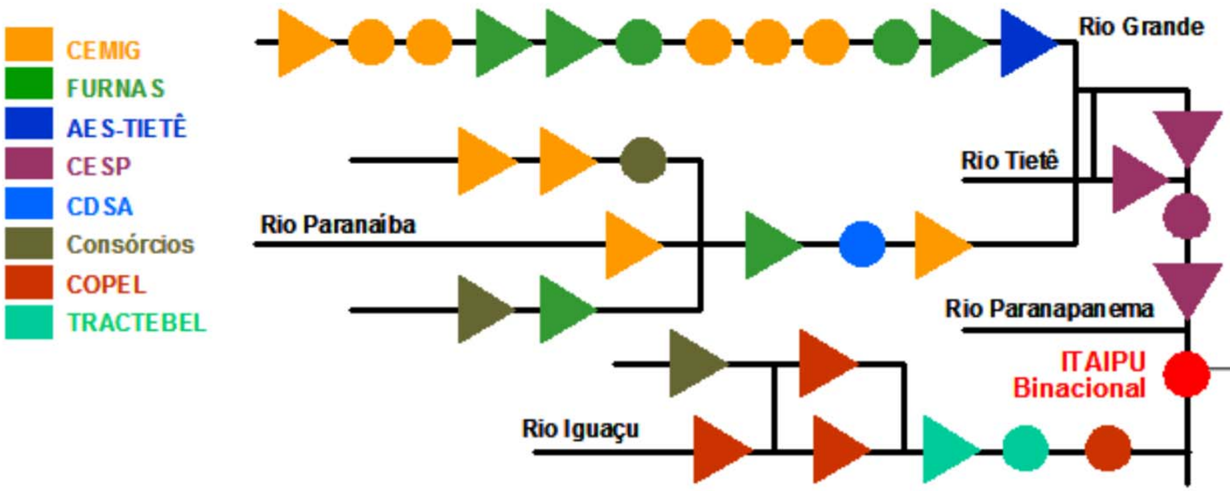
- Nested CVaR objective functions [Philpott 2011] involve a cost function represented as a convex combination of the expected value and CVaR of future costs

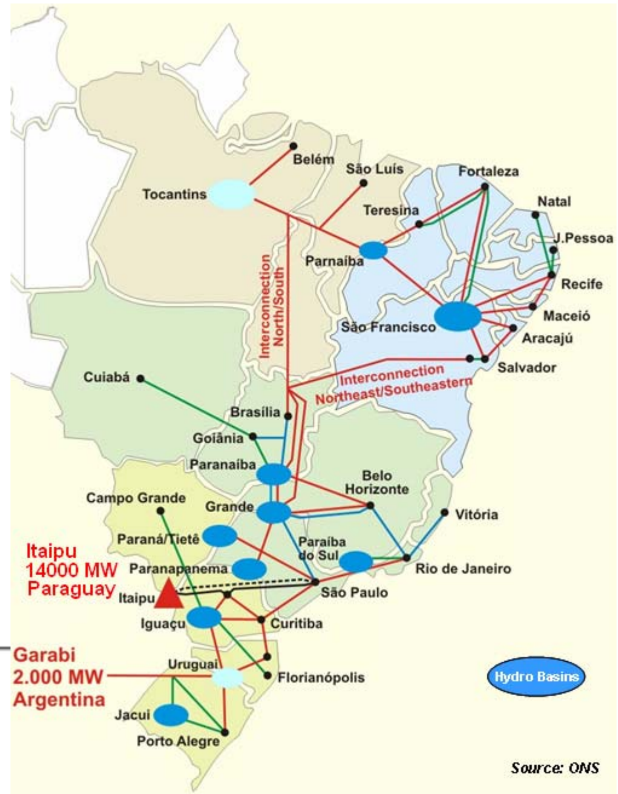
$$\alpha_t(z_t) = \min_{g_t} C(g_t, z_t) + \underbrace{(1-\hat{\lambda}) \cdot \mathbb{E}(\alpha_{t+1}(z_{t+1}) | g_t, z_t) + \hat{\lambda} \cdot CVaR_{\hat{q}}(\alpha_{t+1}(z_{t+1}) | g_t, z_t)}_{\text{risk measure}}$$

- Even though this type of objective function can be easily represented in SDDP problems, the calculation of the upper bound has proved to be a challenge in many practical applications

Application: nested CVaR objective function

- The nested CVaR problem was of particular practical interest because it has been used since 2013 as the official methodology for the hydrothermal dispatch problem in Brazil
  - Although Brazilian authorities do not solve the investment and operation problems in an integrated fashion, it should be imperative to implement consistent objective functions





Interpretation as a dynamic set of weights

$$(1-\hat{\lambda}) \cdot \mathbb{E}(\alpha_{t+1}^{l}) + \hat{\lambda} \cdot CVaR_{\hat{q}}(\alpha_{t+1}^{l}) =$$

- Rockafeller linear representation:

$$\underbrace{(1-\hat{\lambda}) \cdot \frac{1}{|\mathbb{L}|}\sum_{l\in\mathbb{L}} \alpha_{t+1}^{l}}_{\mathbb{E}(\alpha_{t+1}^{l})} + \hat{\lambda} \cdot \underbrace{\left[\gamma + \frac{1}{\hat{q}\times|\mathbb{L}|}\sum_{l\in\mathbb{L}} y_{t+1}^{l}\right]}_{CVaR_{\hat{q}}(\alpha_{t+1}^{l})} \quad s.t. \quad \begin{matrix} y_{t+1}^{l} \geq \alpha_{t+1}^{l} - \gamma \\ y_{t+1}^{l} \geq 0 \end{matrix}$$

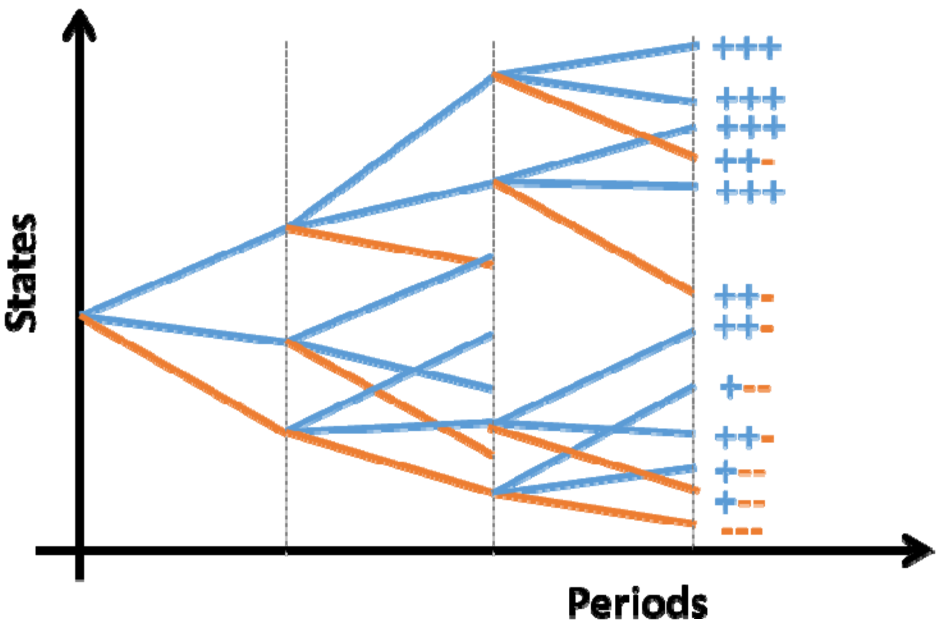
- Representation as probability weights:

$$\frac{1}{|\mathbb{L}|} \cdot \left[\left[\sum_{l\in\mathbb{L}^{+}} (1-\hat{\lambda}) \cdot \alpha_{t+1}^{l}\right] + \left[\sum_{l\in\mathbb{L}^{-}} \left(1-\hat{\lambda}+\frac{\hat{\lambda}}{\hat{q}}\right) \cdot \alpha_{t+1}^{l}\right] + \left[\sum_{l\in\{l^{*}\}} \left(1-\hat{\lambda}+\frac{\hat{\lambda}}{\hat{q}} \cdot \underbrace{[\hat{q}\cdot|\mathbb{L}| - |\mathbb{L}^{-}|]}_{\in(0,1]}\right) \cdot \underbrace{\alpha_{t+1}^{l}}_{=\gamma}\right]\right]$$

$\{\mathbb{L}^{+}, \mathbb{L}^{-}, \{l^{*}\}\}$ a partition of $\mathbb{L}$ such that $\begin{cases} y_{t+1}^{l^{*}} = \alpha_{t+1}^{l^{*}} - \gamma = 0 \\ l \in \mathbb{L}^{+} \Rightarrow y_{t+1}^{l} = 0 \\ l \in \mathbb{L}^{-} \Rightarrow y_{t+1}^{l} = \alpha_{t+1}^{l} - \gamma \\ |\mathbb{L}^{-}| \in [\hat{q}\cdot|\mathbb{L}| - 1, \hat{q}\cdot|\mathbb{L}|) \end{cases}$

Interpretation as a dynamic set of weights

- The upper bound to the nested CVaR objective function must consider the conditional weighing of each branch of the tree, taking each node as a starting point
  - If each state transition in $\mathbb{T} \times \mathbb{S}$ is among the backward openings $\mathbb{L}_{t,s}$, it is straightfoward to identify the associated weighing parameter



Tentative solutions for the upper bound problem

- If this condition can be ensured, there are two “natural” ways to fix the nested CVaR upper bound calculation:

- **Markov weighing**: maintain the sampling strategy but weigh the forward scenarios

$$UB = \sum_{t \in \mathbb{T}} \sum_{s \in \mathbb{S}} \frac{1}{|\mathbb{S}|} C(t,s) \cdot w(t,s)$$

$$w(t,s) = (1-\hat{\lambda})^{n_+} \left(1-\hat{\lambda}+\frac{\hat{\lambda}}{\hat{q}}\right)^{n_-} \left(1-\hat{\lambda}+\frac{\hat{\lambda}}{\hat{q}}\hat{a}\right)^{n_0}$$

$$n_+ + n_- + n_0 = t$$

- **Resampling strategy**: reselect a sample s among the openings l_t at each iteration

$$UB_i = \sum_{t \in \mathbb{T}} \sum_{s \in \mathbb{S}_i} \frac{1}{|\mathbb{S}_i|} C(t,s_i)$$

$$\Pr(s_{i,t} = \hat{s}_{i,t}) = 1-\hat{\lambda}+\frac{\hat{\lambda}}{\hat{q}} \cdot \mathbb{I}_{\hat{s}_{i,t}}$$

Application: nested CVaR investment problem

- In practice, however, properly and efficiently estimating the nested CVaR's upper bound remains an open problem
- As a consequence, calculating the contributions from installed capacity decisions to operative costs using the "classical" upper bound formulation could lead to unreliable results
  - Applying the same upper bound correction strategies to the Lagrange multipliers for the investment problem would result in better estimates
- However, we may sidestep this problem entirely by using the lower bound cuts presented earlier to construct the expansion strategy for the nested CVaR

Application: nested CVaR investment problem

- We compared our optimal investment solution to a "naïve" strategy that does not incorporate either of our proposed strategies to correct the upper bound

$$w \geq w_0 + \sum_{t \in \mathbb{T}} \sum_{s \in \mathbb{S}} \frac{1}{|\mathbb{S}|} \mu_t^s \cdot x_t$$

- Even though it is clearly suboptimal, the naïve strategy is what one would obtain if applying a risk-neutral investment model to the operation outputs from a nested CVaR implementation
- In addition, it is the market-driven expansion outcome if risk-neutral and price-taking agents are remunerated according to the marginal cost of electricity at each stage

Case study

- We applied those two strategies to the same Costa Rican system presented earlier, using a nested CVaR formulation with $\hat{\lambda} = 0.7$ and $\hat{q} = 0.2$
- The naïve expansion strategy finds an investment solution that is very similar to the risk-neutral case

Problem	Cut generation strategy	Investment cost (M$)	Operative cost LB (M$)	Naïve operative cost (M$)	Total cost LB (M$)
Risk-neutral	Lower Bound	499.22	491.2	500.42	990.42
Nested CVaR	Naïve	498.72	1645.5	539.35	2144.22

Case study

- Using our proposed methodology, we find a substantially better solution by increasing investment costs by around 70%
  - Total installed capacity nearly doubled, from 540 to 980 MW
- This solution results in an improvement of nearly 20% with respect to the naïve strategy

Problem	Cut generation strategy	Investment cost (M$)	Operative cost LB (M$)	Naïve operative cost (M$)	Total cost LB (M$)
Risk-neutral	Lower Bound	499.22	491.2	500.42	990.42
Nested CVaR	Naïve	498.72	1645.5	539.35	2144.22
Nested CVaR	Lower bound	854.16	899.49	394.21	1753.65

Conclusions

- In this study, we propose an alternative method to obtain cuts for the investment problem based on the operation problem's lower bound rather than the upper bound
  - Simulations showed consistent results in both implementations
- We also highlight one important application of this formulation in calculating optimal expansion for a problem using a nested CVaR objective function
  - Our formulation sidesteps the issue of upper bound estimation
  - The resulting expansion decision is substantially different from the one obtained from a "naïve" expansion strategy, resulting in 20% lower cost

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Thank you