PSR

Extension of the SDDP algorithm to determine an integrated stochastic investment & operations strategy

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Planning = investment + operation

- **Investment problem** – Determine the reinforcements (generation capacity and transmission) required for an *economic and reliable supply* of predicted load

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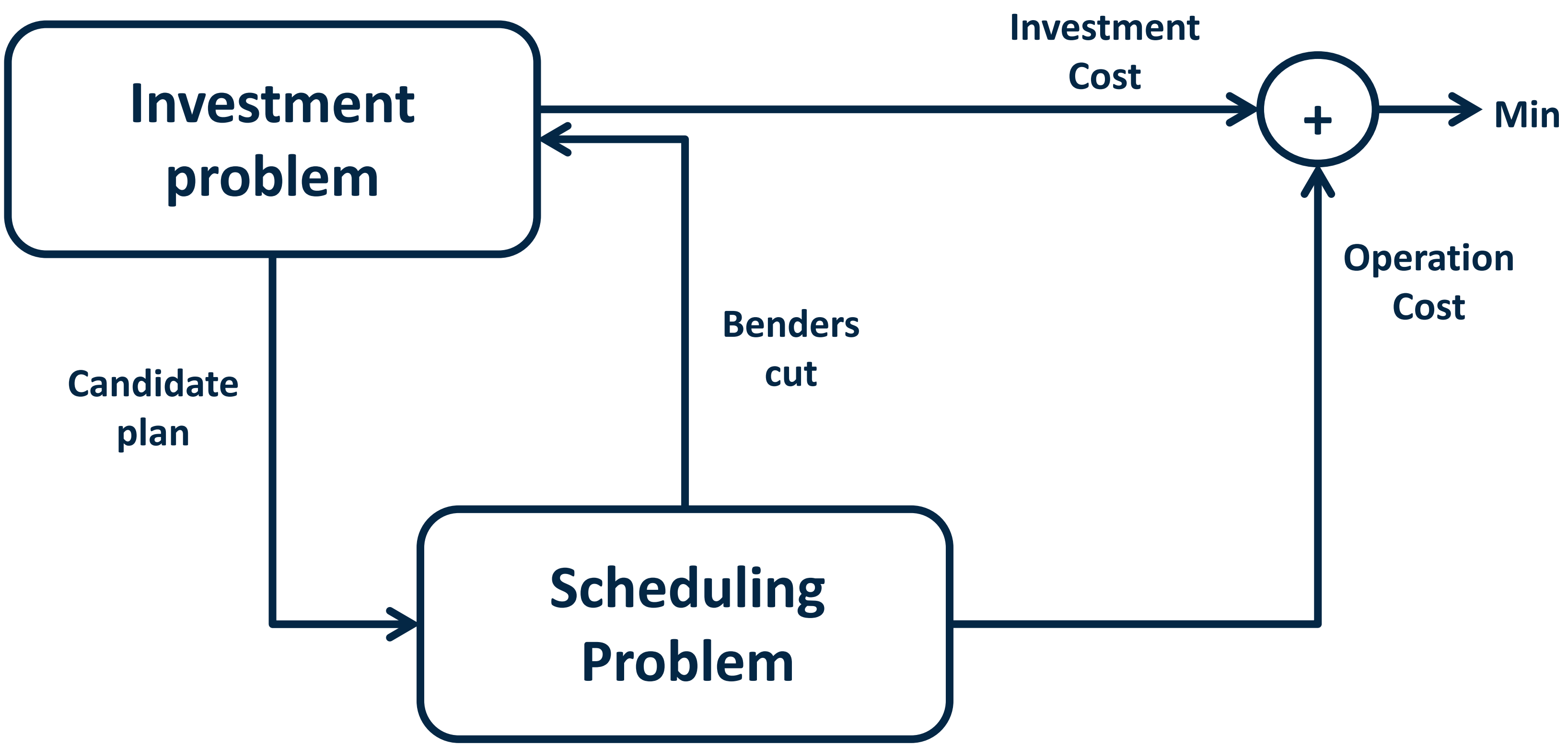
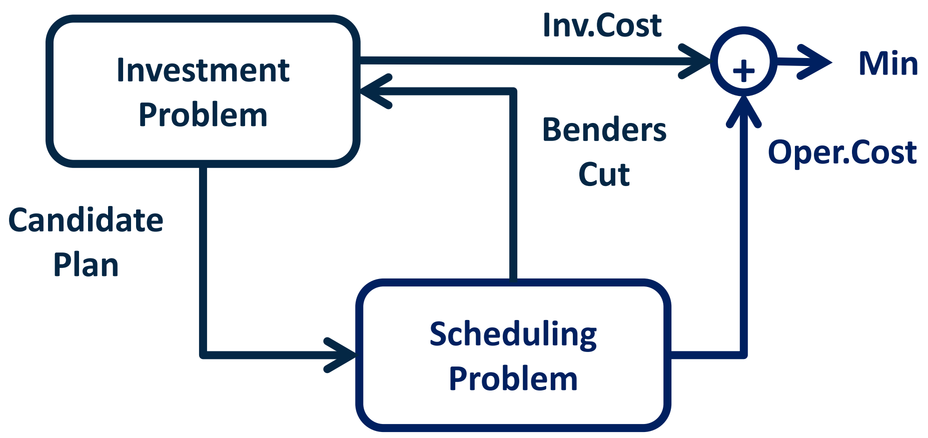
- **System operation (generation scheduling)** - Optimize the use of existing resources (hydro, natural gas, renewables etc.)

Benders decomposition for capacity planning



Benders decomposition for capacity planning



Benders decomposition: investment problem

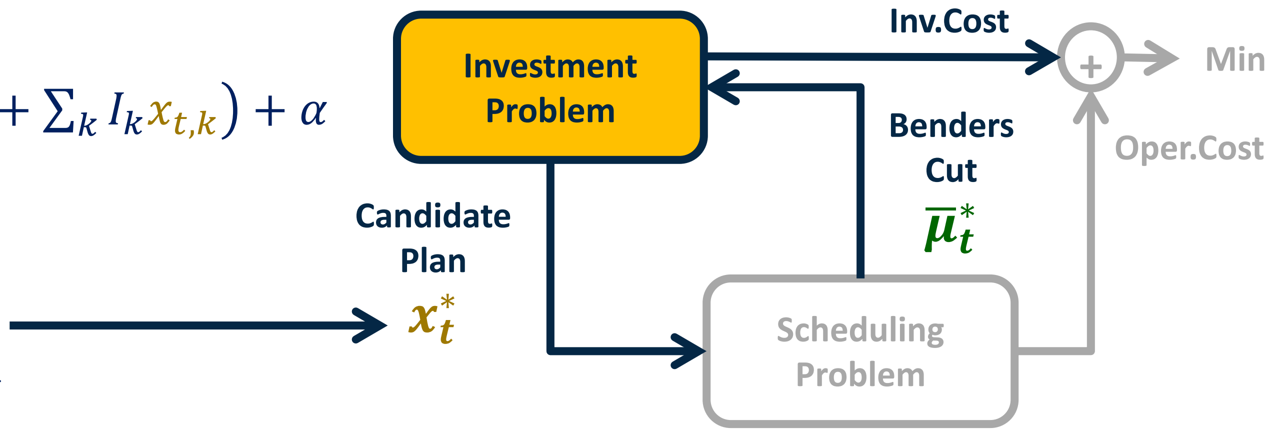
$$Min\ \sum_t \left(\sum_i I_i x_{t,i} + \sum_j I_j x_{t,j} + \sum_k I_k x_{t,k} \right) + \alpha$$

s/t

$$x_{t,i} \geq x_{t-1,i}, \forall t, i$$

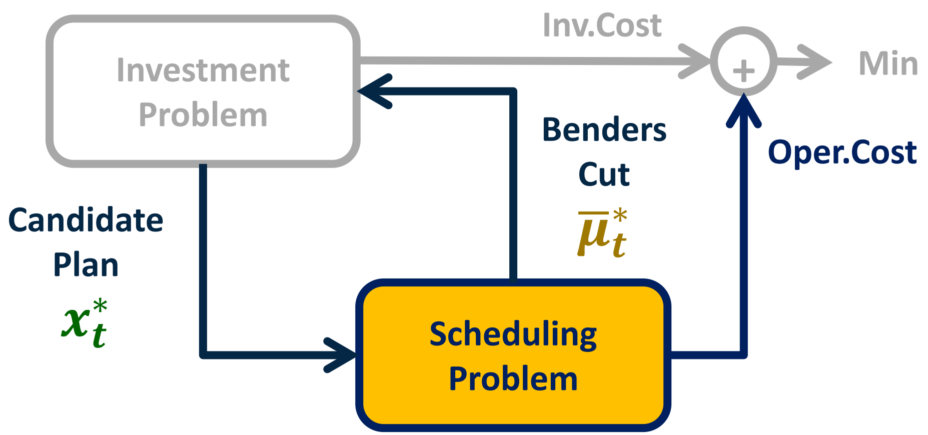
$$x_{t,j} \geq x_{t-1,j}, \forall t, j$$

$$x_{t,k} \geq x_{t-1,k}, \forall t, k$$

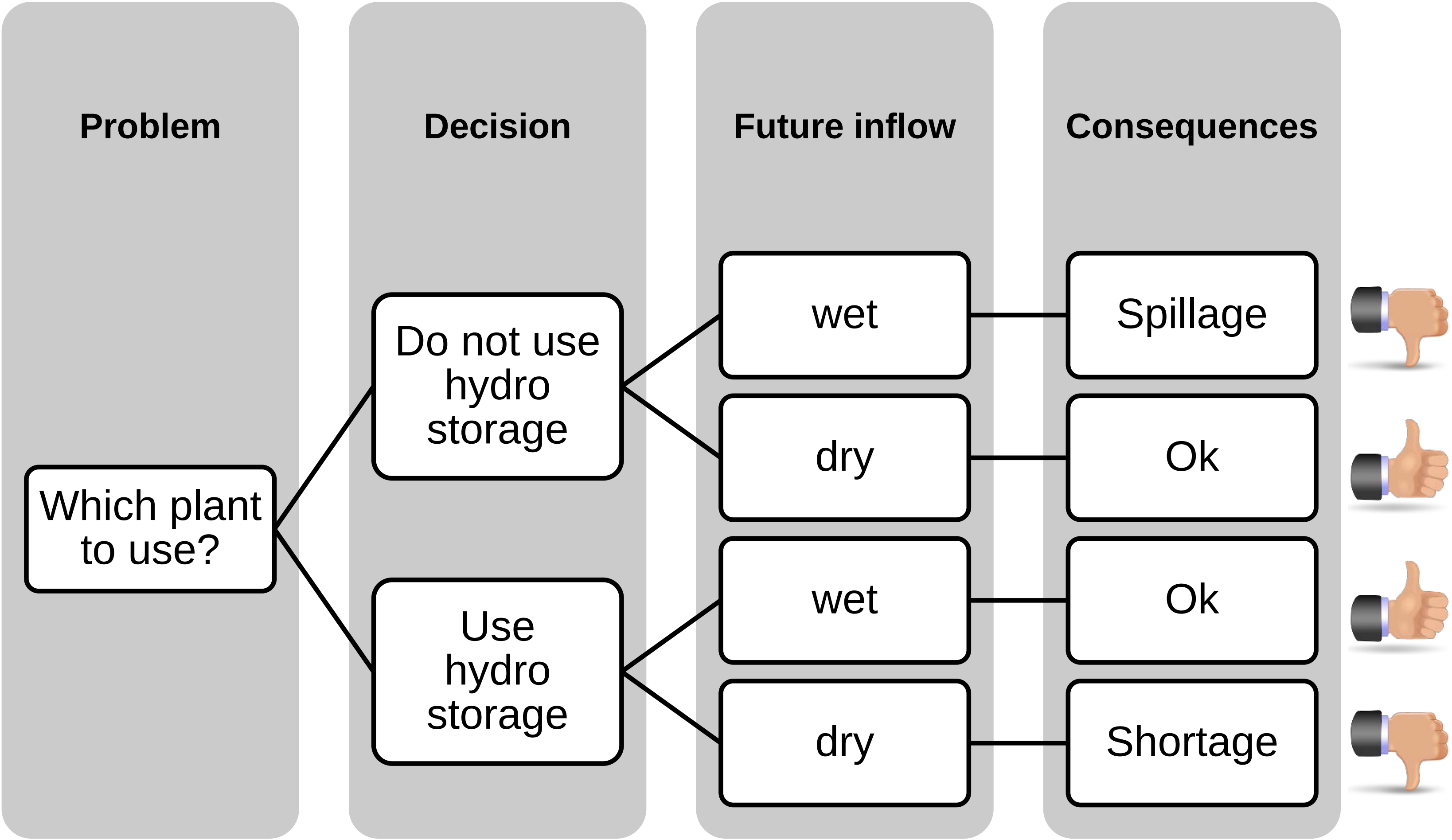


$$\alpha \geq \bar{\alpha}^m + \sum_t \left(\sum_i \bar{\mu}_{t,i}^m \left(x_{t,i} - x_{t,i}^m \right) + \sum_j \bar{\mu}_{t,j}^m \left(x_{t,j} - x_{t,j}^m \right) + \sum_k \bar{\mu}_{t,k}^m \left(x_{t,k} - x_{t,k}^m \right) \right), \forall m$$

Benders decomposition: scheduling problem



Operating decision under uncertainty



Stochastic optimization model

Solution algorithm: stochastic dual dynamic programming (SDDP)

- Avoids “curse of dimensionality” of traditional SDP $\Rightarrow$ handles large systems
- Suitable for distributed processing

Stochastic parameters

- Hydro inflows and renewable generation (wind, solar, biomass etc.)
  - Multivariate stochastic model (PAR(p))
    - Inflows: macroclimatic events (El Niño), snowmelt and others
    - Spatial correlation of wind, solar and hydro
    - External renewable models can be used to produce scenarios
- Uncertainty on fuel costs
  - Markov chains (hybrid SDDP/SDP model)
- Wholesale energy market prices
  - Markov chains
- Load variability and equipment outages
  - Monte Carlo sampling

SDDP characteristics

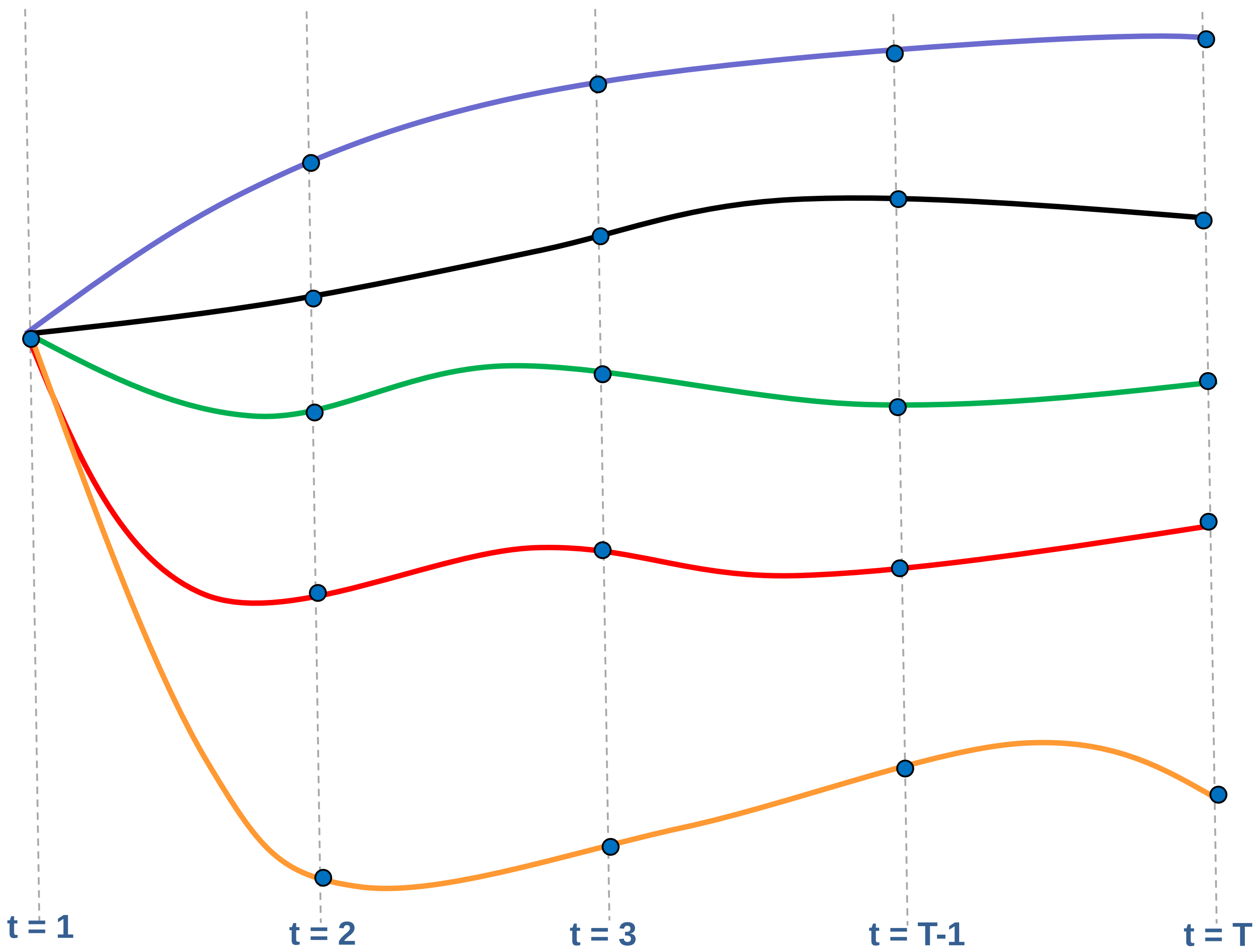
Iterative procedure

1. forward simulation: finds new states and provides upper bound
2. backward recursion: updates FCFs and provides lower bound
3. convergence check (LB in UP confidence interval)

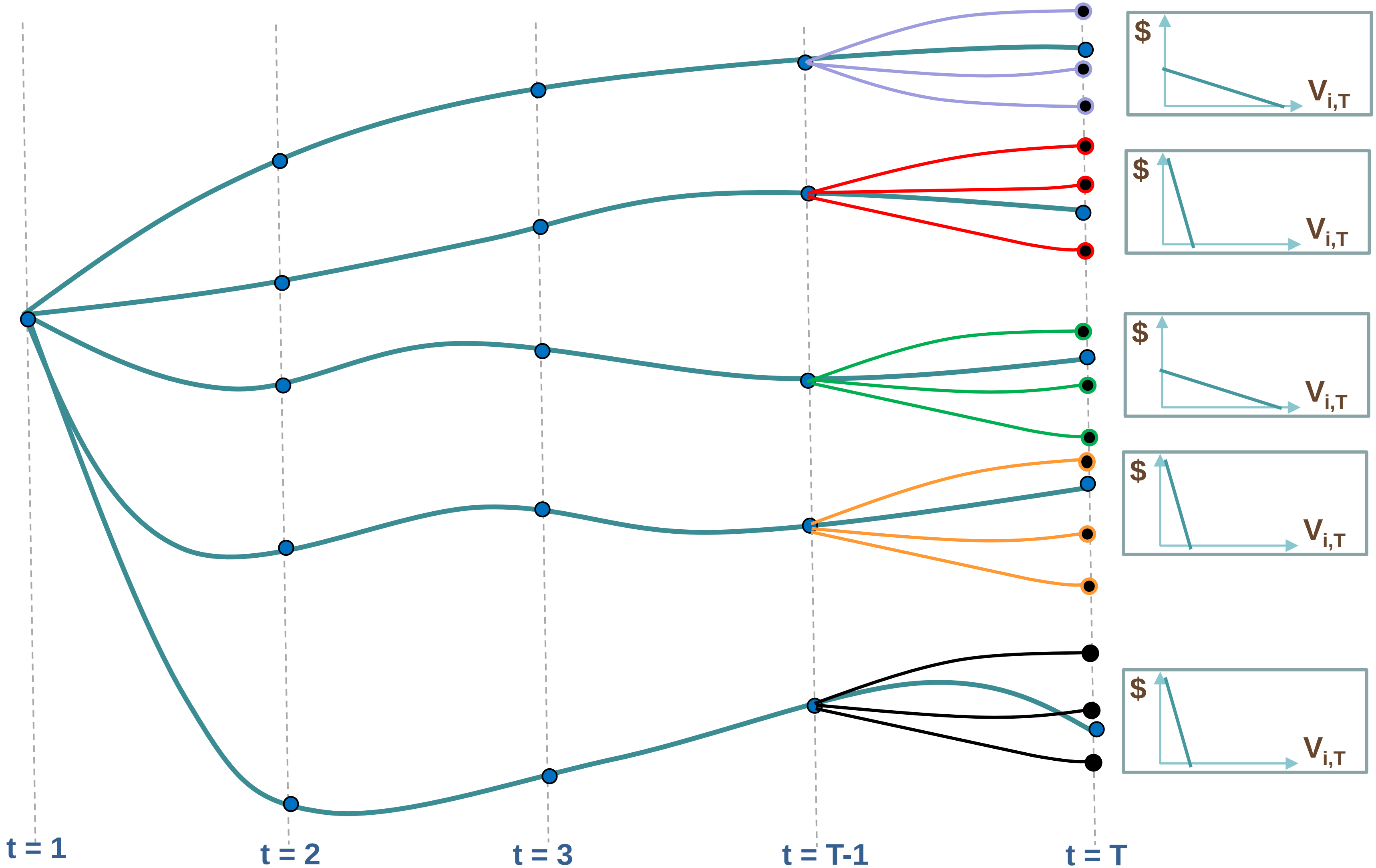
Distributed processing

- The one-stage subproblems in both forward and backward steps can be solved simultaneously, which allows the application of distributed processing
- SDDP has been running on computer networks since 2001; from 2006, in a cloud system with AWS
  - We currently have 500 virtual servers with 16 CPUs and 900 GPUs each

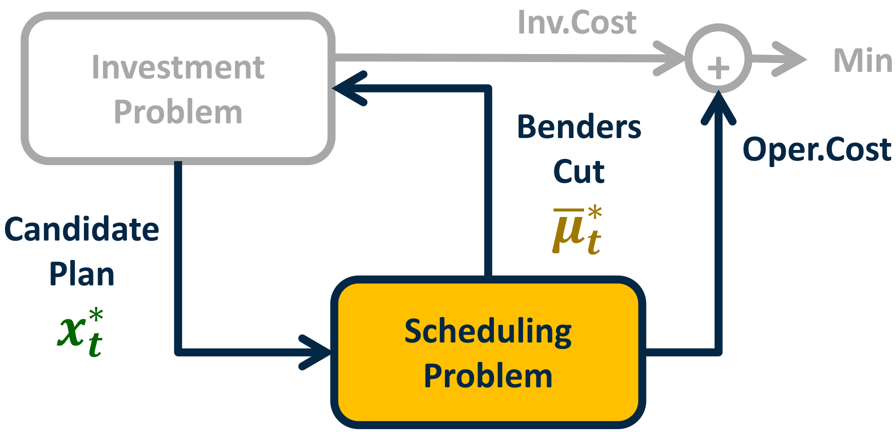
SDDP: distributed processing of forward step



SDDP: distributed processing of backward step



Benders decomposition for capacity planning



$$\bar{\mu}_{t,i}^{m+1} = \mathbb{E}\left[\bar{v}_i \pi_{t,i}^{v,s} + \bar{u}_i \pi_{t,i}^{u,s}\right]$$

$$\bar{\mu}_{t,j}^{m+1} = \mathbb{E}\left[\bar{g}_j \pi_{t,j}^{g,s}\right]$$

$$\bar{\mu}_{t,k}^{m+1} = \mathbb{E}\left[r_{t,k}^{s} \pi_{t}^{d,s}\right]$$

$$\alpha_t(v_t^s) = Min \ \textstyle\sum_j c_j g_{t,j} + \frac{1}{L} \sum_l \alpha_{t+1}^l$$

s/t

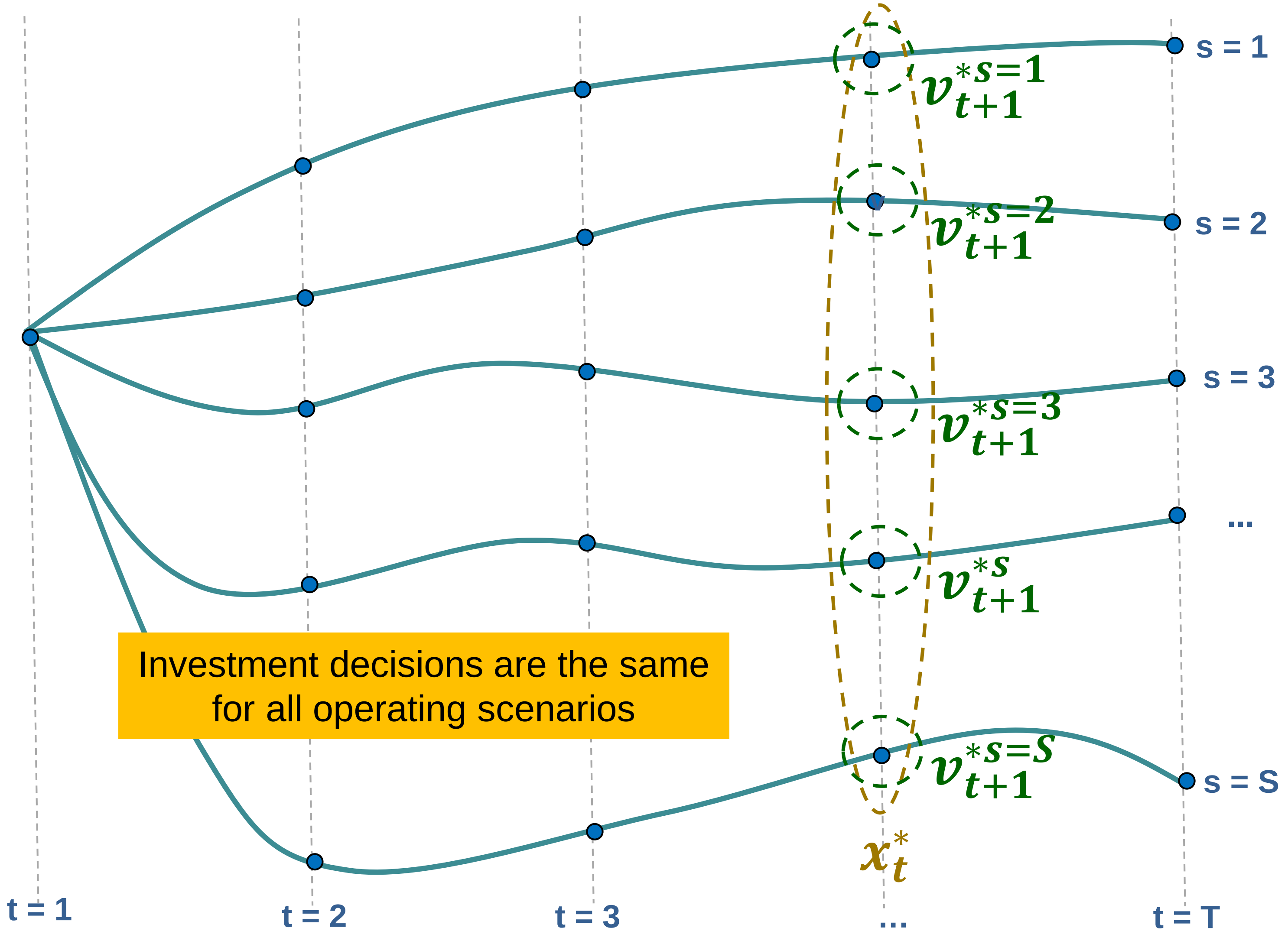
$v_{t+1,i} = v_{t,i}^s + a_{t,i} - u_{t,i},$	$\forall i$	$\pi_{t,i}^{h,s}$
$v_{t+1,i} \le \bar{v}_i x_{t,i}^{m+1},$	$\forall i$	$\pi_{t,i}^{v,s}$
$u_{t,i} \le \bar{u}_i x_{t,i}^{m+1},$	$\forall i$	$\pi_{t,i}^{u,s}$
$g_{t,j} \le \bar{g}_j x_{t,j}^{m+1},$	$\forall j$	$\pi_{t,i}^{g,s}$
$\sum_i \rho_i u_{t,i} + \sum_j g_{t,j} = d_t - \sum_k r_{t,k}^s \, x_{t,k}^{m+1}$		$\pi_t^{d,s}$
$\alpha_{t+1}^l \ge \alpha_{t+1}^* + \sum_i \pi_{t+1,i}^{h,p}\left(v_{t+1,i} - v_{t+1,i}^s\right),$		$\forall l, p$

Expansion strategy

- Expansion plans (construction schedule determined by the investment module) do not capture well an important attribute for the evaluation of generation technologies: construction time
  - Hydro: 6+ years; thermal (gas): 4+ years; renewables: 1-2 years
- The ability to adjust construction of new generation to evolving conditions is especially relevant for developing economies
  - High, but very uncertain, load growth
  - In some countries, hydro storage is an important factor for reinforcements (e.g. recent three-year drought in Brazil)
  - Uncertainty on fuel costs also favors renewables

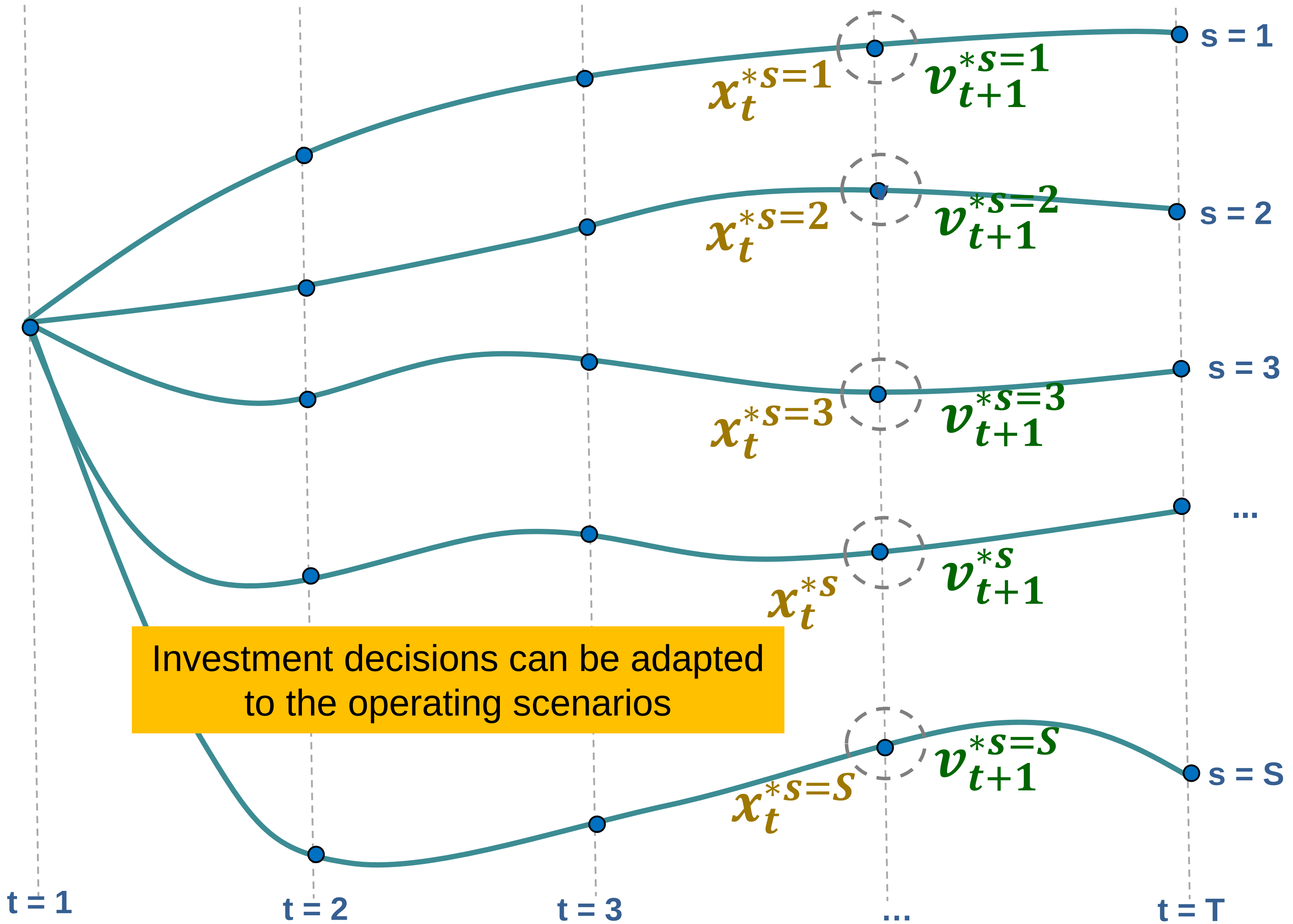
⇒ In this work, we extend SDDP to determine a capacity expansion strategy, where investment decisions (w/ time construction lags) in each stage depend on the system state

Modeling expansion plans



Investment decisions are the same for all operating scenarios

Modeling expansion strategies



Operation strategy – stage t

$$\alpha_t(v_t^s) = Min \sum_j c_j g_{t,j} + \frac{1}{L}\sum_l \alpha_{t+1}^l$$

s/t

s/t	$v_{t+1,i} = v_{t,i}^s + a_{t,i} - u_{t,i},$	$\forall i$	$\pi_{t,i}^h$
	$v_{t+1,i} \leq \bar{v}_i x_{t,i}^m,$	$\forall i$	$\pi_{t,i}^v$
	$u_{t,i} \leq \bar{u}_i x_{t,i}^m,$	$\forall i$	$\pi_{t,i}^u$
	$g_{t,j} \leq \bar{g}_j x_{t,j}^m,$	$\forall j$	$\pi_{t,j}^g$
	$\sum_i \rho_i u_{t,i} + \sum_j g_{t,j} = d_t - \sum_k r_{t,k}^s x_{t,k}^m$		π_t^d
	$\alpha_{t+1}^l \geq \alpha_{t+1}^* + \sum_i \pi_{t+1,i}^{h,p}\left(v_{t+1,i} - v_{t+1,i}^s\right),$	$\forall l,p$	

Operation strategy – stage t

$$\alpha_t(\boldsymbol{v}_t^s) = Min \sum_j c_j g_{t,j} + \frac{1}{L}\sum_l \alpha_{t+1}^l$$

s/t

$v_{t+1,i} = \boldsymbol{v}_{t,i}^s + a_{t,i} - u_{t,i},$	$\forall i$	$\boldsymbol{\pi}_{t,i}^h$
$v_{t+1,i} \le \bar{v}_i x_{t,i}^m,$	$\forall i$	$\pi_{t,i}^v$
$u_{t,i} \le \bar{u}_i x_{t,i}^m,$	$\forall i$	$\pi_{t,i}^u$
$g_{t,j} \le \bar{g}_j x_{t,j}^m,$	$\forall j$	$\pi_{t,j}^g$
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$\alpha_{t+1}^l \ge \alpha_{t+1}^* + \sum_i \pi_{t+1,i}^{h,p}(v_{t+1,i} - v_{t+1,i}^s),$	$\forall l, p$	

$$\alpha_t^l \ge \alpha_t^* + \sum_i \boldsymbol{\pi}_{t,i}^{h,p}(v_{t,i} - \boldsymbol{v}_{t,i}^s)$$

Benders cut for stage t–1

Expansion strategy – stage t

$$\alpha_t(v_t^s, x_{t-1}^s) = Min \sum_i I_i x_{t,i} + \sum_j I_j x_{t,j} + \sum_k I_k x_{t,k} + \sum_j c_j g_{t,j} + \frac{1}{L}\sum_l \alpha_{t+1}^l$$

s/t

$v_{t+1,i} = v_{t,i}^s + a_{t,i} - u_{t,i},$	$\forall i$	$\pi_{t,i}^h$
$v_{t+1,i} - \bar{v}_i x_{t,i} \leq 0,$	$\forall i$	$\pi_{t,i}^v$
$u_{t,i} - \bar{u}_i x_{t,i} \leq 0,$	$\forall i$	$\pi_{t,i}^u$
$g_{t,j} - \bar{g}_j x_{t,j} \leq 0,$	$\forall j$	$\pi_{t,j}^g$
$\sum_i \rho_i u_{t,i} + \sum_j g_{t,j} + \sum_k r_{t,k} = d_t$		π_t^d
$r_{t,k} - r_{t,k}^s x_{t,k} \leq 0,$	$\forall k$	$\pi_{t,k}^r$
$x_{t,i} \geq x_{t-1,i}^s,$	$\forall i$	$\pi_{t,i}^x$
$x_{t,j} \geq x_{t-1,j}^s,$	$\forall j$	$\pi_{t,j}^x$
$x_{t,k} \geq x_{t-1,k}^s,$	$\forall k$	$\pi_{t,k}^x$

$$\alpha_{t+1}^l \geq \alpha_{t+1}^* + \sum_i \pi_{t+1,i}^{h,p}\left(v_{t+1,i} - v_{t+1,i}^s\right) +$$

$$\sum_i \pi_{t+1,i}^{x,p}\left(x_{t,i} - x_{t,i}^s\right) + \sum_j \pi_{t+1,j}^{x,p}\left(x_{t,j} - x_{t,j}^s\right) + \sum_k \pi_{t+1,k}^{x,p}\left(x_{t,k} - x_{t,k}^s\right), \forall p$$

Expansion strategy – stage t

$$\alpha_t(v_t^s, \boxed{\boldsymbol{x_{t-1}^s}}) = Min \sum_i I_i x_{t,i} + \sum_j I_j x_{t,j} + \sum_k I_k x_{t,k} + \sum_j c_j g_{t,j} + \frac{1}{L}\sum_l \alpha_{t+1}^l$$

s/t

$$v_{t+1,i} = v_{t,i}^s + a_{t,i} - u_{t,i}, \quad \forall i \quad \pi_{t,i}^h$$

$$v_{t+1,i} - \bar{v}_i x_{t,i} \leq 0, \quad \forall i \quad \pi_{t,i}^v$$

$$u_{t,i} - \bar{u}_i x_{t,i} \leq 0, \quad \forall i \quad \pi_{t,i}^u$$

$$g_{t,j} - \bar{g}_j x_{t,j} \leq 0, \quad \forall j \quad \pi_{t,j}^g$$

$$\sum_i \rho_i u_{t,i} + \sum_j g_{t,j} + \sum_k r_{t,k} = d_t \quad \pi_t^d$$

$$r_{t,k} - r_{t,k}^s x_{t,k} \leq 0, \quad \forall k \quad \pi_{t,k}^r$$

$$x_{t,i} \geq \boldsymbol{x_{t-1,i}^s}, \quad \forall i \quad \pi_{t,i}^x$$

$$x_{t,j} \geq \boldsymbol{x_{t-1,j}^s}, \quad \forall j \quad \pi_{t,j}^x$$

$$x_{t,k} \geq \boldsymbol{x_{t-1,k}^s}, \quad \forall k \quad \pi_{t,k}^x$$

$$\alpha_{t+1}^l \geq \alpha_{t+1}^* + \sum_i \pi_{t+1,i}^{h,p}\left(v_{t+1,i} - v_{t+1,i}^s\right) +$$

$$\boxed{\sum_i \pi_{t+1,i}^{x,p}\left(x_{t,i} - x_{t,i}^s\right) + \sum_j \pi_{t+1,j}^{x,p}\left(x_{t,j} - x_{t,j}^s\right) + \sum_k \pi_{t+1,k}^{x,p}\left(x_{t,k} - x_{t,k}^s\right)}, \forall p$$

Expansion strategy – stage t

Benders cut for stage t–1

$$\alpha_t^l \geq \alpha_t^* + \sum_i \pi_{t,i}^{h,p}\left(v_{t,i} - v_{t,i}^s\right) +$$

$$\sum_i \pi_{t,i}^{x,p}\left(x_{t-1,i} - x_{t-1,i}^s\right) + \sum_j \pi_{t,j}^{x,p}\left(x_{t-1,j} - x_{t-1,j}^s\right) + \sum_k \pi_{t,k}^{x,p}\left(x_{t-1,k} - x_{t-1,k}^s\right)$$

Expansion strategy – construction time

$$\alpha_t(v_t^s, x_{t-1}^s, x_{t-2}^s, x_{t-3}^s) = Min \sum_i I_i x_{t,i} + \sum_j c_j g_{t,j} + \frac{1}{L}\sum_l \alpha_{t+1}^l$$

s/t

$$v_{t+1,i} = v_{t,i}^s + a_{t,i} - u_{t,i}, \quad \forall i \quad \pi_{t,i}^h$$

$$v_{t+1,i} \le \bar{v}_i x_{t-3,i}^s, \quad \forall i \quad \pi_{t,i}^v$$

$$u_{t,i} \le \bar{u}_i x_{t-3,i}^s, \quad \forall i \quad \pi_{t,i}^u$$

$$g_{t,j} \le \bar{g}_j, \quad \forall j \quad \pi_{t,j}^g$$

$$\sum_i \rho_i u_{t,i} + \sum_j g_{t,j} = d_t - \sum_k r_{t,k}^s \quad \pi_t^d$$

$$x_{t,i} \ge x_{t-1,i}^s, \quad \forall i \quad \pi_{t,i}^x$$

$$y_{t-1,i} = x_{t-1,i}^s, \quad \forall j \quad \pi_{t-1,i}^y$$

$$y_{t-2,i} = x_{t-2,i}^s, \quad \forall j \quad \pi_{t-2,i}^y$$

$$\alpha_{t+1}^l \ge \alpha_{t+1}^* + \sum_i \pi_{t+1,i}^{h,p}\left(v_{t+1,i} - v_{t+1,i}^s\right) +$$

$$\sum_i \left(\varphi_{t,i}^{x,p}\left(x_{t,i} - x_{t,i}^s\right) + \varphi_{t-1,i}^{x,p}\left(y_{t-1,i} - x_{t-1,i}^s\right) + \varphi_{t-2,i}^{x,p}\left(y_{t-2,i} - x_{t-2,i}^s\right)\right), \forall p$$

Expansion strategy – construction time

$$\alpha_t(v_t^s, \boldsymbol{x_{t-1}^s}, \boldsymbol{x_{t-2}^s}, \boldsymbol{x_{t-3}^s}) = Min \sum_i I_i x_{t,i} + \sum_j c_j g_{t,j} + \frac{1}{L}\sum_l \alpha_{t+1}^l$$

s/t

	$v_{t+1,i} = v_{t,i}^s + a_{t,i} - u_{t,i},$	$\forall i$	$\pi_{t,i}^h$
	$v_{t+1,i} \leq \bar{v}_i \boldsymbol{x_{t-3,i}^s},$	$\forall i$	$\pi_{t,i}^v$
	$u_{t,i} \leq \bar{u}_i \boldsymbol{x_{t-3,i}^s},$	$\forall i$	$\pi_{t,i}^u$
	$g_{t,j} \leq \bar{g}_j,$	$\forall j$	$\pi_{t,j}^g$
	$\sum_i \rho_i u_{t,i} + \sum_j g_{t,j} = d_t - \sum_k r_{t,k}^s$		π_t^d
	$x_{t,i} \geq \boldsymbol{x_{t-1,i}^s},$	$\forall i$	$\pi_{t,i}^x$
	$y_{t-1,i} = \boldsymbol{x_{t-1,i}^s},$	$\forall j$	$\pi_{t-1,i}^y$
	$y_{t-2,i} = \boldsymbol{x_{t-2,i}^s},$	$\forall j$	$\pi_{t-2,i}^y$

$$\alpha_{t+1}^l \geq \alpha_{t+1}^* + \sum_i \pi_{t+1,i}^{h,p}\left(v_{t+1,i} - v_{t+1,i}^s\right) +$$

$$\sum_i \left(\varphi_{t,i}^{x,p}\left(x_{t,i} - x_{t,i}^s\right) + \varphi_{t-1,i}^{x,p}\left(y_{t-1,i} - x_{t-1,i}^s\right) + \varphi_{t-2,i}^{x,p}\left(y_{t-2,i} - x_{t-2,i}^s\right)\right), \forall p$$

Expansion strategy – construction time

Benders cut for stage t–1

$$\alpha_t^l \geq \alpha_t^* + \sum_i \pi_{t,i}^{h,p}\left(v_{t,i} - v_{t,i}^s\right) +$$

$$\sum_i \left(\varphi_{t-1,i}^{x,p}\left(x_{t-1,i} - x_{t-1,i}^s\right) + \varphi_{t-2,i}^{x,p}\left(y_{t-2,i} - x_{t-2,i}^s\right) + \varphi_{t-3,i}^{x,p}\left(y_{t-3,i} - x_{t-3,i}^s\right)\right)$$

Where

$$\varphi_{t-1,i}^{x,p} = \pi_{t,i}^x + \pi_{t-1,i}^y$$

$$\varphi_{t-2,i}^{x,p} = \pi_{t-2,i}^y$$

$$\varphi_{t-3,i}^{x,p} = \bar{v}_i \pi_{t,i}^v + \bar{u}_i \pi_{t,i}^u$$

Expansion strategy – integer decision

- Continuous expansion decision is usually reasonable for smaller plants such as wind, biomass and solar
- Binary variables are almost always necessary for large hydro and thermal plants, which cannot be scaled continuously
- However, SDDP algorithm requires convexity in the multistage recursion. What are the alternatives?

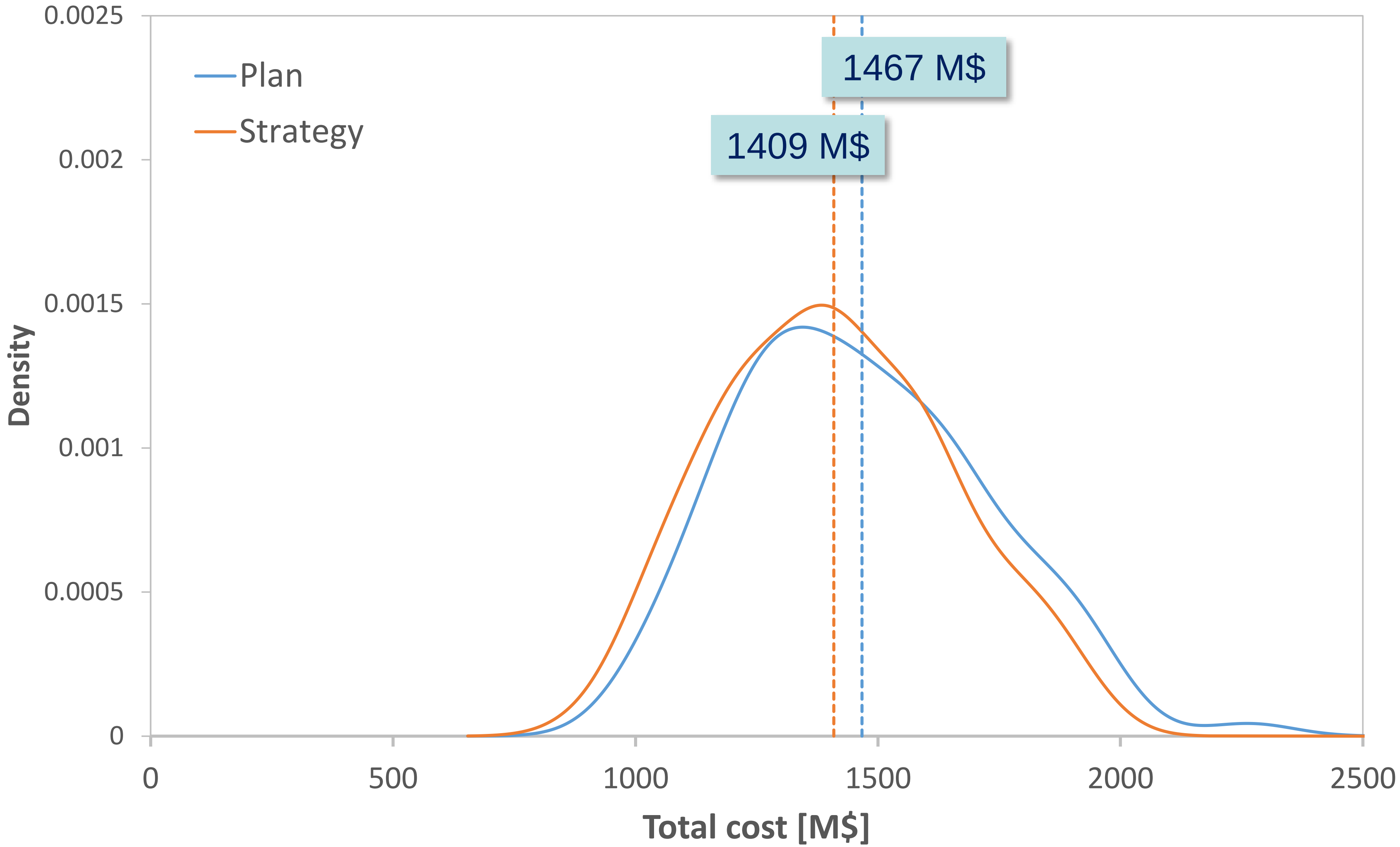
Expansion strategy – integer decision

- Relaxation scheme applied to each stage and scenario of backward recursion to produce convex Benders cuts
  - Linear relaxation
  - [Thome et al. 2013] Customized Lagrangian relaxation scheme
  - [Zou et al. 2016] Interesting results for purely binary multistage problems
- Hybrid plan/strategy
  - The longer the project construction time (e.g. hydro), the less the current system state affects the project results when it starts operation
  - As a consequence, PLAN decisions = STRATEGY decisions, so integer decisions can be made in the first stage problem

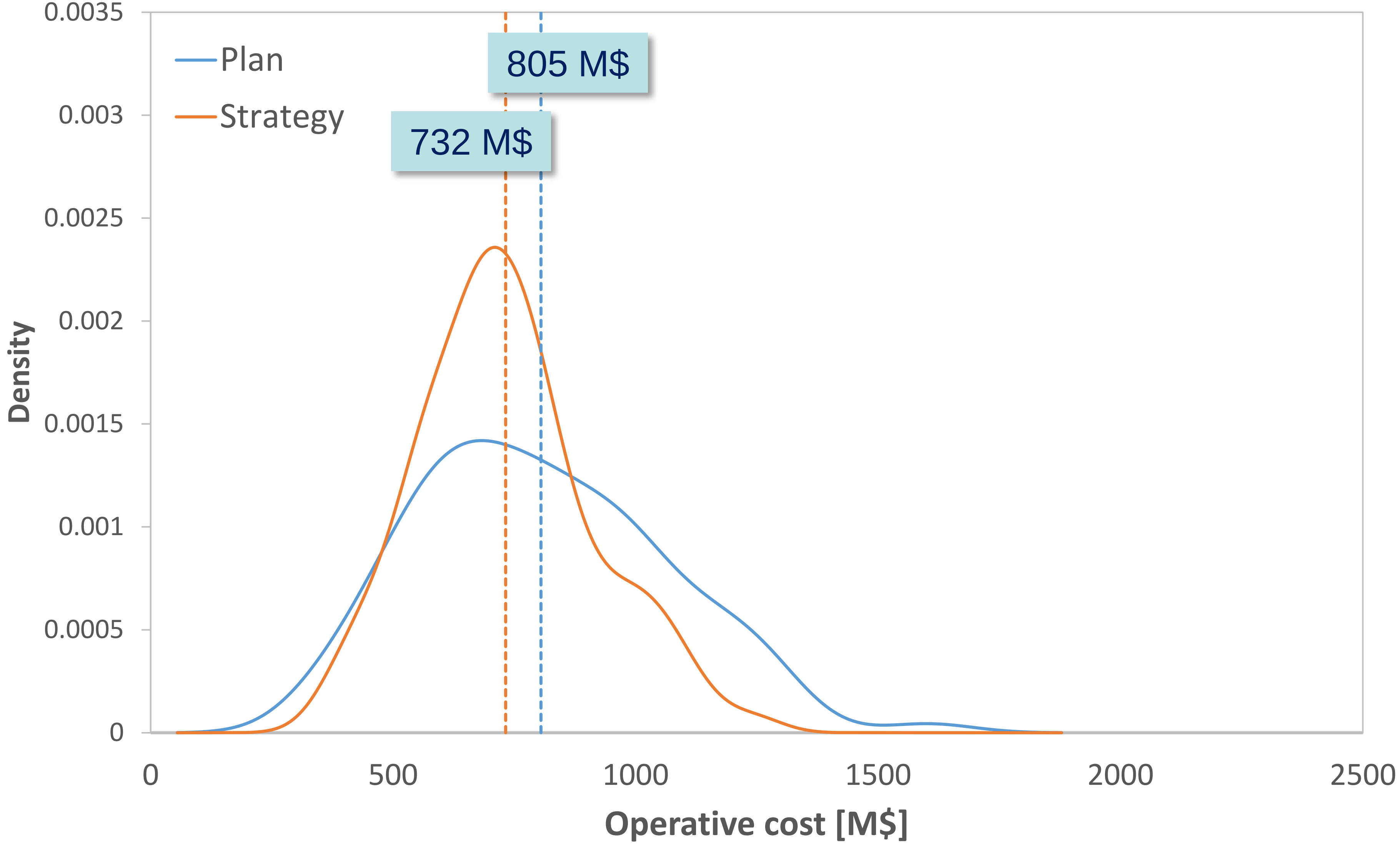
Case study: plan x strategy

- System: Costa Rica
- Horizon: 2016 – 2025
- Annual investment decisions
  - Hydro: large construction time & integer decision
  - Renewables: short construction time & continuous decision
- Operation model
  - Monthly operation decisions
  - SDDP parameters: 100 forward scenarios, 30 backward openings

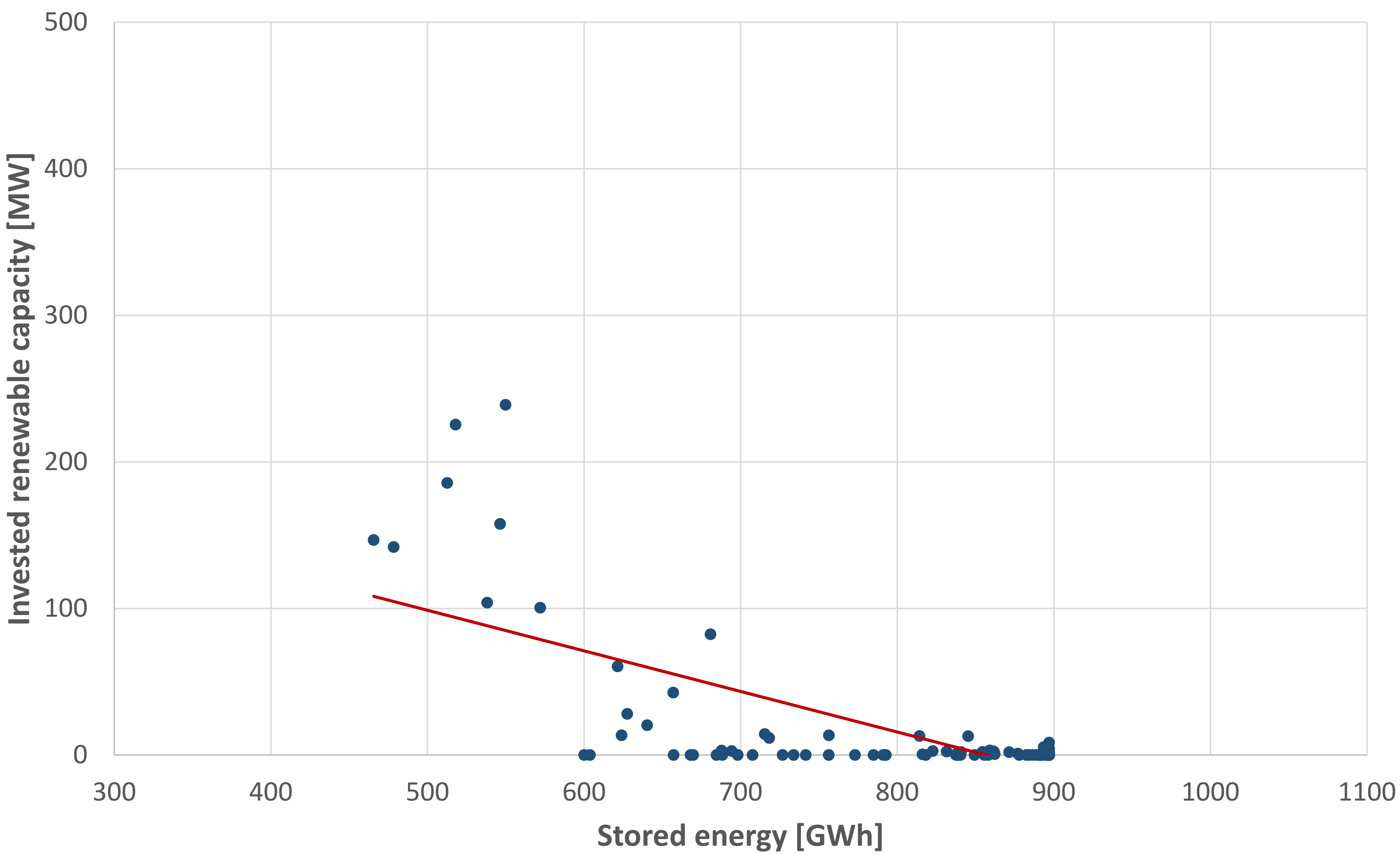
Total expected cost (investment + operation)



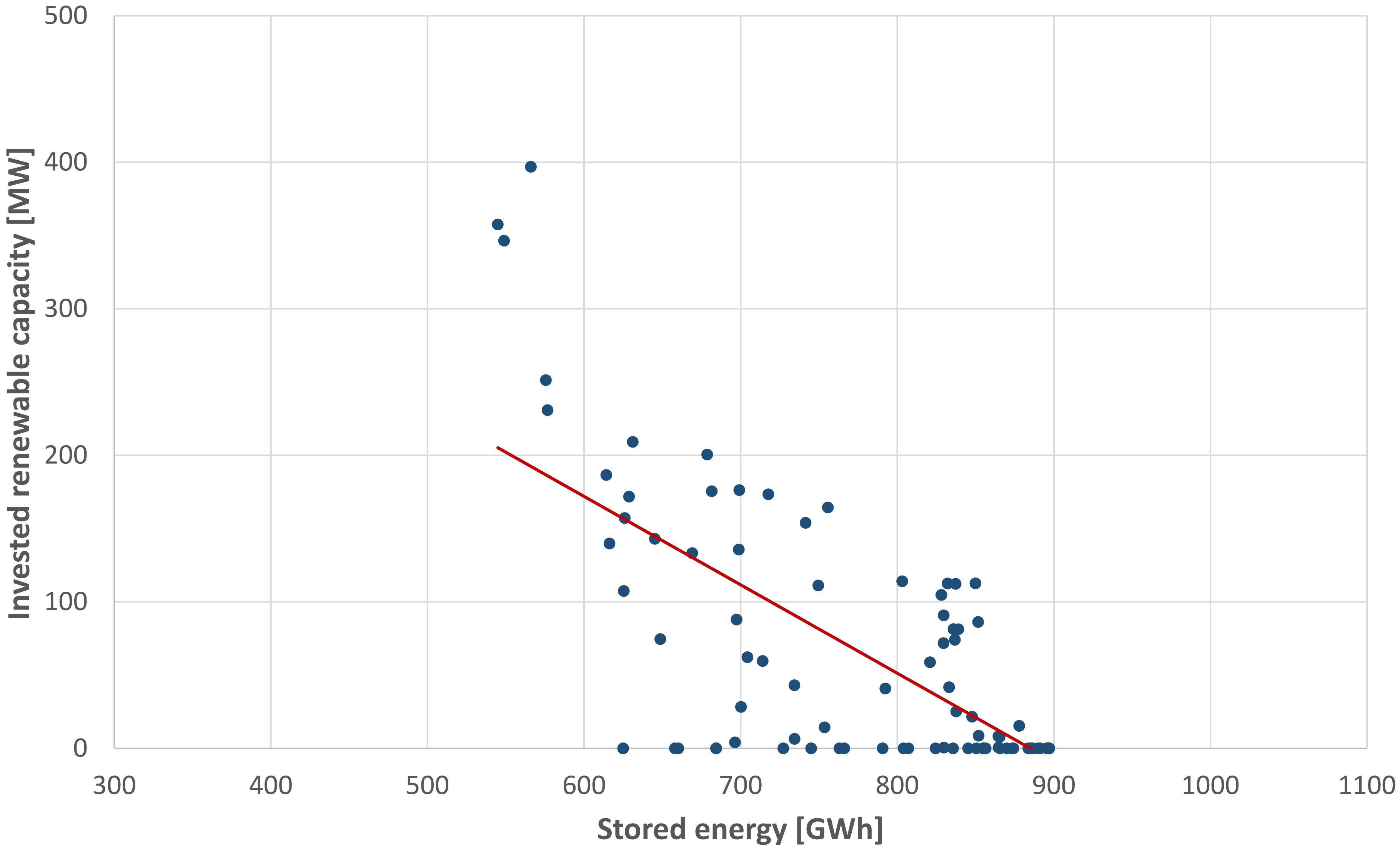
Operation cost



Investment vs. Stored energy (2017)



Investment vs. Stored energy (2020)



Conclusions

- Uncertainties play a very important role in the decision making process of the power system expansion
- As in the operation problem, it is possible to formulate an expansion strategy, where investment decisions take into account the system conditions

Thank you!
Questions?

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