

Modelling power markets with multi-stage stochastic Nash equilibria

Joaquim Dias Garcia, Raphael Chabar

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- Key concepts
- Motivation and Literature Review
- Multi-stage economic dispatch and SDDP
- Revenue maximization and the MC-SDDP (MAXREV)
- Optimal day-ahead bidding (OPTBID)
- Single agent revenue (NASHBID)
- Multi-stage nash equilibria
- Case study: Panama



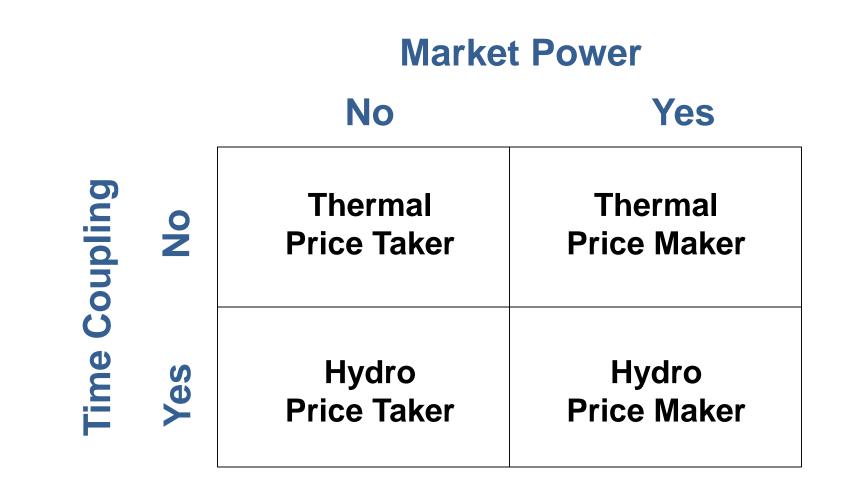
Key concepts

Cost Based Power Markets X Bid Based Power Markets

- Bid: Market Power
- Cost: Monitoring costs
- Day ahead bidding
 - Bidding
 - Clearing
 - Pricing
- Price Takers X Price Makers
 - S.Borestein. J.Bushnell, F. Wolak, *Diagnosing Market Power in California's Restructured Wholesale Electricity Market*, August 2000
 - F.Wolak, An Effective Regulator is Needed for New Zealand Electricity Industry, New Zealand Herald, April 2014.



Motivation: Problem Classes





Motivation: the bidding problem

Thermal Price Taker:

- Gross and Finaly (2000)
 - Agents offer energy at their production cost

Thermal Price Maker:

- Barroso et al. (2006)
 - Bilevel problems converted to MPEC
 - Solved by MILP for instance



Motivation: the bidding problem

- Hydro Price Taker
 - Gjelsvik et al. (1999), Fosso et al. (1999)
 - SDDP+SDP
 - Offering costs for thermals and opportunity costs for hydros
 - Lino et al. (2003)
 - Water market for agents in the same cascade

Hydro Price Maker

- Flach et al. (2010)
 - Convexify the problem and use SDDP (3 to 5 years)
 - One Agent, quantity offer, needs Price Takers
- Other works using shorter horizons with deterministic inflows



Simulate the Hydro Price Maker:

Multi Agent

- Price and Quantity Bid
- Do not require Price Takers



- Classical dispatch problem:
 - Objective:

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min\sum c_jg_j + \beta(v_{t+1}, a_{t+1})
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- Classical dipatch problem:
 - Objective:

 $min\sum c_jg_j + \beta(v_{t+1}, a_{t+1})$

Water Balance:

$$v_{t+1} = v_t + a_t - u - s$$



- Classical dipatch problem:
 - Objective:

 $min\sum c_jg_j + \beta(v_{t+1}, a_{t+1})$

Water Balance:

$$v_{t+1} = v_t + a_t - u - s$$

Load Balance

$$\sum g_j + \sum \rho_i u_i = d - \sum r_k$$



- Classical dipatch problem:
 - Objective:

 $min\sum c_jg_j + \beta(v_{t+1}, a_{t+1})$

Water Balance:

$$v_{t+1} = v_t + a_t - u - s$$

Load Balance:

$$\sum g_j + \sum \rho_i u_i = d - \sum r_k$$

AR model (Inflow temporal dynamics):

$$a_{t+1} = \phi_1 a_t + \phi_2 a_{t-1} + \xi_{t+1}$$



Revenue maximization (MAXREV)

► New term and state in the Objective function:

$$min - \pi_t^s E + \sum c_j g_j + \beta(v_{t+1}, a_{t+1}, \pi_{t+1}^s)$$

Changing the load balance

$$\sum g_j + \sum \rho_i u_i + \sum r_k = E$$



Revenue maximization (MAXREV)

New term and state in the Objective function:

$$min - \pi_t^s E + \sum c_j g_j + \beta(v_{t+1}, a_{t+1}, \pi_{t+1}^s)$$

Changing the load balance

$$\sum g_j + \sum \rho_i u_i + \sum r_k = E$$

- π^{s} depends of scenarios
 - \Rightarrow objective is saddle shaped

Solution: represent prices by a Markov Model



MAXREV and the MC-SDDP

Markov process for price: $\begin{pmatrix}
p_{11} & p_{12} & p_{13} \\
p_{21} & p_{22} & p_{23} \\
p_{31} & p_{32} & p_{33}
\end{pmatrix}$

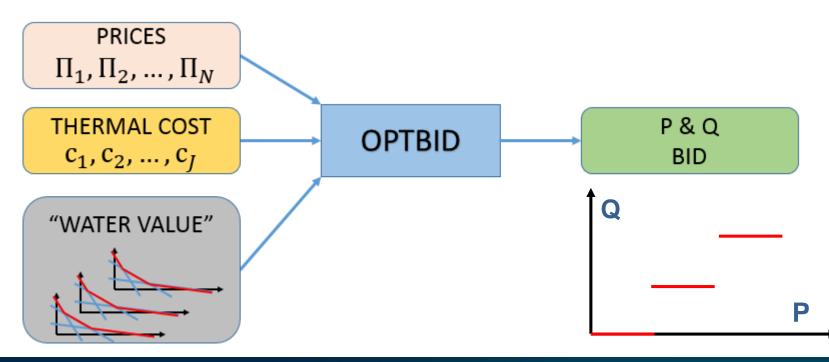
New Objective function with multiple future costs:

$$\begin{split} \min &-\pi^{s} E + \sum c_{j} g_{j} + p_{1} \beta^{k(s) \to k1}(v_{t+1}, a_{t+1}) \\ &+ p_{2} \beta^{k(s) \to k2}(v_{t+1}, a_{t+1}) \\ &+ p_{3} \beta^{k(s) \to k3}(v_{t+1}, a_{t+1}) \end{split}$$



Optimal day-ahead bidding (OPTBID)

- Detail the bidding strategy under uncertainty
 - Convert energy Quantity bid into Price and Quantity bid
- Procedure
 - Pre define a set of prices
 - Optimize the quantity of energy allocated to each price

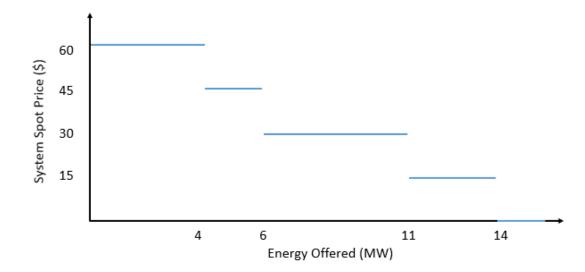




Single (price maker) agent revenue

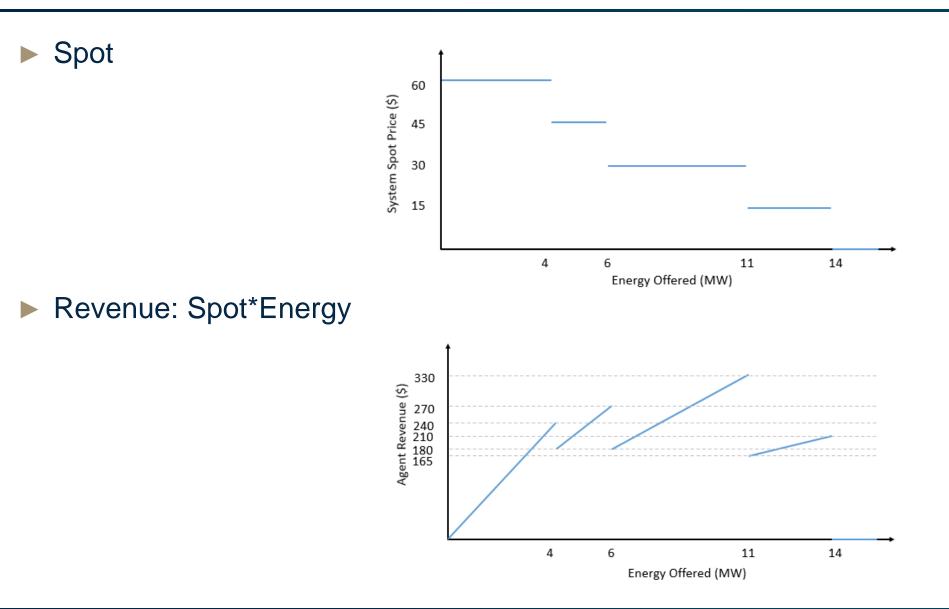
Market Clearing $z(E) = min \sum_{i \in -a} p_i e_i$ s.t. $\sum e_i \leq d - E \leftarrow \pi(E)$ $e_i \leq q_i$

► Spot Price is affected by Price Maker offer





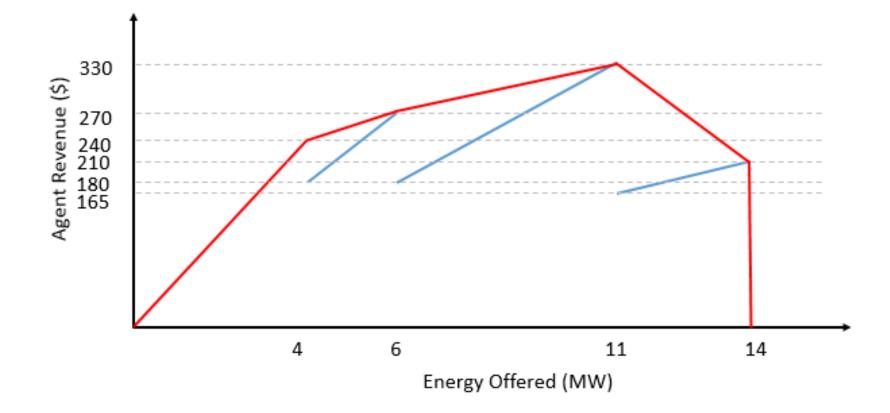
Single (price maker) agent revenue





Single (price maker) agent revenue







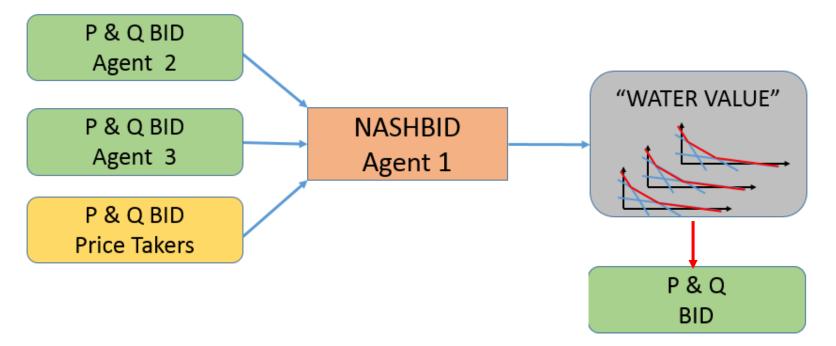
Same problem of the price taker produnction but now spot is a function of energy offered (also scenario dependent)

$$\begin{split} \min &- R^{s}(E) + \sum c_{j}g_{j} + p_{1}\beta^{k(s) \to k1}(v_{t+1}, a_{t+1}) \\ &+ p_{2}\beta^{k(s) \to k2}(v_{t+1}, a_{t+1}) \\ &+ p_{3}\beta^{k(s) \to k3}(v_{t+1}, a_{t+1}) \end{split}$$



Multi-stage Nash equilibria

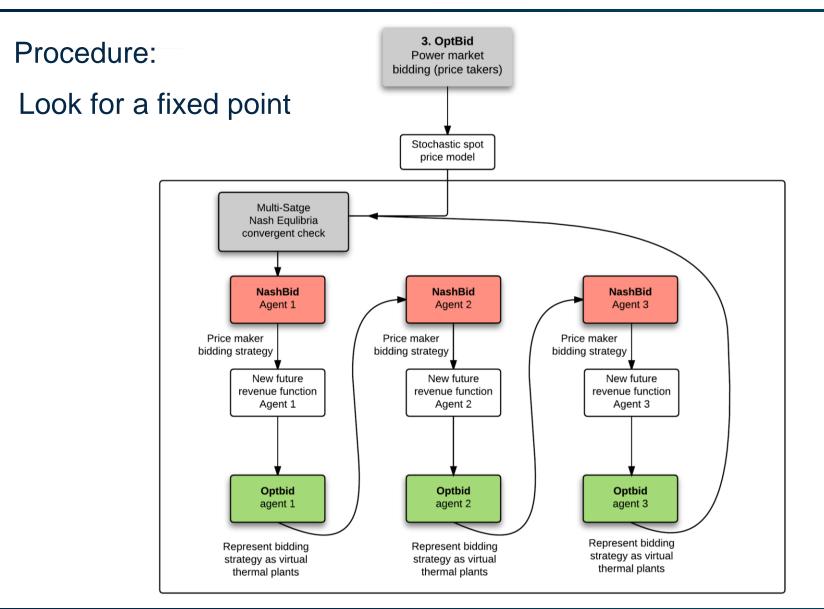
Use NASHBID revenue maximization model to optimize some agent quantity bid strategy given the bids of the other agents



Quantity bid are converted into price and quantity bids by OPTBID

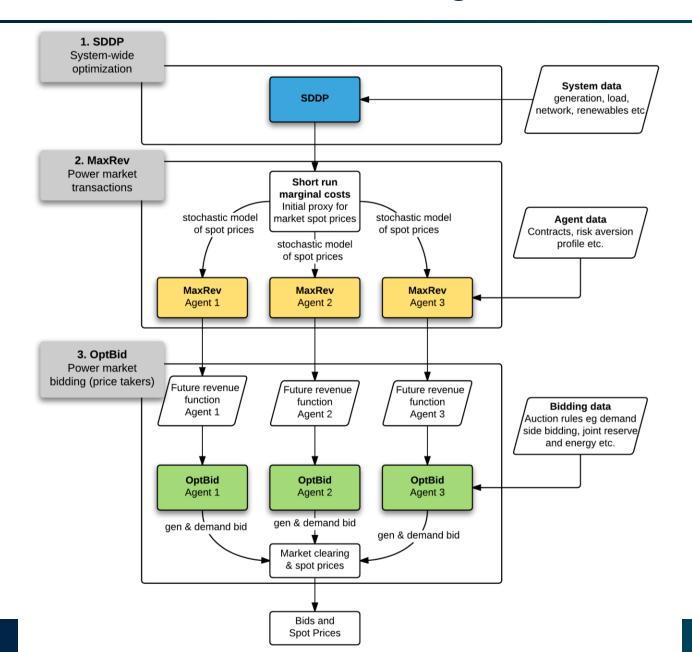


Simulating Multi-stage Nash equilibrium





How do we Initialize bid strategies?

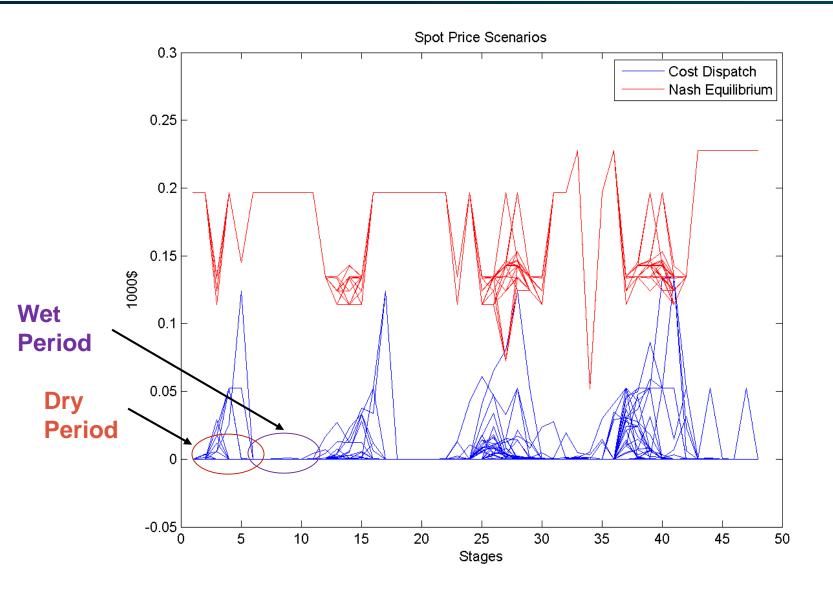




- Panama
- 42 hydros with one cascade
- 22 thermal plants
- We built 3 agents evenly distributing hydro plants
- Considering a smaller demand than the real one

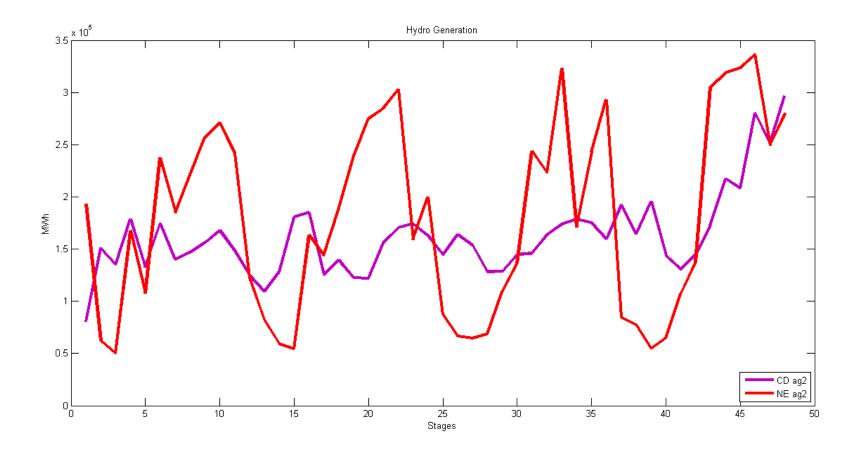


Results: Spot Prices



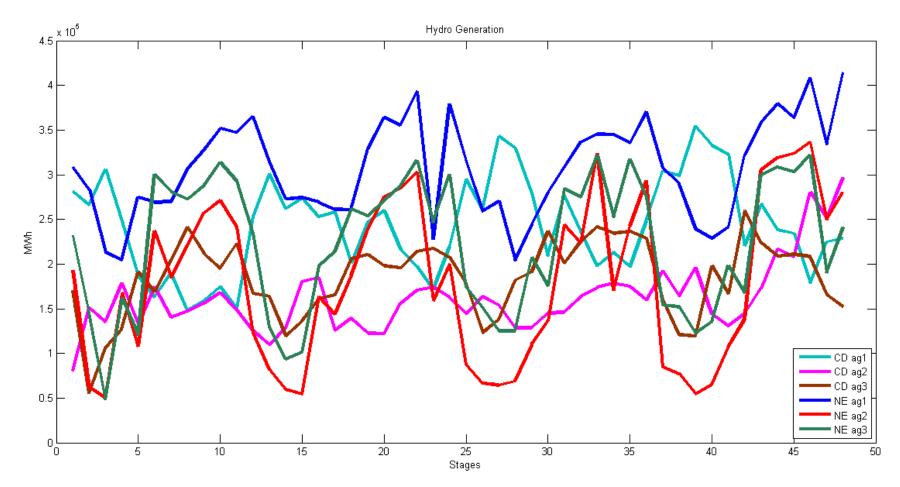


Agent 2 hydro generation





All agents hydro generation





Concluision

Simulation procedure

Price makers move water to optimize their revenues

Spot prices significantly modified

Future work:

- Multiple equilibria
- Agents with hydros in the same cascade
- Simulating even larger power systems





- **>** +55 21 3906-2100
- g psr@psr-inc.com



Thanks!

