PSR

Implementation and assessment of alternative risk aversion methodologies for large scale stochastic hydrothermal scheduling

Luiz Carlos da Costa Jr

luizcarlos@psr-inc.com

Motivation

- Hydrothermal generation scheduling
  - Objective: optimize the use of existing resources (hydro, natural gas, renewables etc.) over a planning horizon

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- Characteristics:
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  - Strong stochastic components
- Trade-off: minimum cost x supply reliability
  - Minimum cost operation ➔ least reliable
  - Most reliable operation ➔ most expensive

Stochastic optimization model

Stochastic parameters

- Hydro inflows and renewable generation (wind, solar, biomass etc.)
  - Multivariate stochastic model (PAR(p))
    - Inflows: macroclimatic events (El Niño), snowmelt and others
    - Spatial correlation of wind, solar and hydro
    - External renewable models can be used to produce scenarios
- Uncertainty on fuel costs
  - Markov chains (hybrid SDDP/SDP model)
- Wholesale energy market prices
  - Markov chains
- Load variability and equipment outages
  - Monte Carlo sampling

Solution algorithm: stochastic dual dynamic programming (SDDP)

- Avoids "curse of dimensionality" of traditional SDP $\Rightarrow$ handles large systems
- Suitable for distributed processing

SDDP characteristics [Pereira & Pinto, 1991]

Iterative procedure

1. forward simulation: finds new states and provides upper bound
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Distributed processing

- The one-stage subproblems in both forward and backward steps can be solved simultaneously, which allows the application of distributed processing
- SDDP has been running on computer networks since 2001; from 2006, in a cloud system with AWS
  - We currently have 500 virtual servers with 16 CPUs and 900 GPUs each

One-stage problem (simplified)

- Objective function (min immediate cost + future cost)

$$\alpha_t(\{v_{t,i}\}) = Min \sum_\tau \sum_j c_j g_{t\tau j} + \sum_\tau \delta r_{t\tau} + \alpha_{t+1}(\{v_{t+1,i}\})$$

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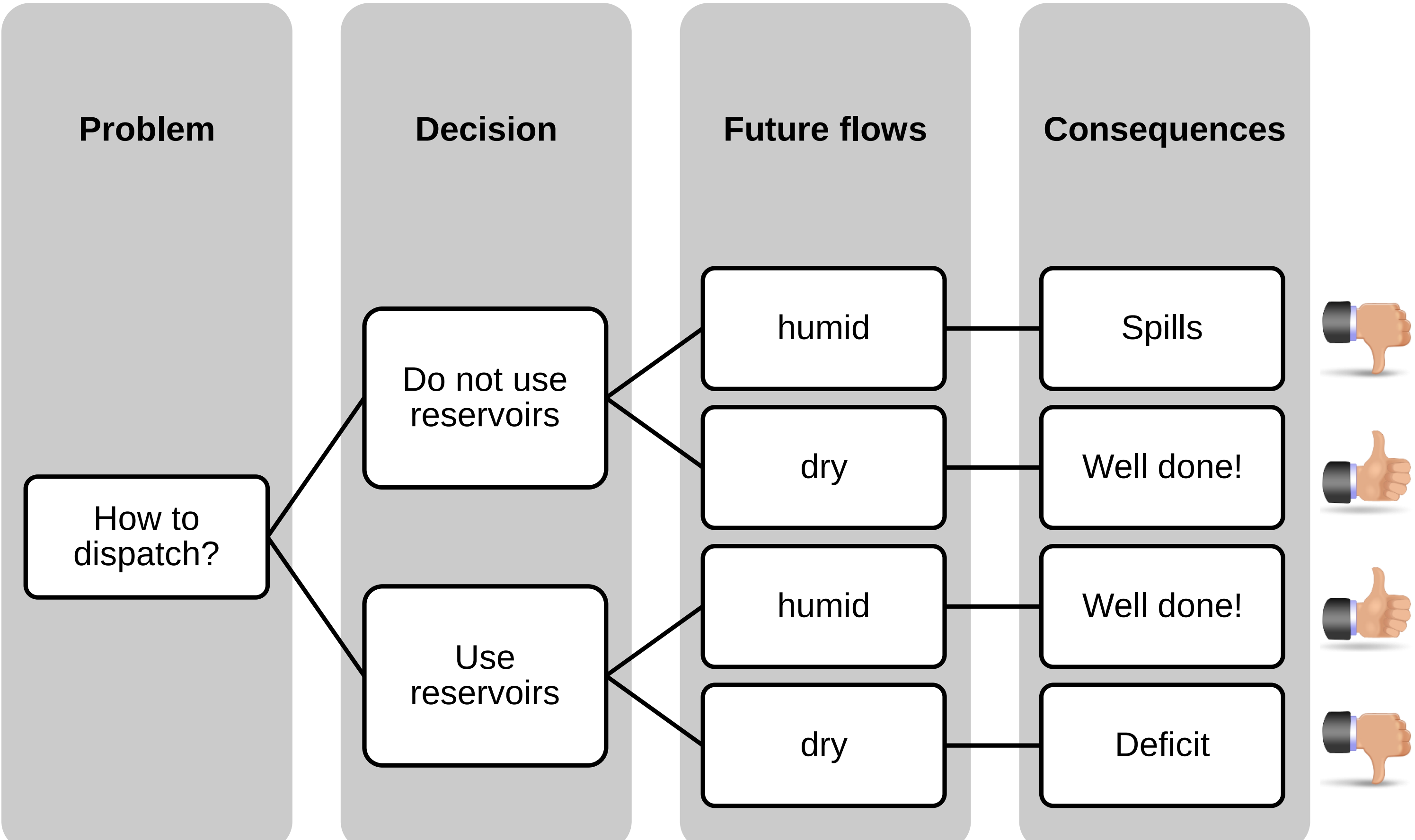
- Future cost function (FCF)

$$\alpha_{t+1} \geq \sum_i \pi_{vi}^k v_{t+1,i} + \sum_i \pi_{ai}^k a_{t+1,i} + \delta^k \quad \forall k = 1, \dots, K$$

Stochastic optimization with risk aversion



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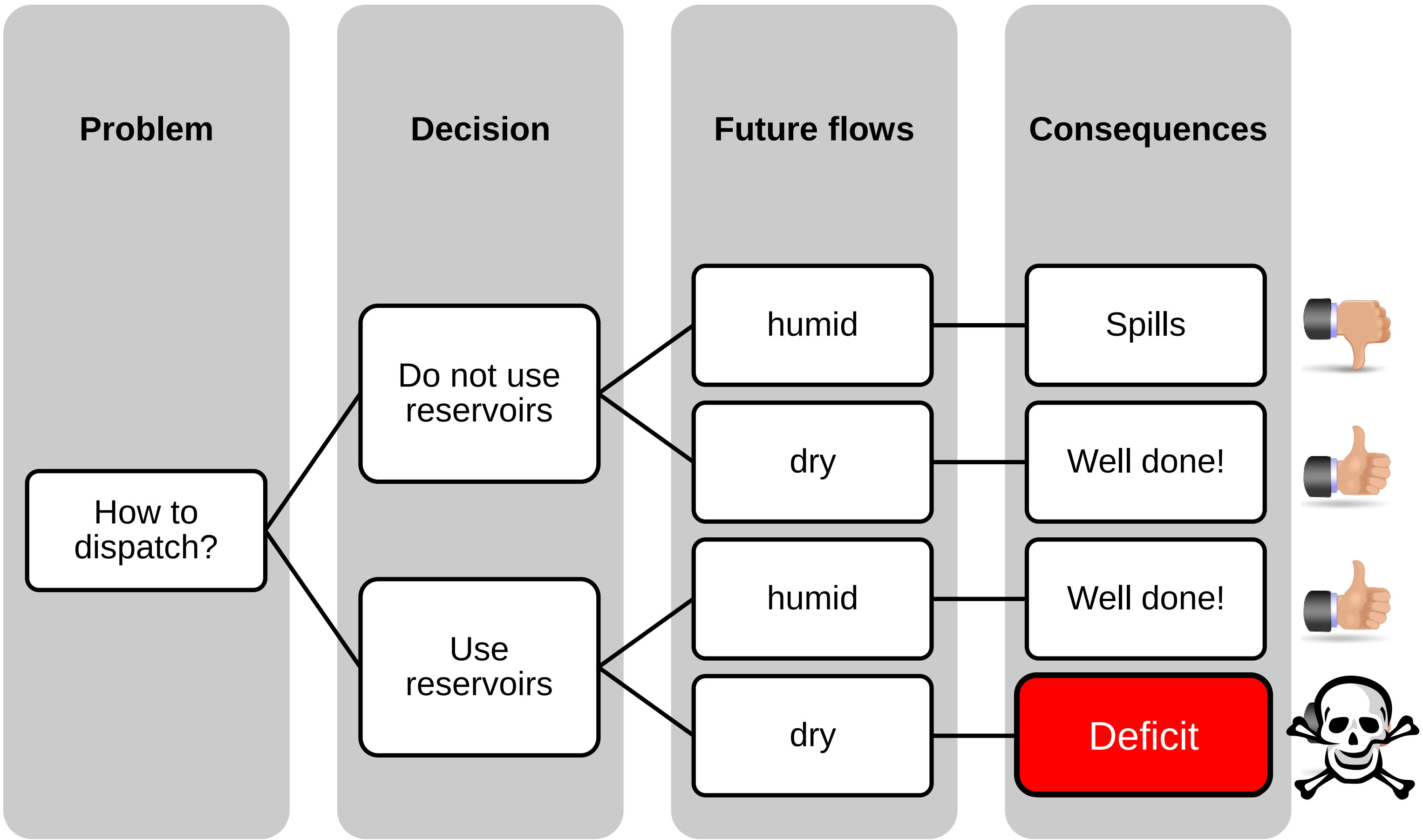

Three approaches to risk aversion

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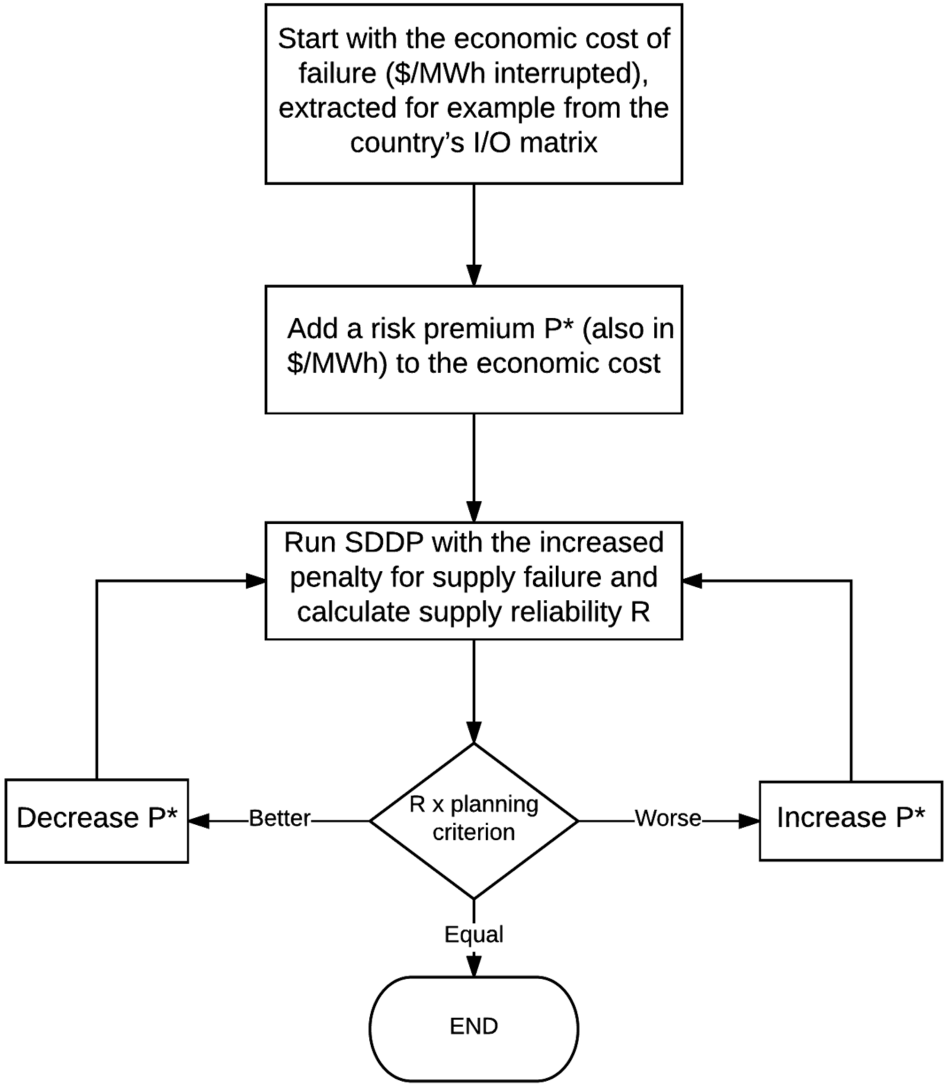
1. Penalize supply failures
   - Economic cost of failure + “risk premium”
2. Ensure feasibility for a set of critical scenarios
   - Hybrid robust/stochastic optimization
3. Give more weight to higher costs in the SDDP recursion
   - Equivalent to skewing the conditioned inflow distribution in SDDP’s backward step

Approach #1: penalize supply failures



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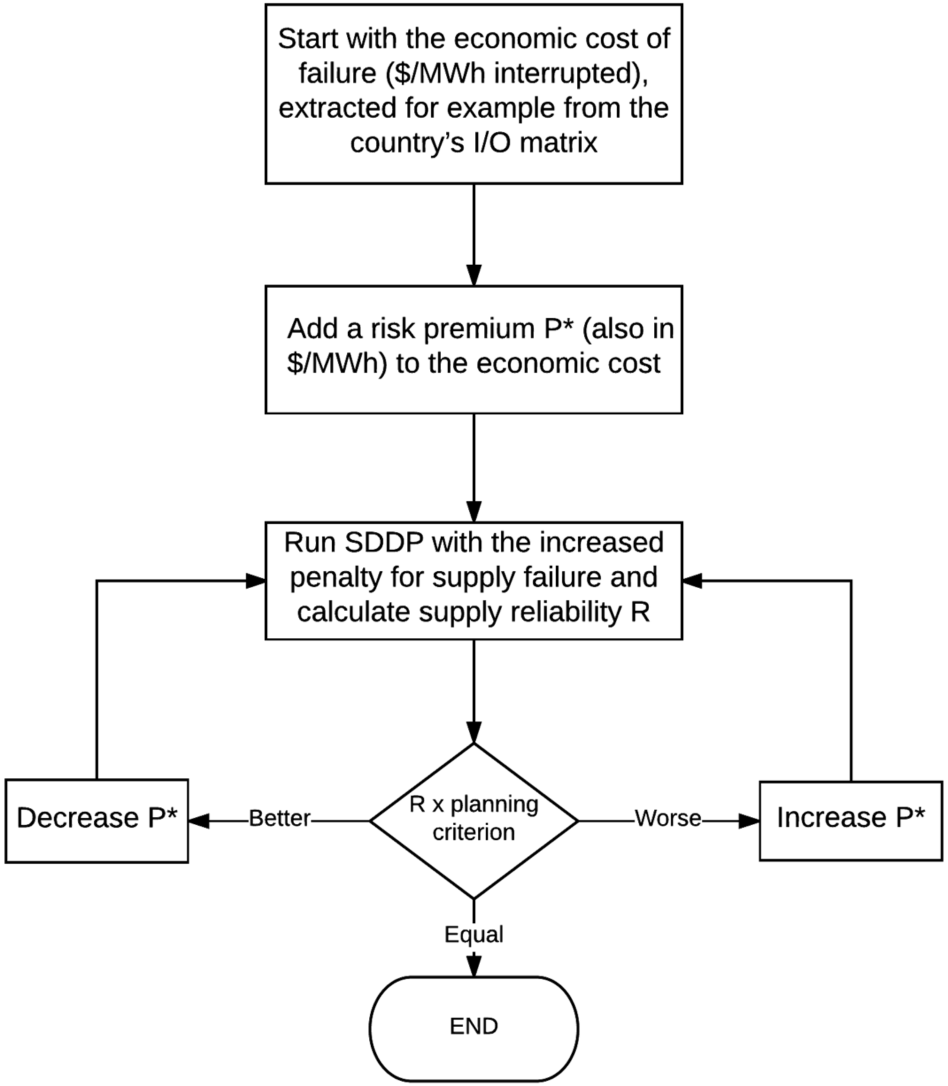

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$$R \le \eta \qquad \mu$$

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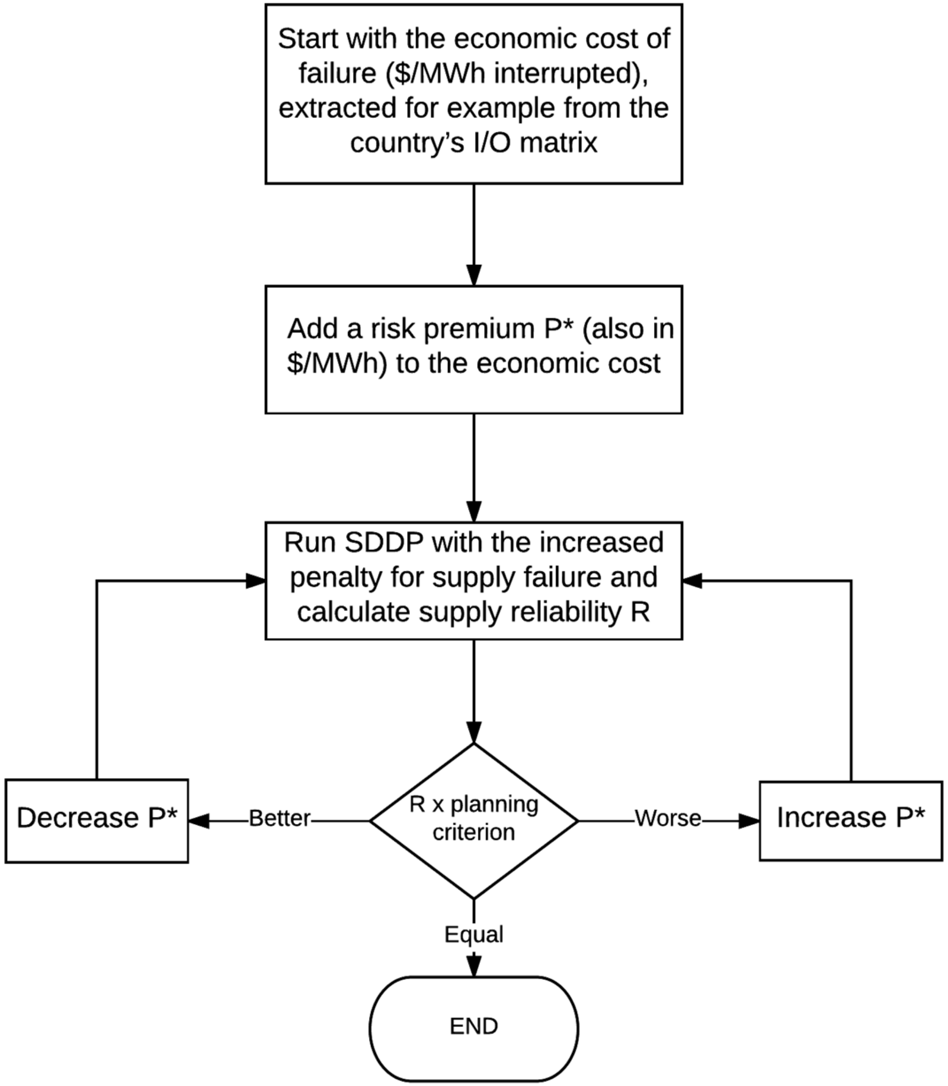

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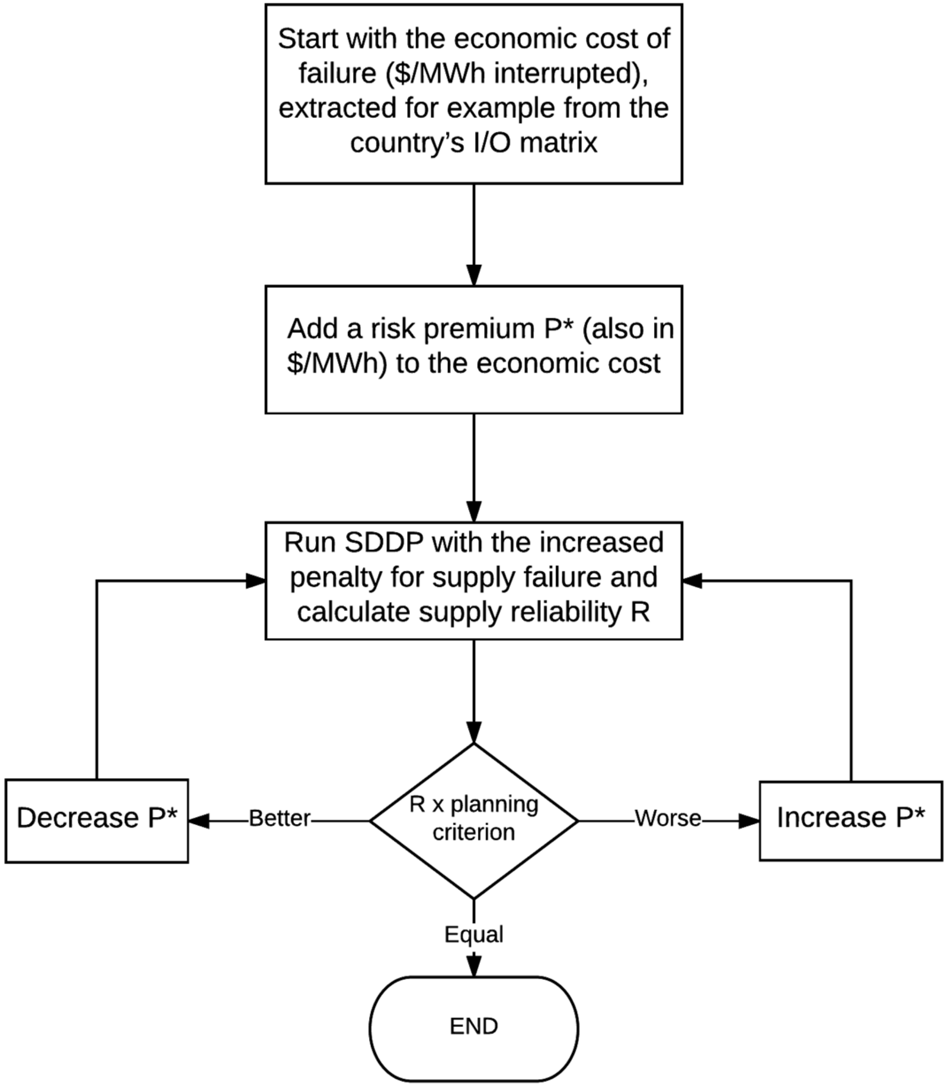

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Challenge: reliability criterion

1. Expected energy not supplied (EENS) does not reflect risk of failure

2. On the other hand, risk of failure does not capture *severity*

Proposed criterion: CVaR of ENS

- Lagrangian relaxation ensures problem separability

$$\underset{\mu}{\text{Max}}\ \text{Min}\, E[z] + \mu(\text{CVaR}_\alpha[r] - \eta)$$

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$$\text{CVaR}_{\alpha}[r_{\tau}] = \inf_{b}\left\{b + \frac{1}{\alpha}E[r - b]^{+}\right\}$$

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$$\alpha_t(\{v_{t,i}\}, b) = Min\ \sum_\tau \sum_j c_j g_{t\tau j} + \sum_\tau \delta r_{t\tau} + \frac{\mu}{\alpha} y_t + \alpha_{t+1}(\{v_{t+1,i}\})$$

- Storage balance

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- Power balance

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- Second deficit segment

$$\mathrm{y_t} \geq \sum_\tau r_{t\tau} - b$$

- Future cost function (FCF)

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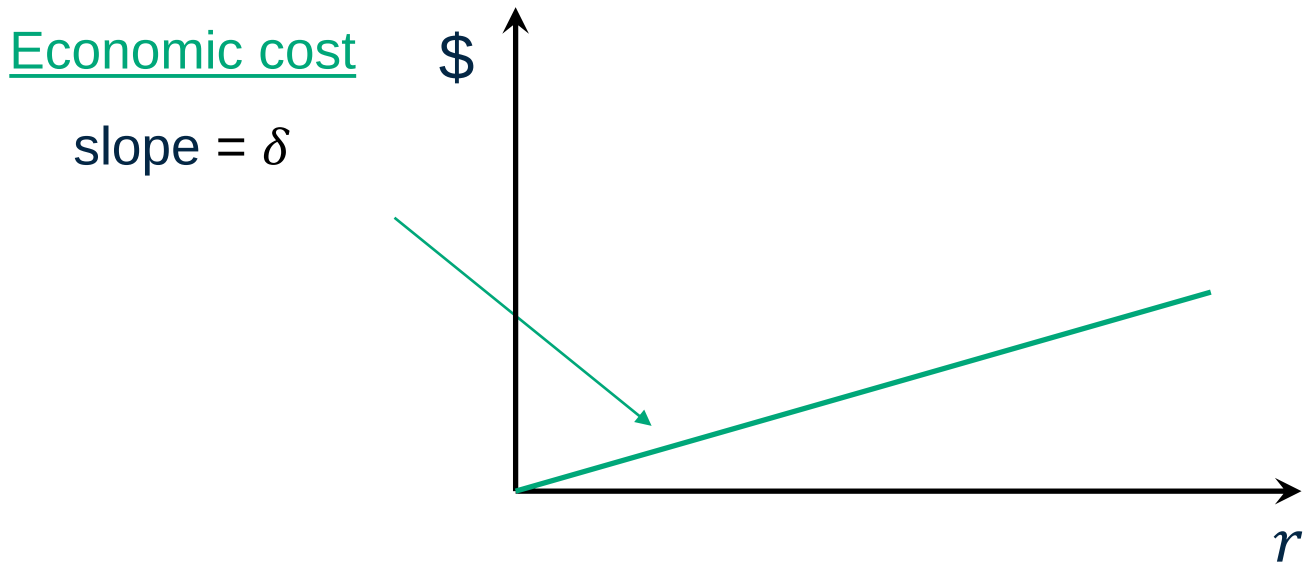
Economic interpretation for CVaR of ENS

- For example, the expected energy not supplied in the 1% quantile should not exceed 5% of load



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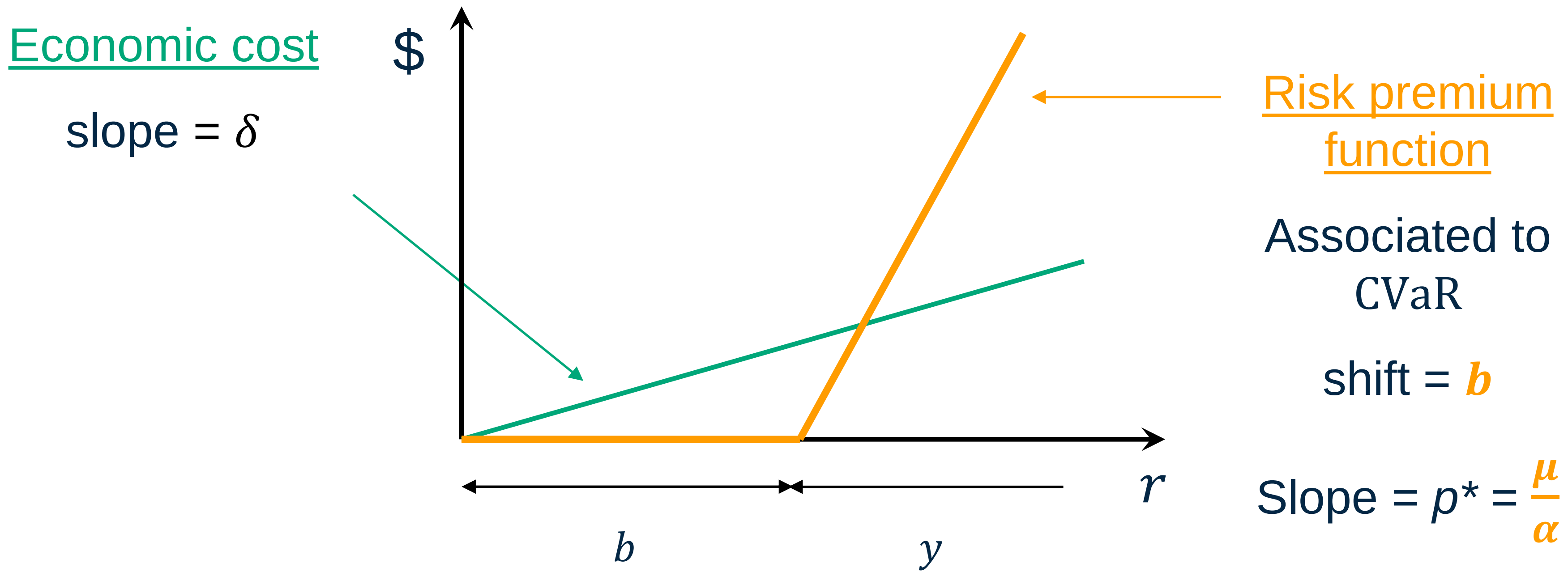

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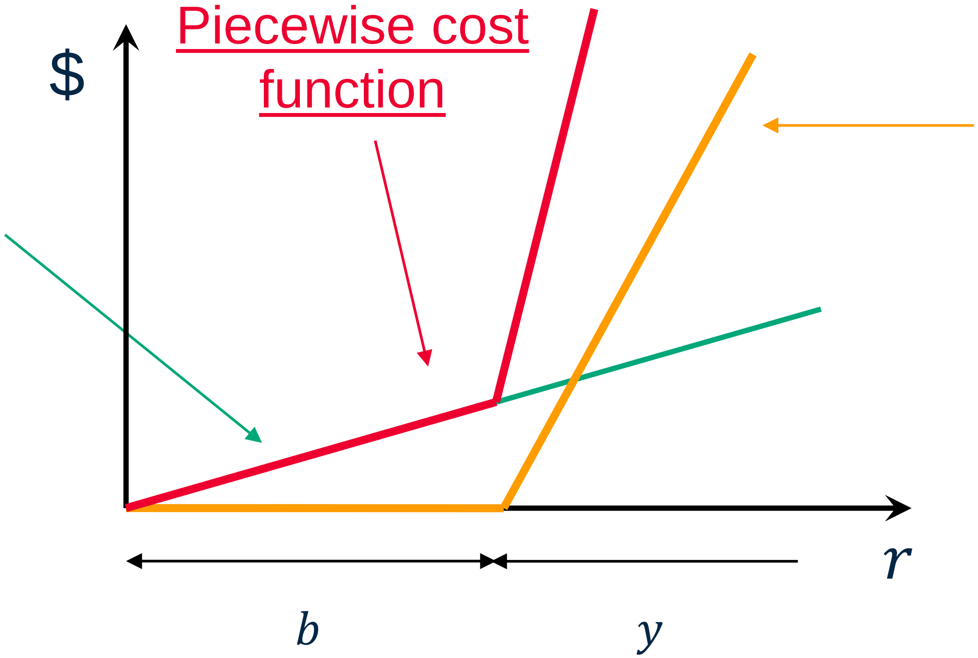
Economic cost

slope = δ



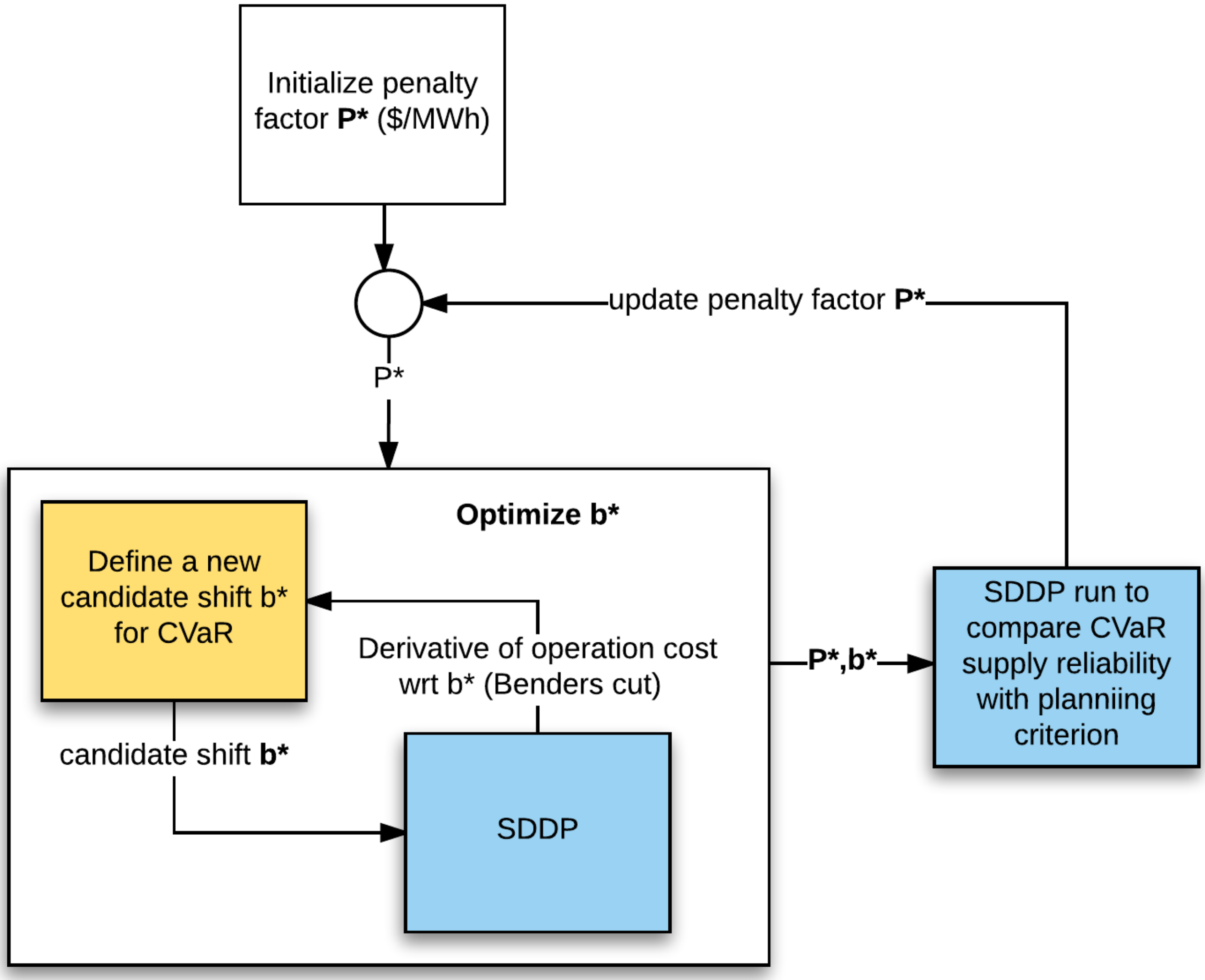
Risk premium function

Associated to CVaR

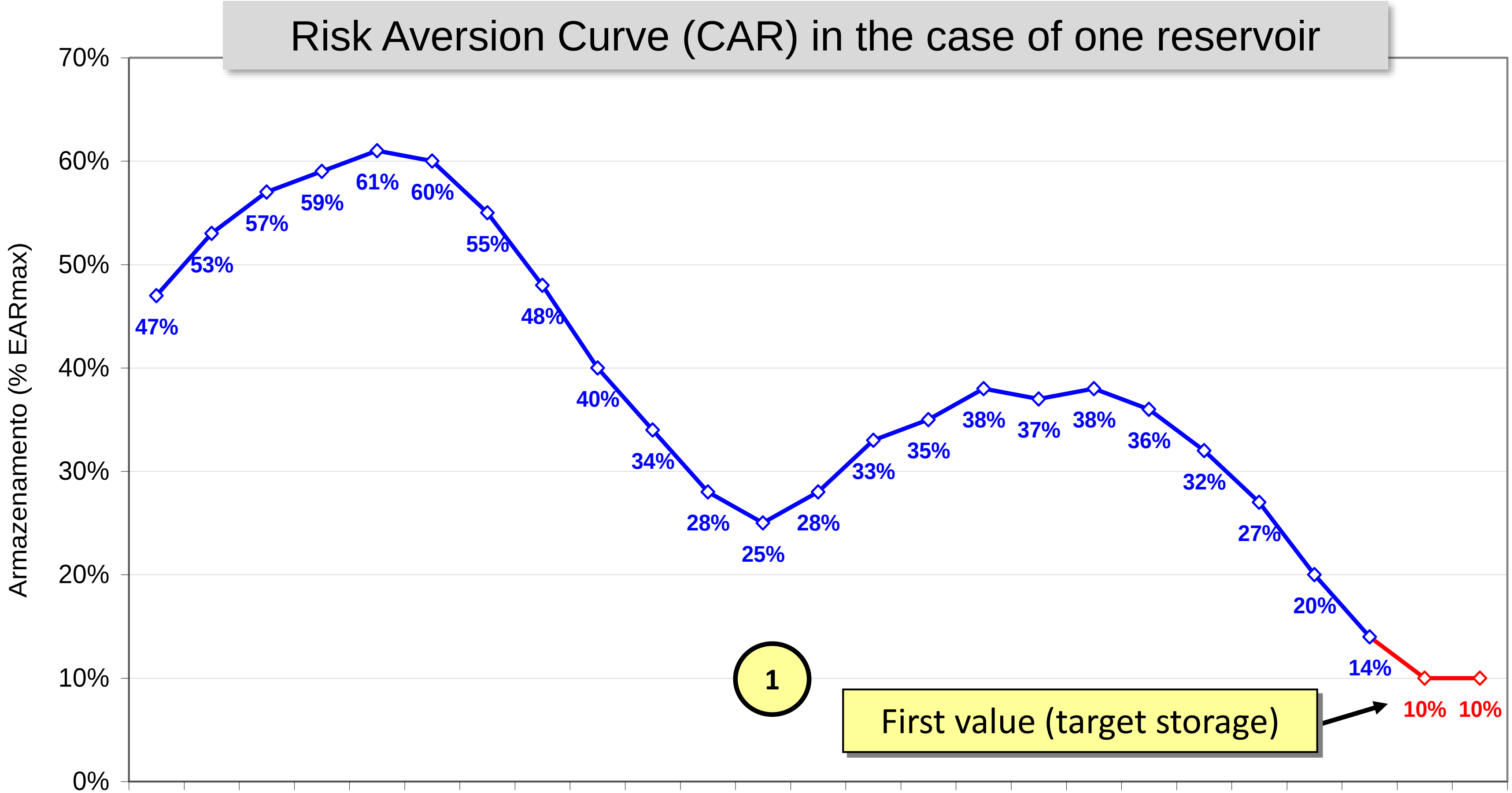
shift = $\boldsymbol{b}$

Slope = $p^* = \frac{\boldsymbol{\mu}}{\boldsymbol{\alpha}}$

SDDP with CVaR on supply reliability



Approach #2: protection against critical scenarios



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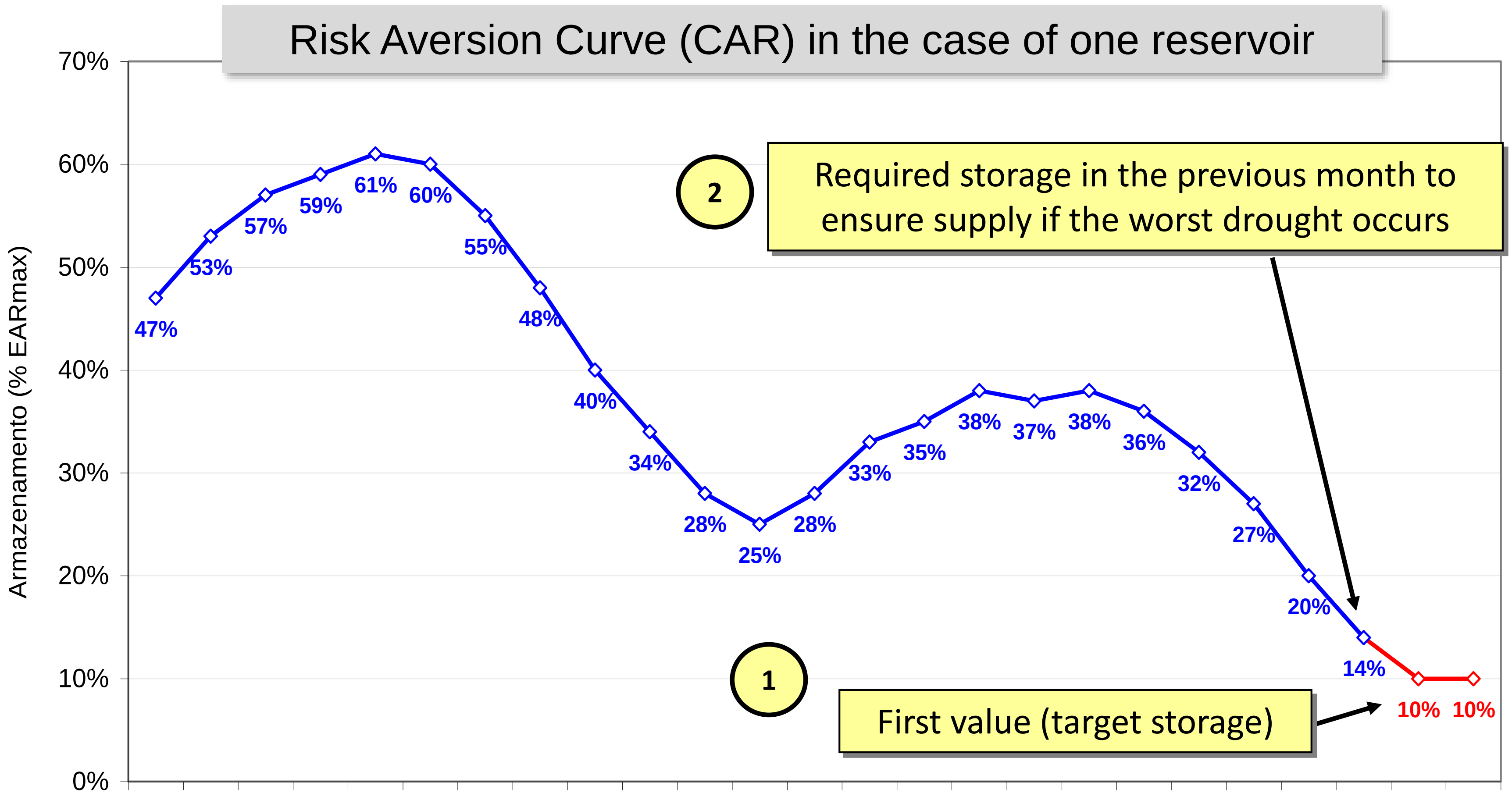

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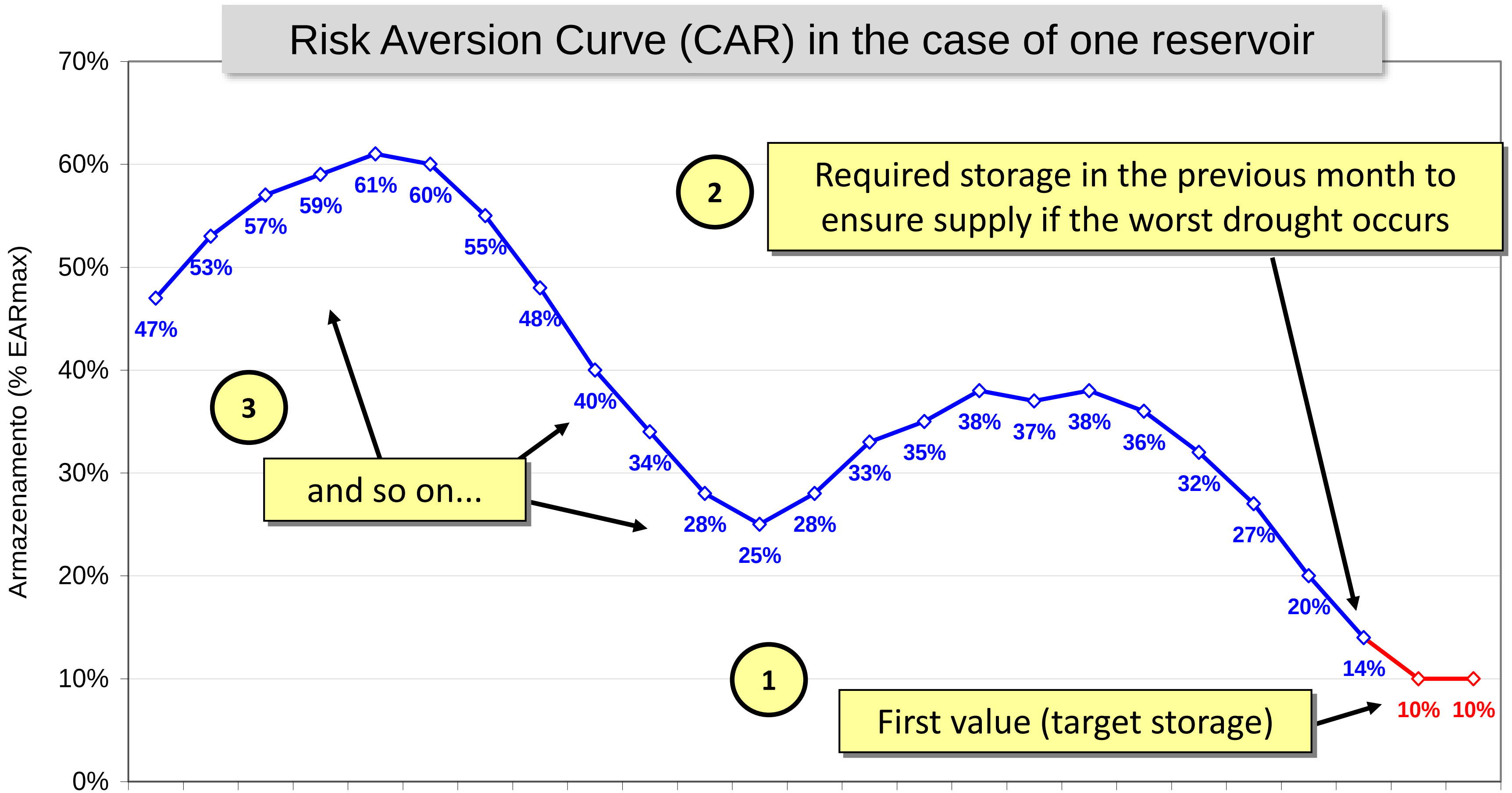

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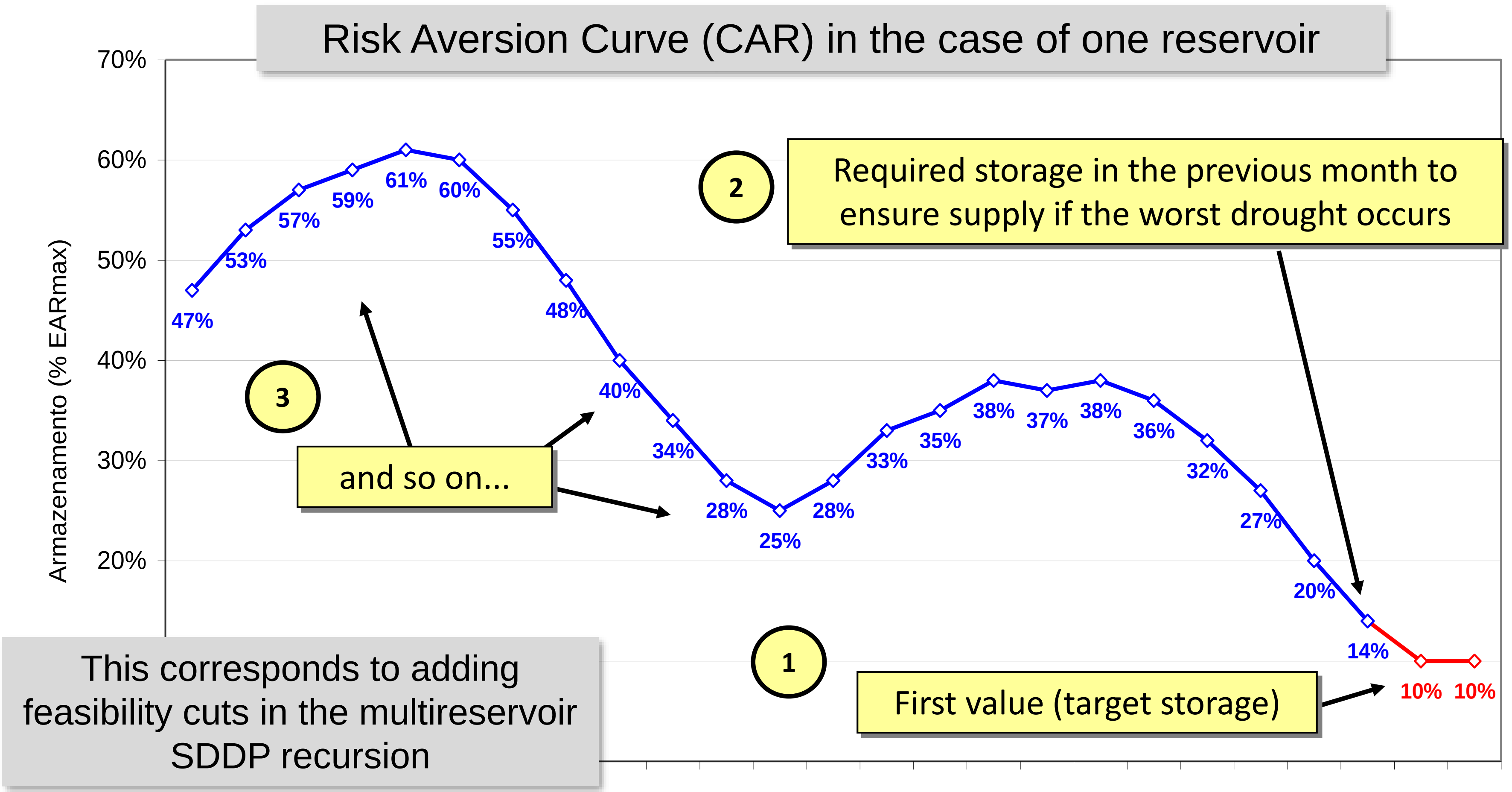
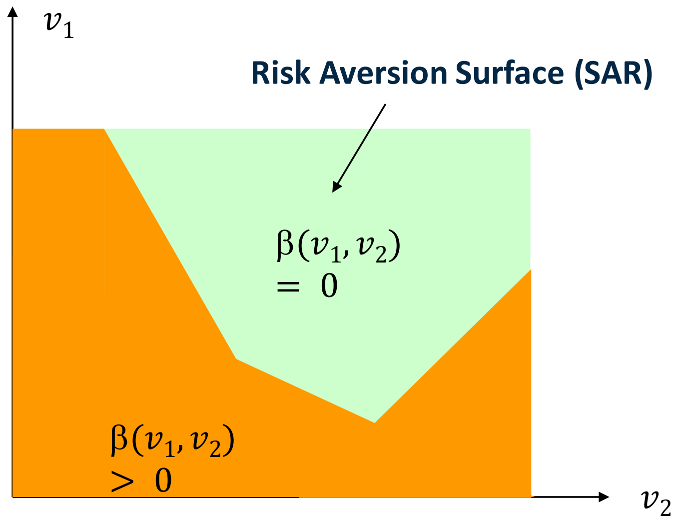
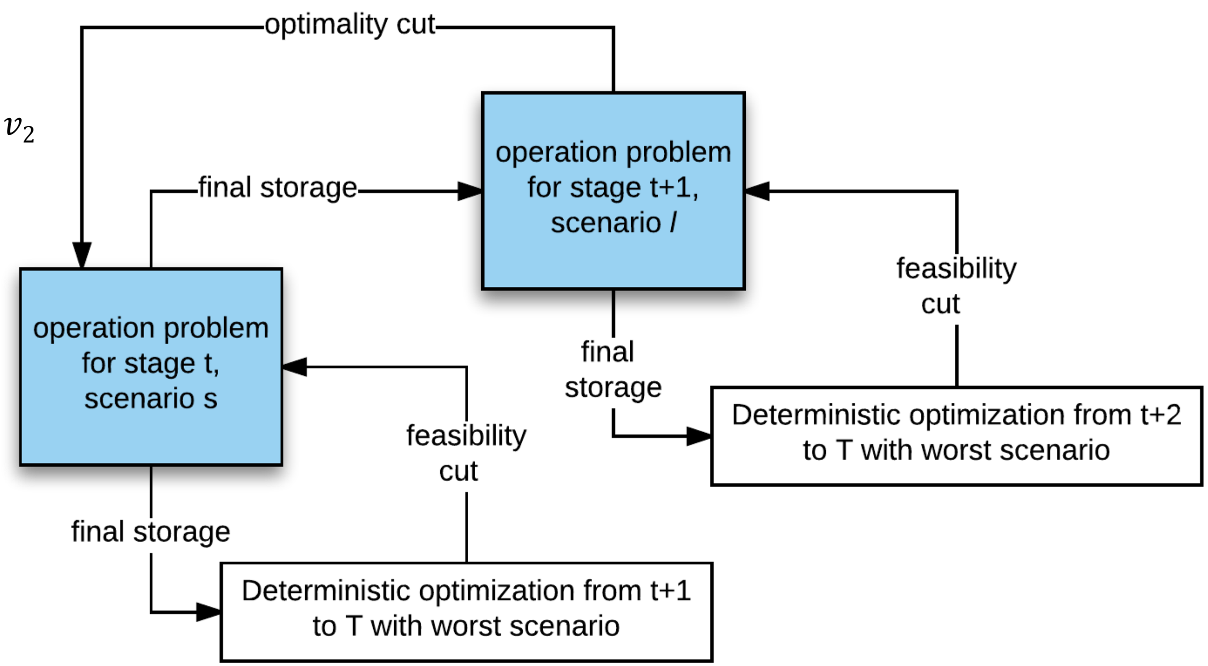


Risk Aversion Surface (SAR)



CAR and (partially) SAR were used for several years as a risk aversion criterion in Brazil's operation



Approach #3: CVaR on operation cost

- New objective function of the one-stage problem

$$Min\ \lambda E(z) + (1-\lambda)CVaR_q(z)$$

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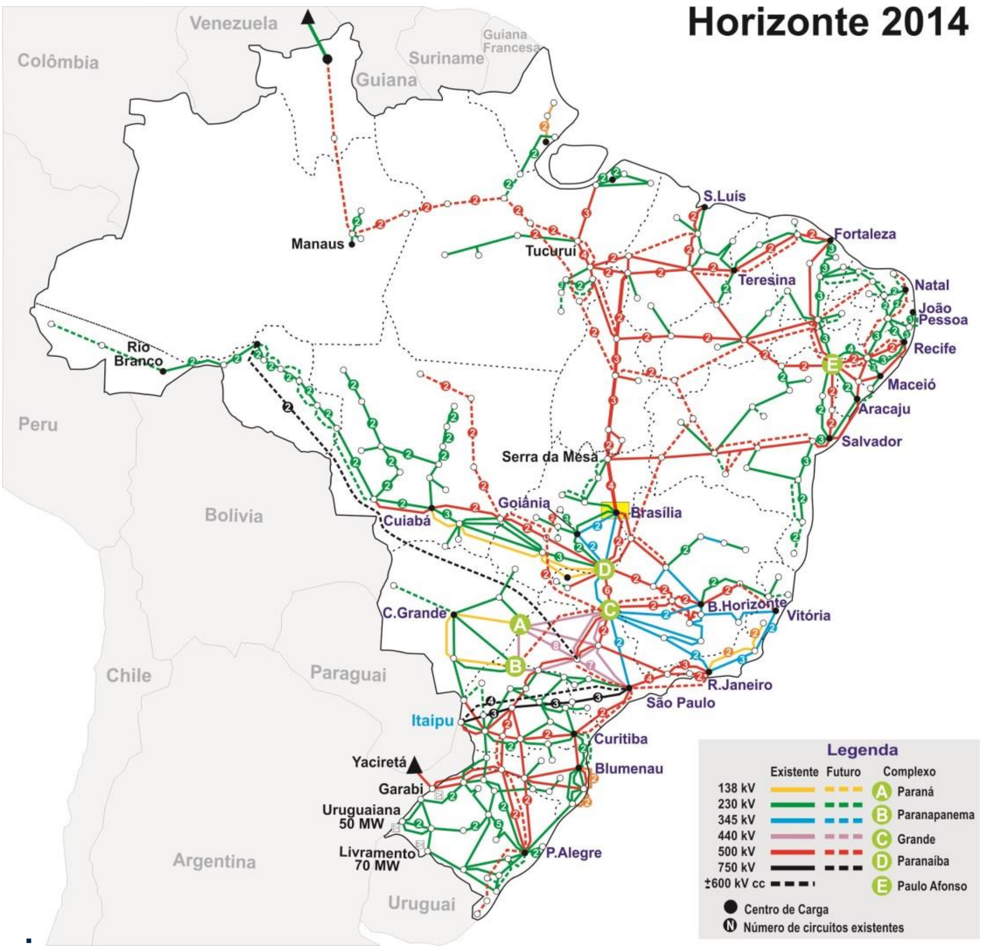
- Nested form
- The CVaR-cost criterion is easy to implement in SDDP, because it is equivalent to changing the weights of the conditioned inflow scenarios in the backward recursion
  - This interpretation also allows a simple and exact calculation of the upper bound in the SDDP algorithm with CVaR, which had been a concern for some time

Case study: Brazil

- Characteristics:
  - 157 hydro plants / reservoirs
  - 119 thermal plants
  - 5 nodes
- Horizon (static):
  - 5 years + 5 years buffer
  - monthly stages
- Uncertainty representation
  - 30 branching backward
  - simulation 1,200 forward scenarios



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  - 12 iterations to converge
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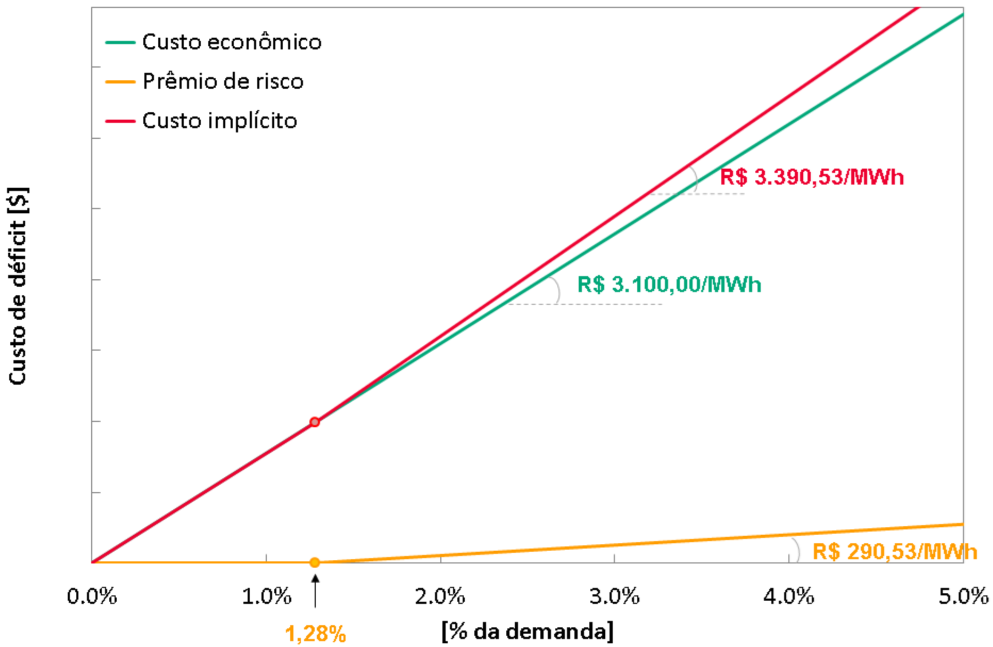
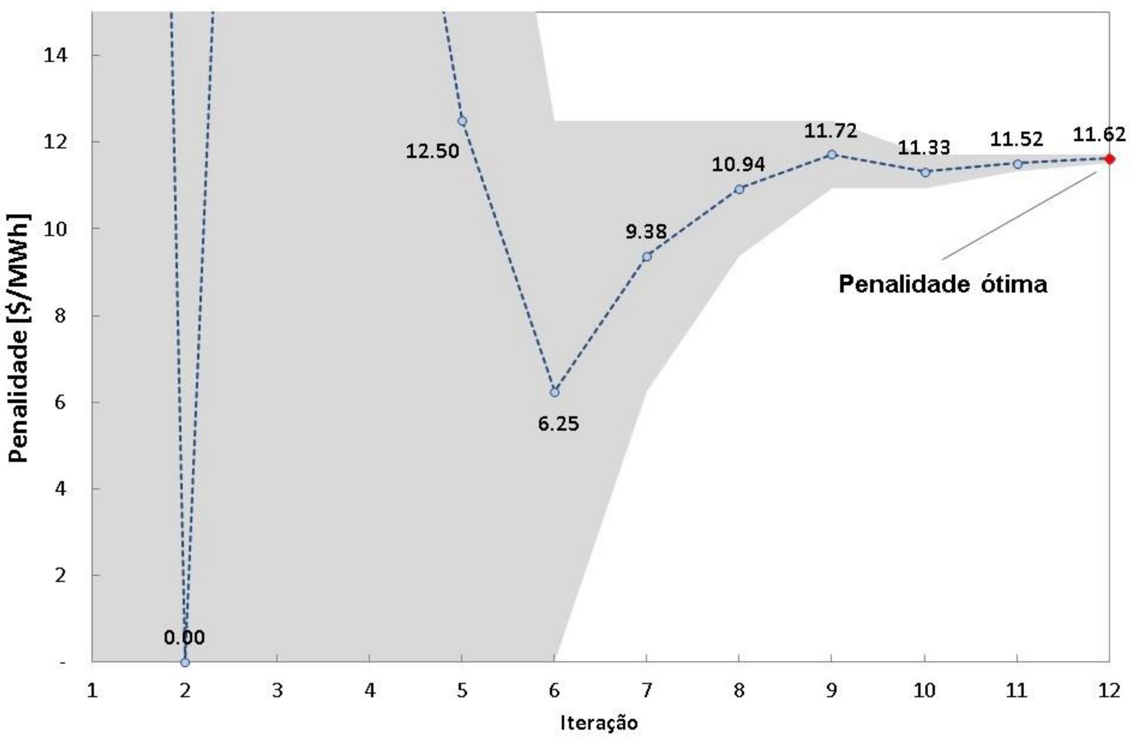


Lagrangian function



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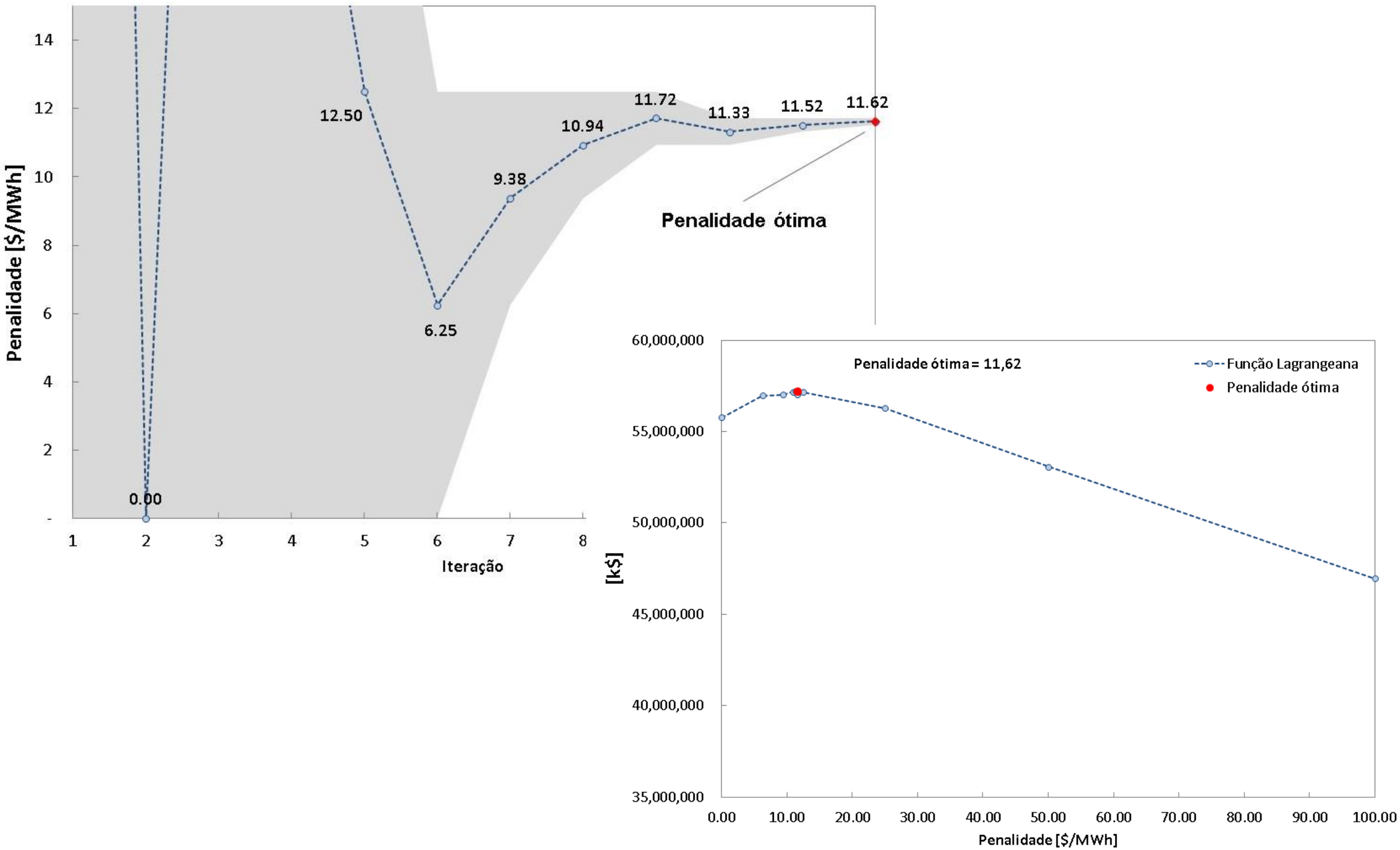
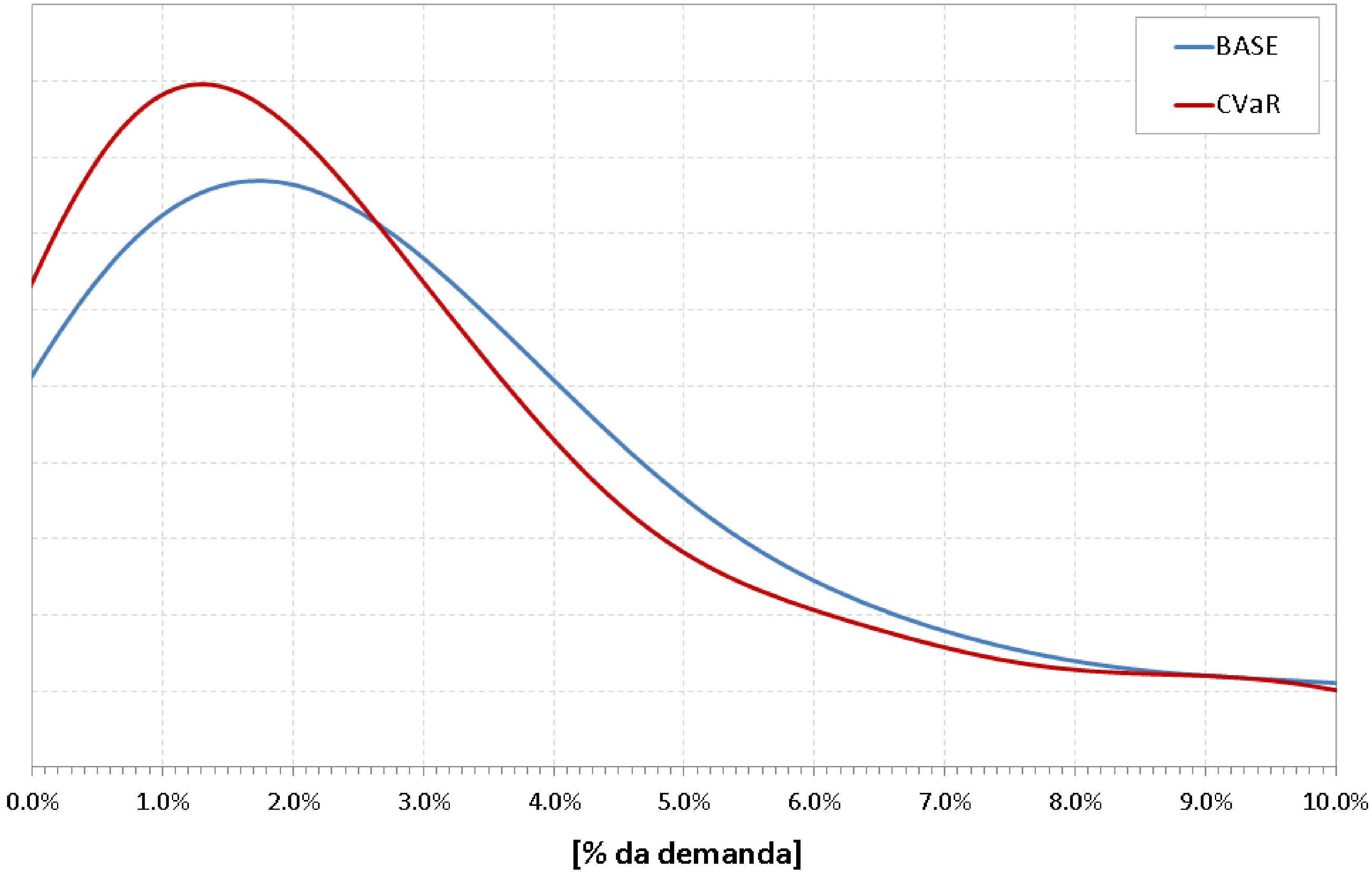
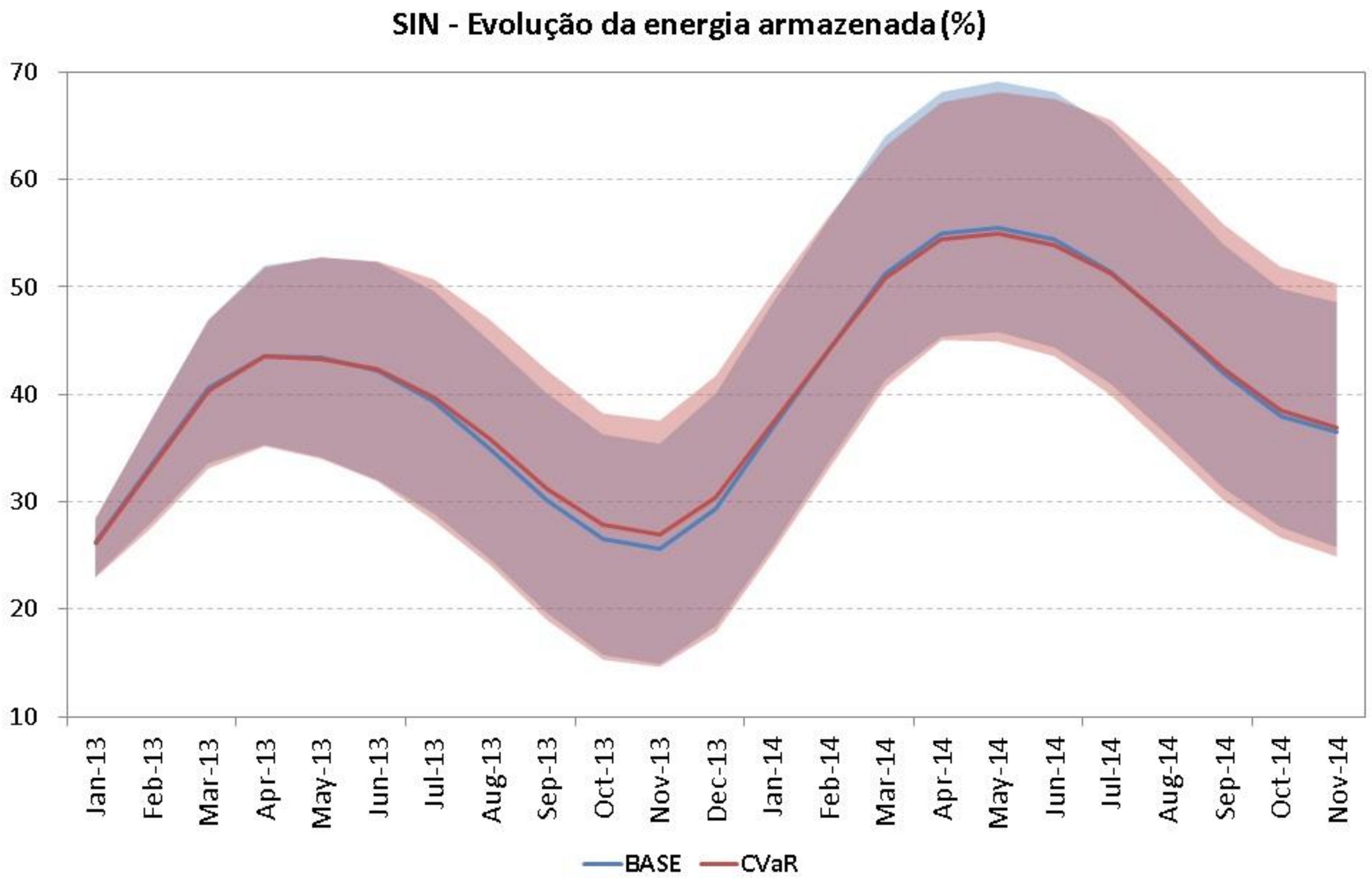


Accumulated supply failure distribution

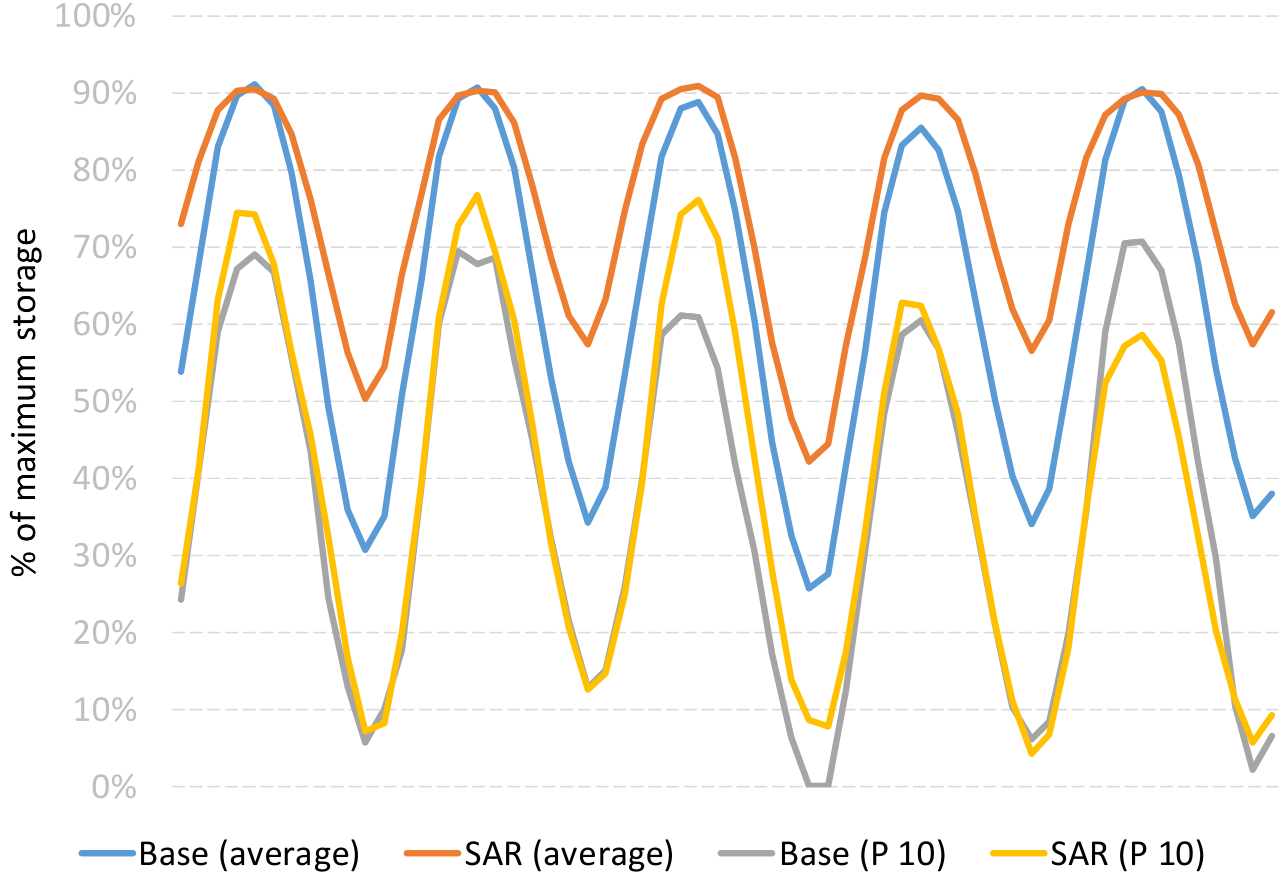


Stored energy (years 1 and 2)



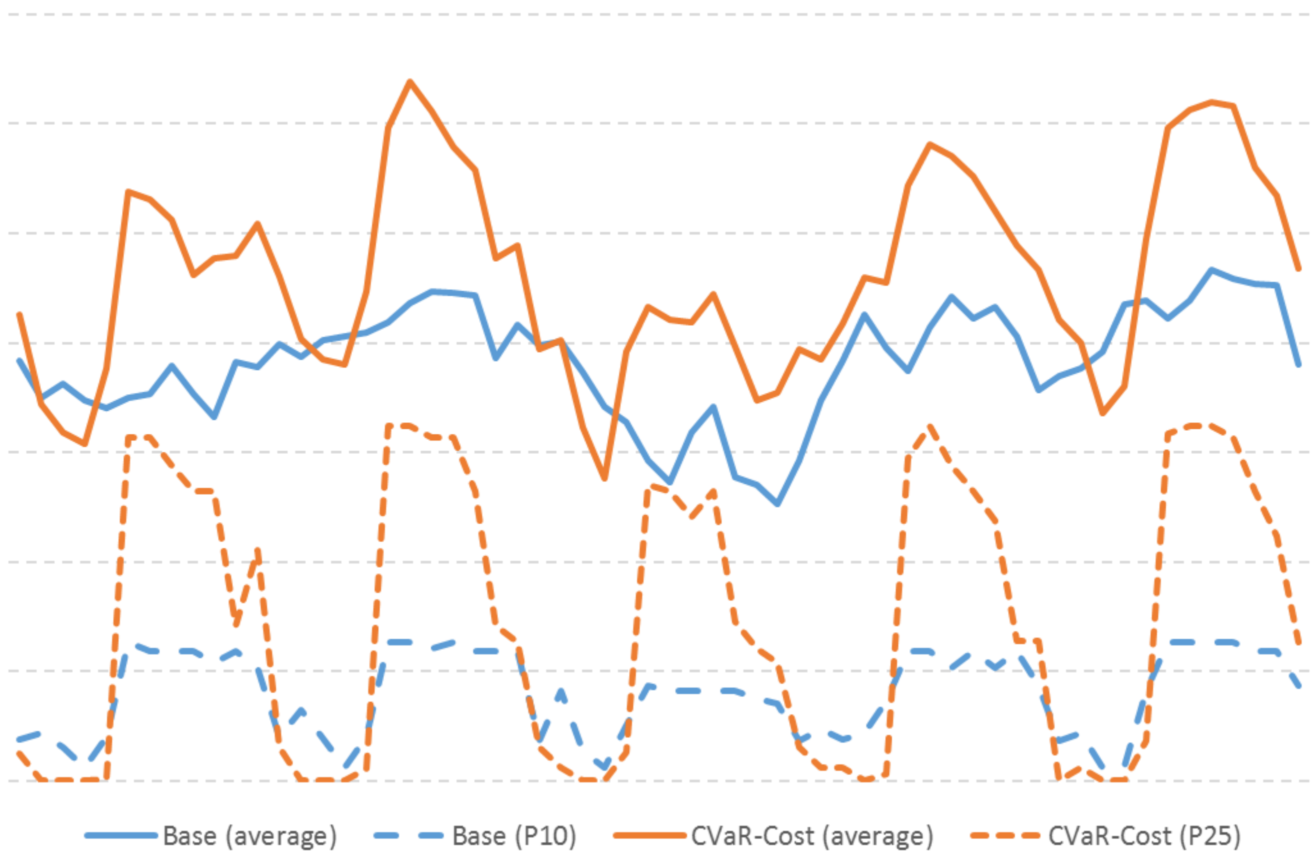
Approach #2: hybrid robust/stochastic optimization

- Stored energy



Approach #3: CVaR on objective function

- Operating costs



Comparison of risk aversion approaches

Approach→ Attribute↓	CVaR-EENS	SAR	CVaR-cost
Easy to understand?	Yes	Yes	Sort of…
Represents risk aversion directly?	Yes	Yes	No
Easy to calibrate?	Medium	Yes	No
Additional computational effort with respect to standard SDDP	High to calculate segment, low after it	Medium	Low

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  - The risk level may not depend on other costs (for example, fuel costs)
- Expectation can be a “naive” (risk neutral) measure:
  - For example, it cannot distinguish between two energy shortages of 100 MW or one of 200 MW
- CVaR is a good (risk averse) alternative:
  - Sensitive to the tail of the distribution, representing a protection against extreme scenarios
  - Coherent risk measure
  - Convex: can be incorporated in decomposition schemes like SDDP

Conclusions - Risk aversion approaches

- Approach #1: minimization of E[.] considering a CVaR constraint
  - CVaR constraint is directly applied over the supply failure variable to ensure the desired risk level
  - The definition of the acceptable risk level is a pre-defined criteria
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  - The definition of the acceptable risk level is a pre-defined criteria
  - Implicit cost function for supply failures is a result
- Approach #2: aversion curve surface
  - Constraints directly apply over reservoir levels to indirect control the risk of energy shortage
  - The definition of minimum storage curve may be a challenging task

Conclusions - Risk aversion approaches (cont'd)

- Approach #3: minimization of a risk measure (CVaR, for example)
  - Requires the definition of a weight for the CVaR on the objective function (parameter definition is a challenging task)
  - Protection against higher costs, risk of supply failures is a consequence

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THANK YOU

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