SDDP-CORAL – Composite Reliability Analysis and Resource Adequacy Assessment

Methodology Manual

PSR

Contents

1 INTRODUCTION

SDDP-CORAL is PSR's Reliability and Resource Adequacy (RA) model which is available inside SDDP at no additional cost for SDDP licensed users. CORAL evaluates the composite (generation-transmission) supply reliability of large-scale systems, taking into account:

- Several power systems' elements, such as: thermal, hydro, non-conventional renewables (such as wind, solar, biomass, etc.) transmission lines, storage devices among others;
- Generation and transmission outages;
- Hydrological uncertainty;
- The effect of hydrological uncertainty on reservoir storage levels – and hence, on hydro production capacity;
- Production uncertainty of renewable generation such as wind, solar, biomass, and small hydro;
- Load variation;
- State of the art methodology:
    - Efficient implementation of the main reliability evaluation algorithms;
    - Novel simulation algorithm that allows a more realistic representation of small storage devices, such as batteries and Concentrated Solar Power plants (CSPs).

CORAL presents three different hierarchical levels:

- **Generation system:** evaluates the existence of sufficient generators within the system to meet the demand taking into consideration generation failures;
- **Transmission system:** evaluates the integrity of the transmission system and its ability to transport the necessary power from the generation to the load taking into consideration transmission failures;
- **Composite system:** evaluates the reliability of the given power system taking into consideration both generation and transmission failures.

It is worth mentioning that CORAL performs **system adequacy assessment** that is related to the existence of sufficient facilities within the system to meet the demand not considering aspects related to **system security assessment**, i.e., the ability of the system to respond to disturbances, including the minor and major disturbances that result in dynamic, transient or voltage instability of power systems.

Furthermore, in terms of the solution approach, CORAL performs Monte Carlo-based simulation methods, in which random statistical sampling mechanisms are utilized in the contingency selection, considering, additionally, stochastic processes that describe the uncertainties associated with the demand, hydrological state, renewable sources, etc. The figure below illustrates in a didactic way the objectives of these types of studies:

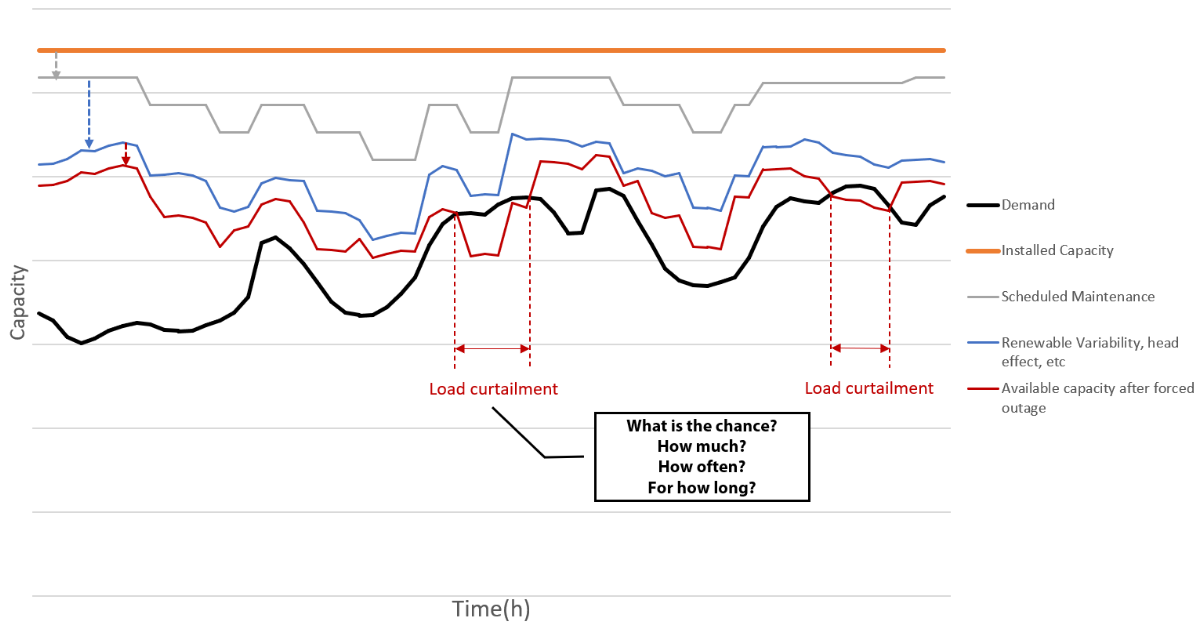

Figure 1.1 – Exemplification of a system adequacy assessment study

Furthermore, it is worth mentioning that CORAL presents three different solution methods:

- **Monte Carlo Non-Sequential:** in this solution strategy, a state sampling approach is applied, in which case the state space is randomly sampled without reference to the system operation chronology;
- **Monte Carlo Pseudo-Sequential:** in this solution strategy, the non-sequential method is used to sample the state spaces and after that, a chronological phase is added to investigate the sub-sequences of the loss-of-load states. In other words, the non-sequential strategy is used to identify failure states. Then, for each contingency scenario (or reliability scenario that will be defined in detail in the next section), if a failure state is identified, the chronological phase is done. Otherwise, the model moves on to the next contingency scenario. This approach allows the calculation of the frequency and duration reliability indices.
- **Monte Carlo Pseudo-Interval:** in this solution strategy, first, the Pseudo-Sequential approach is applied to identify the region around the failures and after that, a new phase is added to reoptimize the small storage devices (such as batteries) within the period to identify whether the system would have conditions to prevent the failure.

The three solution methods will be explained in detail throughout this document, however, the diagram below is intended to provide a general and initial overview of SDDP-CORAL:

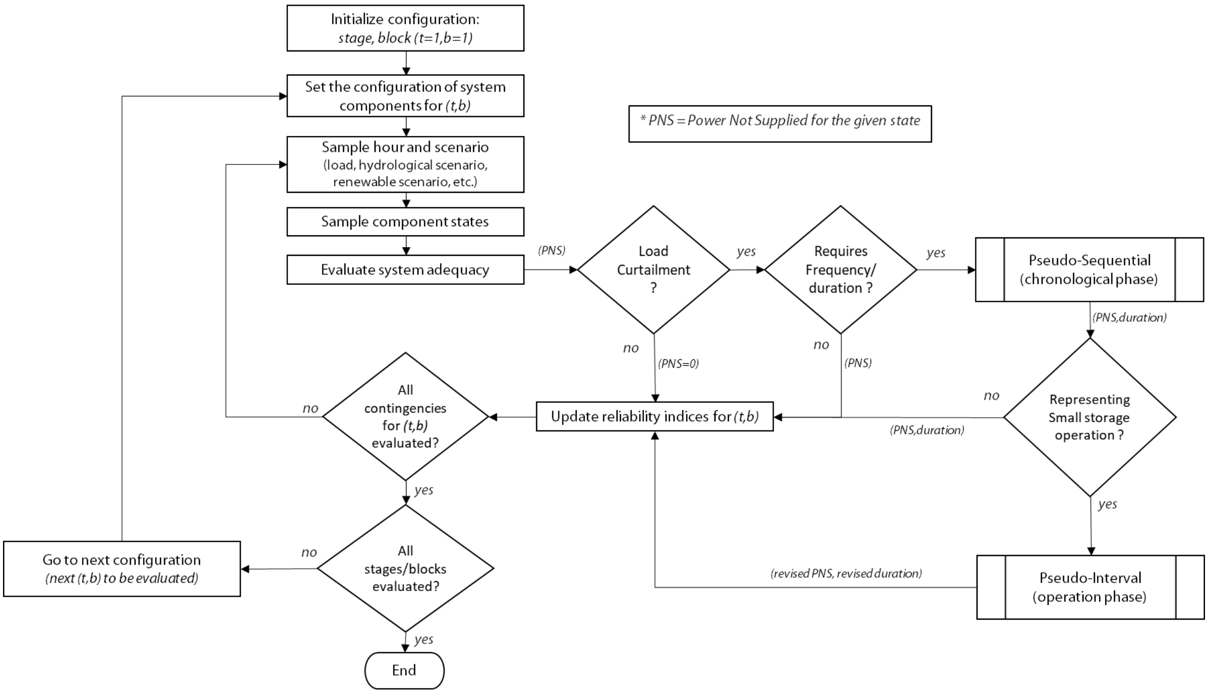

Figure 1.2 – General overview of the SDDP-CORAL model

2 BASIC CONCEPTS AND DEFINITIONS

2.1 Two-state Markov model components

The two-state Markov model is largely applied in stochastic simulations due to the facts that (i) many power system components (generation units, transmission lines, transformers, etc.) can be represented as a random binary variable (1 = operating, 0 = failed) and (ii) the assumption that the operating states of these components follow an exponential distribution ("no memory" properties).

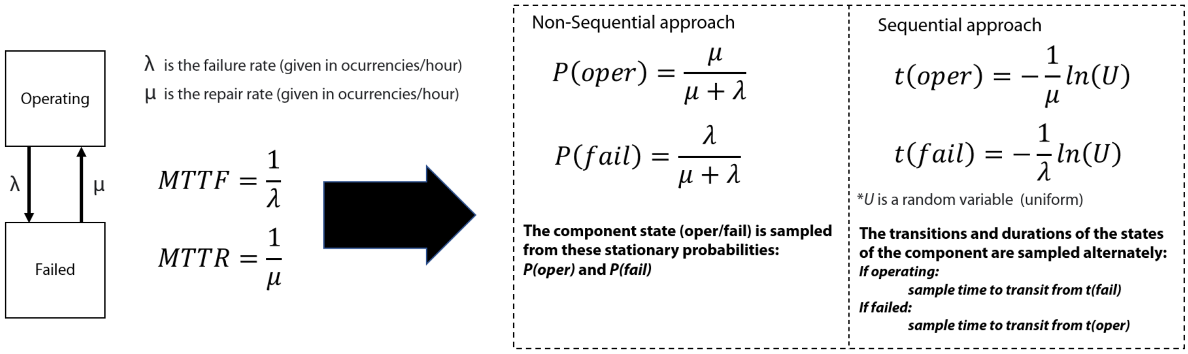

Figure 2.1 – Two-state Markov model components[1]

2.2 Generation contingency scenario

A generation contingency scenario corresponds to the power availability of every generation plant given the outages of a number of generating units as a function of the forced outage rate, which is an input data for the CORAL model.

If the two-state Markov model would be applied in the task, the procedure to evaluate the production capacity of a plant (considering outages in the generating units) would consist in sampling a random number between 0 and 1 from a uniform distribution and comparing it to the outage rate given by the user. If the number is smaller than the rate, then the generating unit is not available at the scenario. On the other hand, if the number is greater than the rate, then the unit is available for operating. The available power of each plant is obtained by multiplying the number of available units by the capacity of each one of them.

Let η_i be the number of generating units, τ_i be the outage rate of the plant i and λ be the number of operating units. Then, to obtain plant i capacity, we would perform the following steps:

Initialize the number of operating units, $k = 0$

For each unit $n = 1, \dots, \eta_i$

Sample a number δ from a uniform distribution $(0,1)$

If $\delta > \tau_i$, increase the number of operating units: $k = k + 1$

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[1] MTTF: mean time to failure; MTTR: mean time to repair.

The capacity of the generator is given by $\bar{g}^l = \bar{g} \times \frac{k}{\eta_i}$

As can be seen, the two-state Markov model could immediately be applied to represent each generating unit. However, this approach is inefficient when there are many units per plant. A more efficient approach is to sample the total available capacity of the plants from a **binominal distribution**:

$$f(k;n;p) = \binom{n}{k} p^k (1-p)^{n-k} \quad (1)$$

where:

k is the number of available units

n is the number of total units

p is the stationary probability p(failure) of a unit

The performance improvement is due to the fact that (for each plant) the distribution function $f(k;n;p)$ can be used to construct **a discrete cumulative distribution**, given by the table: $[P(0;n;p) \quad P(1;n;p) \quad P(2;n;p) \quad \cdots \quad P(n-1;n;p) \quad P(n;n;p)]$, before the simulation process.

During the simulation process, the number of available units is sampled from the pre-calculated table:

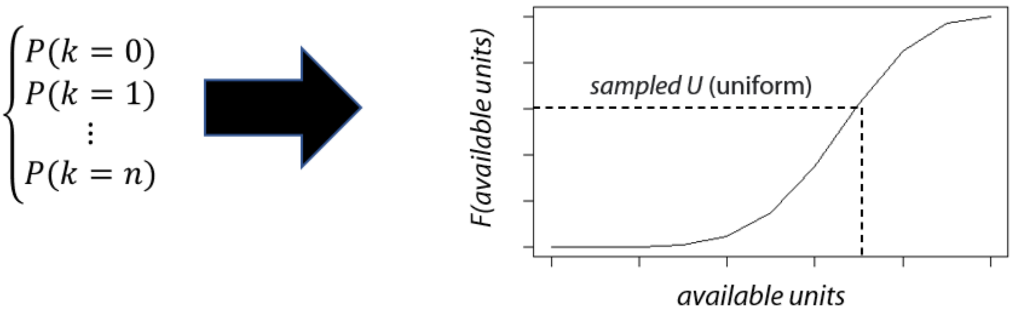

Figure 2.2 – The sampling process of the number of available units

In summary, for the generation contingency scenarios, CORAL uses **a discrete cumulative distribution** for each plant, as explained above.

2.2.1 Considering the effect of the variation of the hydroelectric power with respect to the net head of a reservoir in the calculation of the reliability statistics

In order to use this feature, the user must activate the “Use SDDP hydro capacity limit” option available on the “Reliability analysis > Reliability options” screen. For further details, please check CORAL’s User Manual. This option allows the integration with the results of a previous SDDP run by using the following output: "Hydro Capacity (no outages)". It can be used to represent the variability in the capacity due to the variation of the production coefficient according to the volume stored in the reservoir. If the SDDP run was stochastic, the hydro capacities will vary per scenario.

In other words, depending on the hydrological scenario, the available power of hydroelectric plants may be affected by operative decisions. For hydroelectric plants with reservoirs, the power depends on the net head – difference between forebay and tailwater elevation. As a consequence, to consider the effect of the variation of the hydroelectric power with respect to the net head of a reservoir in the calculation of the reliability statistics, it is required to previously run a stochastic simulation of the system operation using the SDDP model to obtain the scenarios of stored volume in each reservoir of each hydroelectric plant in each stage.

We obtain the available power capacity of the hydroelectric plants with reservoir using the production coefficient, which varies as a function of the net head of the reservoir:

$$\bar{g}_i^h = Min\{\bar{g}_i, \bar{q}_i \times \rho(V_i)\} \qquad (2)$$

Where $\bar{g}_i$ is the installed capacity, $\bar{q}_i$ is the maximum turbining outflow and $\rho(V_i)$ is the production factor (which depends on the storage level V_i) of the hydroelectric plant i.

With respect to run-of-the-river plants (RoRs), first it is important to remind that each RoR has a modulation factor. In the case of a run-of-river plant with modulation factor equal to zero, this means it has free regulation, i.e., all the water that arrives in each load block can be used in any other load block. This factor is continuous from 0 to 1 (it can be 0.2, 0.3, 0.87, etc.) and 1 means that the plant does not have any regulation capacity within the stage, i.e., the water that arrives in load block 1, or it is turbined or spilled in load block 1, there is no way to take this amount of water to any other block. As a consequence, the RoR capacities written in the "Hydro Capacity (no outages)" SDDP output will be as follows:

- **For RoRs with modulation factor = 0:**
    - The same equation applied to hydroelectric plants with reservoir is used:
        - $\bar{g}_i^h = Min\{\bar{g}_i, \bar{q}_i \times \rho(V_i)\}$;
- **For RoRs with modulation factor > 0:**
    - For each load block b, we calculate the capacity associated with the water that cannot be transferred to another block:
        - $\bar{g}_1 = modulation\ factor \times g_b$;
    - Then, we calculate the "free" generation of the other load blocks that can be transferred to the current block b as follows:
        - $\bar{g}_2 = (1 - modulation\ factor) \times \bar{g}_i$;
    - Finally:
        - $\bar{g}_i^h = Min\{(\bar{g}_1 + \bar{g}_2), \bar{g}_i\}$;

Finally, it is worth mentioning that, while using this option, in each reliability scenario, an SDDP forward scenario will be sampled.

2.3 Circuit contingency scenario

While evaluating circuit contingency scenarios, circuit (transmission lines, transformers, etc.) contingencies will be sampled.

Let η_i be the number of circuits that are part of the contingency list, τ_i be the outage probability of the circuit i, λ be the number of circuits under contingency and NC the total number of circuits of the transmission system.

Then, we would perform the following steps:

Initialize the number of circuits under contingency, $c = 0$

For each circuit that is part of the contingency list $n = 1, \dots, \eta_i$

Sample a number δ from a uniform distribution (0,1)

If $\delta < \tau_i$, increase the number of circuits under contingency: $c = c + 1$

And of course, to apply contingency in this circuit, all its data is needed (bus from, bus to, location, electrical parameters, etc.).

After knowing the state of all the circuits, the system adequacy is evaluated using a DC Optimal Power Flow (DC OPF o linearized OPF) modeling. CORAL adopts the linearized active power flow instead of the non-linear power flow due to the following reasons:

- The linearized model provides a good approximation for power flows in meshed high voltage networks. It does not have convergence problems, which are common in non-linear power flow calculations (especially when the network is not reinforced);
- It is easier to be applied while dealing with circuit contingencies and islanding of buses.

2.3.1 Formulation of the linearized power flow

The linearized power flow model of an AC system is expressed by the Kirchhoff's laws presented next.

2.3.1.1 Kirchhoff's first law

This law represents the active power balance in each AC bus (for notational simplicity, we suppose that each bus has generation and load):

$$\sum_{k \in \Omega_i} f_k + g_i = d_i, \forall\, i = 1, \dots, I \tag{3}$$

where:

i	indexes the AC buses
k	indexes the circuits (K is the number of circuits)
Ω_i	is the set of circuits directly connected to bus i
g_i	is the generation of AC bus i (MW)
d_i	is the load of AC bus i (MW)
f_k	is the active power flow in the circuit k (MW)
I	is the number of AC buses

The last equation can be represented in matrix form as:

$$S\,f + g = d \tag{4}$$

where:

- S is the incidence matrix of dimension $I \times K$, whose k-th column has zeros in all rows except those of the bus terminals of the k-th circuit, i_k and j_k
- f is the K-dimensional vector of circuits flows (MW)
- g is the I-dimensional vector of AC bus generations (MW)
- d is the I-dimensional vector of AC bus loads (MW)

2.3.1.2 Kirchhoff's second law

For each AC circuit this law is expressed by:

$$f_k = \gamma_k\big(\theta(i_k) - \theta(j_k)\big) \tag{5}$$

where:

- γ_k is the circuit susceptance (inverse of reactance)
- $\theta(i_k)$ is the voltage angle of the circuit's terminal bus i_k (rad)
- $\theta(j_k)$ is the voltage angle of the circuit's terminal bus j_k (rad)

Last equation can be represented in matrix form as:

$$f = |\gamma|\,S'\,\theta \tag{6}$$

where:

- $|\gamma|$ is the diagonal $K \times K$ matrix of circuit susceptances
- S' is the transpose matrix of s (dimension $K \times I$)
- θ is the I-dimensional vector of AC bus voltage angles (in radians)

2.3.1.3 DC network

In case there are DC links in the network, the linearized power flow also represents balance equations for DC buses. A DC bus has no generation or load, so its balance equation is expressed as:

$$S_D\,f_D + S_I^{DC}\,f_I = 0 \tag{7}$$

where f_D denotes the vector of flow variables for DC links and S_D denotes the incidence matrix of the DC system; f_I is the vector of flow variables for AC/DC Converter and S_I^{DC} is the DC part of the AC-DC incidence matrix, i.e. it contains +1/-1 for DC buses depending if the AC/DC converter is defined as an inverter or a rectifier. Note that the flow in a DC link or AC/DC Converter is not subject to Kirchhoff's second law. The flow variables for DC links and AC/DC Converter have lower and upper limits whose values may vary for each load block.

Since total load matches total generation for each dispatch scenario, the lower and upper flow limits for the DC system components must not block the given AC/DC Converter flows; if this happens load will be shed, as well as re-dispatch of generation needed to restore balance.

2.3.1.4 Solving the linearized power flow equations for AC network

Given the generation vector g and load vector d for a dispatch scenario, the corresponding power flows are calculated as follows. Suppose initially that there are no DC links. Substituting f as defined in (6) into equation (4), results

$$B\,\theta + g = d \tag{8}$$

where $B = S\,|\gamma|\,S'$ is a $I \times I$ matrix known as the susceptance matrix[2]. Next, the linear system is solved and the bus voltage angle vector θ [3] is obtained:

$$\theta = B^{-1}(d - g) \tag{9}$$

Finally, the solution θ is applied to equation (6) and the power flow vector f is obtained.

2.3.1.5 Solving the linearized power flow equations for DC network

If the network has DC links, power flows are obtained solving first the AC network equations, adding to the AC bus balance equations AC/DC Converter flow variables associated to the dispatch scenario:

$$B\,\theta - S_I^{AC}\, f_I + g = d \tag{10}$$

where S_I^{AC} is the AC part of the AC-DC Converter incidence matrix, i.e., it contains -1/+1 for AC buses depending on if the AC/DC converter is defined as an inverter or a rectifier.

Assuming that f_I is known (operation point of the AC/DC Converters), the voltage angle vector θ can be evaluated by the solution of the following linear system:

$$\theta = B^{-1}\left(d - g + S_I^{AC}\, f_I\right) \tag{11}$$

Solved the linear system, AC power flow f can be determined by equation (6) and DC power flow f_D by solving the linear system (7).

On the other hand, if the operating setpoints of the AC/DC Converters are unknown, power flow on both systems are evaluated by solving a LP program that considers equations (6), (7) and the vector f_I as a free variable, as showed next.

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[2] Look that after sampling the outages of the circuits, CORAL will calculate the susceptance matrix based on the states of all circuits of the network.

[3] Since matrix B has rank $I - 1$, calculating its inverse matrix B^{-1} requires eliminating a bus of the matrix B – for example, the bus number ι (this is equivalent to eliminate the row and the column associated to the bus ι). The solution is written as $\tilde{\theta} = \tilde{B}^{-1}(\tilde{d} - \tilde{g})$, where $\sim$ represents matrices and vectors without bus ι, known as the reference bus. The voltage angle of bus ι, θ_ι, is taken as zero. To simplify notation, a zero row and a zero columns are included in matrix $\tilde{B}^{-1}$, corresponding to matrix B^{-1}.

2.3.1.6 Final DC OPF formulation used by SDDP-CORAL

Now, taking the aforementioned subsections into account, the final DC OPF formulation used by SDDP-CORAL will be presented contemplating the linearized power flow equations for the AC and the DC systems simultaneously.

When operating setpoints of AC/DC Converters are unknown, power flows are calculated by the solution of the DC OPF model presented below. It is possible to observe that in each reliability scenario, the DC OPF problem (P) is solved to obtain the reliability results, taking into account the system state (stage/block configuration, sampled generator capacities, sampled circuit states, demand scenario, renewable generation scenario, etc.):

$$z = Min \sum_{i=1}^{I} r_i \tag{12}$$

Subject to:

$$S_D \, f_D + \, S_I^{DC} \, f_I = 0 \tag{13}$$

$$B\theta - \, S_I^{AC} \, f_I + g + r = d \tag{14}$$

$$\underline{f_I} \leq f_I \leq \overline{f}_I \tag{15}$$

$$r \leq d \tag{16}$$

where:

z is the total load shedding (MW)

r is the vector of variables representing the bus load shedding (MW)

$\underline{f_I}, \overline{f}_I$ are the operational limits for AC/DC Converters

As can be seen, since the objective function is to minimize load shedding, after solving the aforementioned optimization problem, the z^*, which is the optimal result of the objective function, can be interpreted as the Expected Power Not Supplied (EPNS) for that given reliability scenario.

3 SOLUTION METHODS

3.1 Monte Carlo Non-Sequential

First of all, as explained in CORAL's User Manual, let NC_{stage} be the number of contingencies per stage, B be the number of selected load blocks and NC_b be the number of contingencies per block. Then:

$$NC_b = \frac{NC_{stage}}{B} \tag{17}$$

Now, for each contingency scenario NC_b, in this solution strategy, a state sampling approach is applied in which the state space is randomly sampled without reference to the system operation chronology. The Monte Carlo Non-Sequential solution method is explained through the script presented below:

|| Enumerate stages of CORAL's study horizon

For each stage $t = 1, \dots, T$ do

\\ For each selected load block

For each load block $b = 1, \dots,$ B:

|| For each contingency scenario (or reliability scenario)

For each contingency scenario state $n = 1, \dots, NC_b$:

Step 1:

Sample one hour if the reliability study has hourly resolution, otherwise go directly to Step 2.

Step 2:

Sample one SDDP forward scenario:

- Always necessary for renewable and demand scenarios (in case they exist);

- Necessary for hydro plants when the "Use SDDP's hydro capacity limit" option is selected;

- Necessary for batteries when one of the following options is selected: "Limit by storage (in addition to the capacity)" or "Use fixed injections calculated by SDDP".

Step 3:

Sample component states.

Step 4:

Evaluate system adequacy.

Step 5:

Update reliability indices.

3.1.1 Flowchart to summarize this solution method

The aforementioned script is summarized in the flowchart presented below:

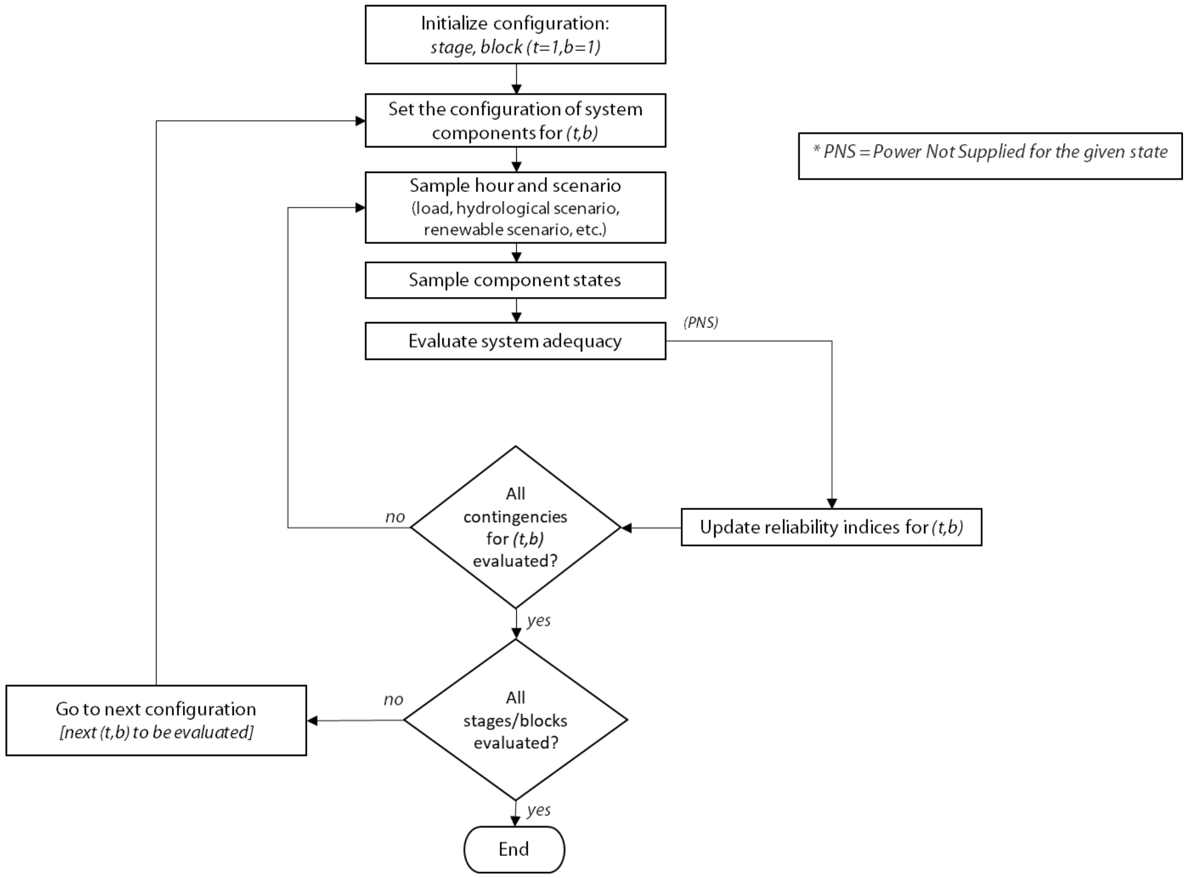

Figure 3.1 – The flowchart of the Monte Carlo Non-Sequential solution method

It is worth noting that:

Even if the reliability study has hourly resolution, the reliability indices will be computed per load block;

- If the hierarchical level selected on the "Execution options > Reliability analysis > Reliability options" screen is "Generation": in Step 4, CORAL will only compare the total available power with the demand of that given stage, scenario and load block (or hour);
- If the user activates the "Transmission" hierarchical level on the "Execution options > Reliability analysis > Reliability options" screen and the "No network or interconnections only" option on the "Execution options > Economic dispatch > Transmission and gas pipeline" screen: in Step 4, CORAL will formulate an optimization problem minimizing load shedding with multi-area representation (i.e., representing only Kirchhoff's first law and interconnection flow limits) in order to check if the total available power meets the demand of that given stage, scenario and load block (or hour);
- If the user activates the "Transmission" hierarchical level on the "Execution options > Reliability analysis > Reliability options" screen and the "Linear power flow" option on the "Execution options > Economic dispatch > Transmission and gas pipeline" screen: in

Step 4, the DC OPF problem (P), described in section "Final DC OPF formulation used by SDDP-CORAL" will be solved.

For more details regarding hierarchical levels and transmission representation options, please check the diagram below:

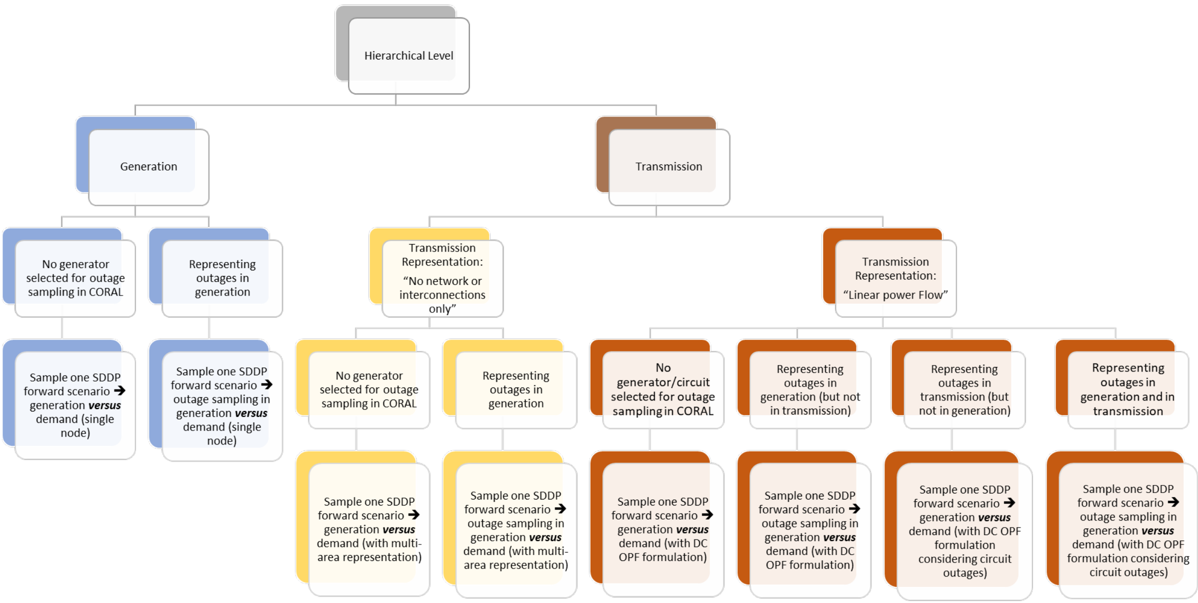

Figure 3.2 – Hierarchical levels and transmission representation options

For more details about the calculation of the reliability indices, please check chapter 4 of SDDP-CORAL's User Manual.

3.2 Monte Carlo Pseudo-Sequential

3.2.1 Introduction

The stochastic simulation method presented in the previous section is the Non-Sequential Monte Carlo Simulation (MCS), which is computationally efficient but, as the name implies, does not represent the chronology of events in the reliability assessment.

However, power systems may have important components that require chronological modeling to produce more realistic results. For example, to assess more accurately the impact of batteries on the reliability of power systems, one must consider the evolution of their charge and discharge along the entire duration of the failures.

A perfect representation of the chronology of events can be achieved with the Sequential MCS. However, the sequential approach has a very high computational cost, which can be prohibitive depending on the system being assessed.

For this reason, we implemented in Coral a hybrid methodology known as Pseudo-Sequential MCS, that achieves the same accuracy as Sequential MCS, but with a much lower computational cost. The Pseudo-Sequential MCS scheme identifies system failure states in a non-sequential way, but evaluates each identified state chronologically, and has been effectively applied, for example, to estimate frequency & duration reliability indices such as LOLF and LOLD.

3.2.2 The Monte Carlo Pseudo-Sequential Approach description

In the pseudo-sequential simulation, the failure states could be identified in a similar way to the non-sequential simulation, however, as it is eventually necessary to analyze the chronology of events around the failure occurrences, it is usual that instead of sampling the states of the system components directly from their operation/failure probabilities, a large number of chronological scenarios are generated a priori for all system components and then, during the simulation, a sample of the system state is taken as a sample of a "specific photograph" within the chronological scenarios generated.

In the method proposed for CORAL, however, it starts from the premise that the operating states of the components follow the exponential distribution, which are "memoryless" distributions, and explores their properties so that the states can be sampled in a way totally analogous to the non-sequential approach, eliminating the need to generate this large set of chronological scenarios in advance.

3.2.2.1 Sampling procedure, first phase

Thus, for this first non-sequential phase, given a stage t, block b, each sample of the state of the system in each "reliability scenario" corresponds to:

1. One sample of a "forward" SDDP scenario s;
2. One sample of an HOUR h belonging to block b;
3. One sample of the system component states:
   a. Thermal plants:

      For each plant i, we sample the number of available units (x_{Ti}), given the total number of units (N_{Ti}) from the failure probability of each unit (p_{Ti}) and the binomial distribution:

      $$x_{Ti} \sim B(N_{Ti}, p_{Ti}) \qquad (18)$$

   b. Hydro plants:

      For each plant i, sample the number of units available (x_{Hi}), given the total number of units (N_{Hi}) from the failure probability of each unit (p_{Hi}) and the binomial distribution:

      $$x_{Hi} \sim B(N_{Hi}, p_{Hi}) \qquad (19)$$

      Obtain the available capacity (without outages) associated to stage t, forward scenario s, block b (resulting from SDDP's system operation):

      $$P_{Hi} = P_{Hi}(t, s, b) \qquad (20)$$

   c. Renewable plants:

For each plant i, we sample the number of units available (x_{Ri}), given the total number of units (N_{Ri}) from the failure probability of each unit (p_{Ri}) and the binomial distribution:

$$x_{Ri} \sim B(N_{Ri}, p_{Ri}) \quad (21)$$

Obtain the renewable energy production associated to stage *t*, forward scenario *s*, hour *h*:

$$fc_i = fc_i(t, s, h) \quad (22)$$

d. Loads:

For each load *i*, obtain its respective value associated to stage *t*, hour *h*:

$$d_i = d_i(t, h) \quad (23)$$

e. Batteries:

For each battery *i*, obtain the net injection associated to stage *t*, forward scenario *s*, hour *h*:

$$I_{bi} = I_{bi}(t, \mathrm{s}, h) \quad (24)$$

From the sampled states of the components, the system adequacy is calculated, verifying whether the current state is a successful state, such that all the demand can be met while respecting the electrical constraints of the system in the state, or whether it is a failure state, requiring load shedding.

If it is a successful state, the reliability indexes are updated, and we move to the next state. If it is a failure state, we move to the chronological analysis process of the failure in order to identify the beginning and duration of the failure.

3.2.2.2 Chronological simulation, second phase

The chronological phase of the method is divided into two sub-phases: (i) backward sub-phase, in which we proceed "backwards" in time, starting from the current state until the first hour of the failure state is identified; and (ii) forward sub-phase, in which one proceeds the direct chronological sequence, until the last hour of the failure state is identified. The result of both sub-phases is the total duration of the failure state.

As mentioned, knowing only the outage probability of each component is not enough in this chronological phase; it is also necessary to know the average repair time of each component, also known as the mean time to repair (MTTR), so as to estimate the failure and repair rates of each component, λ and μ, respectively.

If a component is operating, the time it will take for it to fail, given the exponential distribution, can be sampled from:

$$T_f = -\frac{1}{\lambda}\log(u) \quad (25)$$

Where u s a random number sampled from the uniform distribution $u \sim U(0,1)$

Similarly, if a component is under a failure, the time it will take to repair it, given the exponential distribution, can be sampled from:

$$T_R = -\frac{1}{\mu}\log(u) \qquad (26)$$

Thus, in the proposed approach, given the states $x(i)$ of each of the i components in the "snaptshot" obtained for the system failure state, we carry out for each component:

1. Verify whether $x(i)$ is an "operating" or "failed" state of the component;
2. If $x(i)$ is operating:
   a. Sample the time to failure, $T_f(i)$;
   b. Sample the time $t(i)$ associated to the system failure "snapshot" from the uniform distribution $t(i) \sim U\left(0, T_f(i)\right)$;
   c. Calculate the number of steps (hours) required for the component to transition from the operating state to the failed state in the backward phase as $t(i)$; and the number of steps (hours) required for the component to transition from the failed state to the operating state in the forward phase as $T_f(i)$- $t(i)$.
3. If $x(i)$ is failed:
   a. Sample the time to repair, $T_R(i)$;
   b. Sample the time $t(i)$ associated to the system failure "snapshot" from the uniform distribution $t(i) \sim U\left(0, T_R(i)\right)$;
   c. Calculate the number of steps (hours) required for the component to transition from the operating state to the failed state in the backward phase as $t(i)$; and the number of steps (hours) required for the component to transition from the failed state to the operating state in the forward phase as: $T_R(i)$- $t(i)$.

From this initial definition of the transitions "around" (before/after) the failure, the backward phase of the simulation can be started:

1. Initialize step $t = 1$ as the first hour before the failure state starts;
2. Verify for each component i of the system if it reached the moment of state transition, that is, if $t = t(i)$;
   a. If the component must transition at this moment, its status $x(i)$ is updated and the number of hours required for the next transition is sampled, using either $T_{f(i)}$ or $T_{R(i)}$ depending on the new state of the component.
3. Define the hour associated to the current backward step, $h_1 = h - t$, and obtain the block b_2 associated to the hour h_1;
4. Update the variables of the system components that depend on the hour, or block, as:
   a. Hydro plants:

$$P_{Hi} = P_{Hi}(t, s, b_2) \qquad (27)$$

   b. Renewable plants:

$$fc_i = fc_i(t, s, h_1) \qquad (28)$$

   c. Loads:

$$d_i = d_i(t, h_1) \quad (29)$$

d. Batteries:

$$I_{bi} = I_{bi}(t, \mathrm{s}, h_1) \quad (30)$$

5. Evaluate system supply adequacy for the new operation point:
   a. If the system is still in the failure state, increment the backward step, $t = t + 1$, and return to step 2;
   b. If the system transitions to a "success state", register the total time required to obtain the success state, $T_{back} = t$ and proceed to the forward phase of the do algorithm.

The forward phase is similar to the backward phase; however, the transitions are performed in the direct time of the simulation:

1. Initialize step $t = 1$ as the first hour after the failure state starts;
2. Verify for each component i of the system if it reached the moment of state transition, that is, if $t = T_f(i)$- $t(i)$;
   a. If the component must transition at this moment, its status $x(i)$ is updated and the number of hours required for the next transition is sampled, using either $T_{f(i)}$ or $T_{R(i)}$ depending on the new state of the component.
3. Define the hour associated to the current backward step, $h_2 = h + t$, and obtain the block b_2 associated to the hour h_2;
4. Update the variables of the system components that depend on the hour, or block, as:
   a. Hydro plants:

$$P_{Hi} = P_{Hi}(t, s, b_2) \quad (31)$$

   b. Renewable plants:

$$fc_i = fc_i(t, s, h_2) \quad (32)$$

   c. Loads:

$$d_i = d_i(t, h_2) \quad (33)$$

   d. Batteries:

$$I_{bi} = I_{bi}(t, \mathrm{s}, h_2) \quad (34)$$

5. Evaluate system supply adequacy for the new operation point:
   a. If the system is still in the failure state, increment the forward step, $t = t + 1$, and return to step 2;
   b. If the system transitions to a "success state", register the total time required to obtain the "success state", $T_{forw} = t$ and the chronological phase of the algorithm is finalized.

At the end of the process, the estimated failure duration is given by $D = T_{forw} + T_{back}$.

3.2.3 Flowchart of the Monte Carlo Pseudo-Sequential solution method (chronological phase)

The flowchart presented below summarizes the chronological phase of the Monte Carlo Pseudo-Sequential solution method.

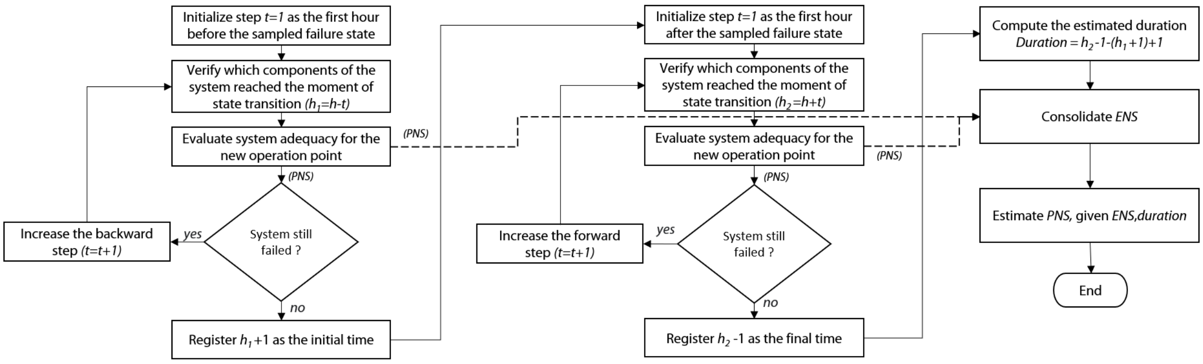

Figure 3.3 – Monte Carlo Pseudo-Sequential (chronological phase)

3.3 Monte Carlo Pseudo-Interval

As could be seen in the previous sections of this document, the Monte Carlo Pseudo-Sequential scheme identifies system failure states in a non-sequential way, but evaluates each identified state chronologically, and can effectively be applied, for example, to estimate frequency & duration reliability indices such as LOLF and LOLD. However, if there are batteries or other small storage devices in the system, their stored energy could avoid or reduce the unserved energy during failures.

In this case, the representation of batteries required a methodological extension to our original Pseudo-Sequential MCS. The reason is that batteries have a fast dynamic response that allows them to be redispatched, but being limited by the storage capacity (in addition to the installed capacity). Therefore, it becomes necessary to optimize the use of their stored energy along the duration of the system component failures. The extended methodology, known as Pseudo-Interval MSC, optimizes the storage operation along the entire duration of the failure state ("perfect forecast"). This allows a more realistic representation of the actions that operators could take in terms of battery redispatching in those moments.

As previously described, the conventional pseudo-sequential simulation, in which the adequacy of the system states is simulated step by step, does not allow representing the real behavior of the system operation when there are batteries or other small storage devices, because in real life, relying on these devices, the operator can use its knowledge of expected system behavior to more intelligently dispatch batteries throughout the duration of a component failure.

In order to have this more realistic modeling, the pseudo-interval methodology adds an extra phase to the pseudo-sequential simulation that solves an hourly optimal dispatch problem for all the failure state duration, that is, for the period between the failure start, previously identified by the backward simulation, and the failure end, identified by the forward simulation. In this phase, the battery is no longer represented as a fixed injection defined by the SDDP hourly simulation, and has its physical characteristics, such as the energy storage balance, represented in more detail.

1. Repeat from $t = 1$ to $t = D,$ corresponding to the first and last stages of the failure state, identified respectively in the backward and forward phases:
    a. Restore the states sampled for the components, during the backward and forward simulations, at time t. That is, if t corresponds to an instant analyzed during the backward phase, the states of the components are restored to that instant analyzed during the backward simulation; if t corresponds to an instant analyzed during the forward phase, the states of the components are restored to that instant analyzed during the forward simulation.
    b. Determine the hour h_2 associated to instant t, and obtain the corresponding block b_2;
    c. The variables of the system components that depend on the hour, or on the block, are updated in a similar way to what we have already presented for the backward and forward simulations;
    d. Add to the problem the variables and constraints that define the system adequacy for instant t;
        i. The main differences in the constraints and variables added to the system adequacy problem at this stage, in relation to the previous analyzes, refer to the battery modeling. At this stage, the batteries are no longer modeled as fixed injections, but through the variables and constraints that represent their operational behavior in more detail:
            1. Battery power production:

$$g(i,t) = g(i,t)^{+} - g(i,t)^{-} \qquad (35)$$

Where $g(i,t)^{+}$indicates battery generation (discharge) and $g(i,t)^{-}$ indicates battery consumption (charge);

2. Maximum capacity:

$$g(i,t) \leq C(i) \qquad (36)$$

Where $C(i)$ is the nominal capacity of battery i;

3. Power balance:

$$e(i,t+1) = e(i,t) - \frac{g(i,t)^{+}}{a(i)} + b(i)g(i,t)^{-} \qquad (37)$$

Where $e(i,t)$ is the energy stored in battery i at instant t, $a(i)$ is the discharge efficiency and $b(i)$ is the charge efficiency;

4. Stored energy capacity:

$$e(i,t) \leq E(i) \qquad (38)$$

Where $E(i)$ is the maximum energy storage of battery i;

5. Up and down ramping constraints:

$$e(i,t+1) - e(i,t) \ \leq R_c(i) \qquad (39)$$

$$e(i,t) - e(i,t+1) \leq R_d(i) \quad (40)$$

Where $R_c(i)$ is the charge ramp and $R_d(i)$ is the discharge ramp of battery i.

The coupled adequacy problem for the entire failure horizon is then solved, and afterwards we verify if the failure state along the entire duration was eliminated by the battery redispatch. If yes, the reliability statistics are updated as a "success state". Otherwise, we update the load curtailment statistics and other supply reliability indices related to failure states.

3.3.1 Flowchart of the Monte Carlo Pseudo-Interval solution method (operation phase)

The flowchart presented below summarizes the operation phase of the Monte Carlo Pseudo-Interval solution method.

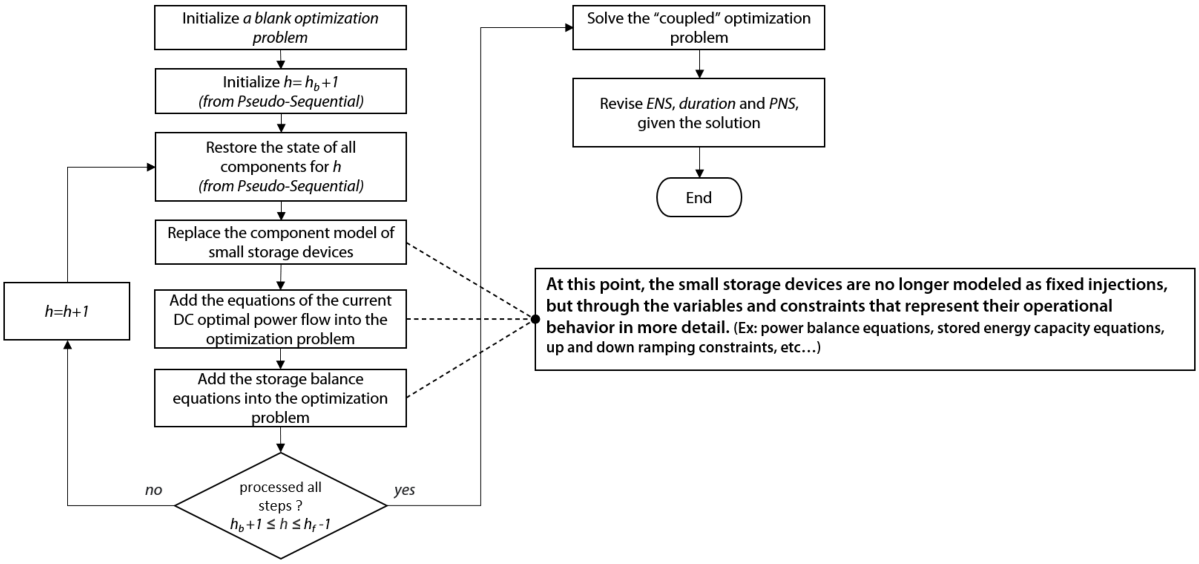

Figure 3.4 – Monte Carlo Pseudo-Interval (operation phase)